Duration Dependence in a Model with Adverse Selection and Information Acquisition

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[1]The views expressed here are my own and do not necessarily reflect those of KIPF.

Motivation

Unemployment duration$\uparrow \rightarrow$ job-finding rate and wage$\downarrow$

- Negative duration dependence

Two popular explanations

- Workers lose their ability while unemployed
- Low ability workers remain unemployed

Mechanism behind the selection?

- Firms screen workers when hiring

The paper revisits this selection theory by focusing on

- Privately-informed workers choose which job to search
- Firms endogenously choose screening intensity

Motivation

Lower hiring probability does not imply longer duration

- Workers can target different firms
- Low-ability to McDonald's vs high-ability to Google

This self-selection can justify firm's screening intensity

- No incentive to screen when wages are low

Previous literature assumed symmetric learning and/or random search

- No self-selection by privately-informed workers

Question: Does the selection theory still explain negative duration dependence? If so, under which conditions?

What this paper does

Propose a labor market model with the following features

- Privately-informed workers choose which job to search (Directed search → Adverse selection)
- Firms endogenously choose informativeness of screening (Information acquisition following Rational inattention)

Characterize an equilibrium, which consists of

- Optimal information acquisition, workers' search, rational belief, and their interaction

Derive the implications of unemployment duration on labor market outcomes

Equilibrium characterization

When $y_H \approx y_L$, the equilibrium is full-separation

- No screening in the equilibrium
- Higher wage/lower job-finding prob for H-type

When $y_H >> y_L$, then the equilibrium exhibits upward-pooling

- High wage/screening job for both types
  + Low wage/no-screening job for L-type

The equilibrium is not constrained efficient if productivity differential > home production differential

- The social planner fully separates types and allocates more firms to H-type without screening
- Higher wage/higher job-finding prob for H-type $\rightarrow$ L-type enters this job

Unemployment duration and wage

Positive duration dependence in a separating eq. ($y_H \approx y_L$)

- H-type searches for a better job and stays at a market longer due to the higher outside option
- Similar to financial market wisdom (DeMarzo and Duffie, 1999; Guerrieri and Shimer, 2014)

In an upward-pooling eq. ($y_H >> y_L$), the sign and magnitude of duration dependence rely on two factors

- Hiring prob of H-type in screening job vs L-type in no screening job ($\pi_H p_H$ vs p_L)
- The fraction of H-type at 0-duration $f(0)$

Relationship between duration and wage

- **Negative** if $\pi_H p_H > p_L$ or $f(0)$ is relatively high
- **Positive** if $\pi_H p_H < p_L$ and $f(0)$ is low

Implications

Negative duration dependence is not always immediate

- Productivity gap needs to exceed a threshold

Duration may affect wage and job-finding rate differently, e.g)

- Higher wage/lower job-finding rate for H-type
- Efficient allocation: higher wage/higher job-finding for H-type (Negative duration dependence)

How should duration and prices be related?

- Unemployment scar vs Sellers hold better assets
- Productivity gap and initial distribution matter

Previous literature

Learning and unemployment duration

- Vishwanath (1989), Lockwood (1991), Gonzalez and Shi (2010), Kroft et al. (2013), Doppelt (2016), Feng et al. (2019), Fewcett and Shi (2021)

(Competitive) Search and adverse selection

- Guerrieri et al. (2010), Guerrieri and Shimer (2014), Chang (2017), Kaya and Kim (2018)

Information acquisition

- Sims (2003), Matejka and McKay (2015), Kim et al. (2021)

Environment

- Continuous time, risk-neutral unemployed workers and firms
- All agents discount future at a rate of $r > 0$
- Privately known types of workers (H and L) with productivity $0 < y_L < y_H$
- The flow value of unemployed is 0 to L-type, b to H-type, where $0 < b < y_H - y_L$
- $f(t)$: the fraction of H-type among unemployment duration t ($f(0)$ is given)
- No separation nor on-the-job search
- All flow variables represent their annuity values
  - e.g) j-type produces a flow payoff ry_j

Search and matching

- Firms post a vacancy specifying w and t
  $\rightarrow$ submarkets $(w, t) \in [0, y_H] \times [0, \infty)$
  - Equivalently, firms post a wage profile contingent on t
- Workers with t choose which wage w to search for
- Market tightness $\theta(w, t)$ is specific to each (w, t)
- "Meeting" rate for workers $p(\theta) = \theta/(1 + \theta)$,
  for vacancies $q(\theta)$ with $q(\theta) = p(\theta)/\theta = 1/(1 + \theta)$
- Wages are non-negotiable after information is revealed

Information acquisition

- After meeting, the firm can conduct interview
- Interview is a signal distribution $G(s,j) \in \Delta S \times \{H, L\}$ that is Bayes-consistent with prior μ
- Examples of the interview
  - Pass the test if and only if H-type: $S = \{0, 1\}$, $G(s = 0|j = L) = G(s = 1|j = H) = 1$
  - Uninformative: $S = \{0\}$
- Firms can freely choose G_j and S with costs proportional to informativeness of the interview

$$C(G, \mu) = \lambda[H(\mu) - E_G(H(\mu_s))]$$

$H(x) \equiv -x\log(x) - (1-x)\log(1-x)$
μ is the prior belief about the worker being of H-type
μ_s is the posterior belief given a signal realization s

Information acquisition

$$C(G,\mu) = \lambda[H(\mu) - E_G(H(\mu_s))]$$

- $H(x)$ is the entropy of binomial distribution with x, which measures "uncertainty" of a distribution
  - Maximum at $x = 1/2$, minimum at $x = 0, 1$
- $H(\mu)$: Initial uncertainty
  $H(\mu_s)$: Uncertainty after a signal s realization
  $E_G(H(\mu_s))$: (Expected) uncertainty induced by interview
- Information cost is proportional to the (expected) reduced entropy

Interview and posterior

Given μ, one can always find an interview that makes $\mu_s = \underline{\mu}$ or $\mu_s = \overline{\mu}$ for any $\underline{\mu} < \mu < \overline{\mu}$ by setting

$$\underline{\mu} = \frac{\mu G(0|H)}{(1-\mu)G(0|L) + \mu G(0|H)}, \quad \overline{\mu} = \frac{\mu G(1|H)}{(1-\mu)G(1|L) + \mu G(1|H)}$$

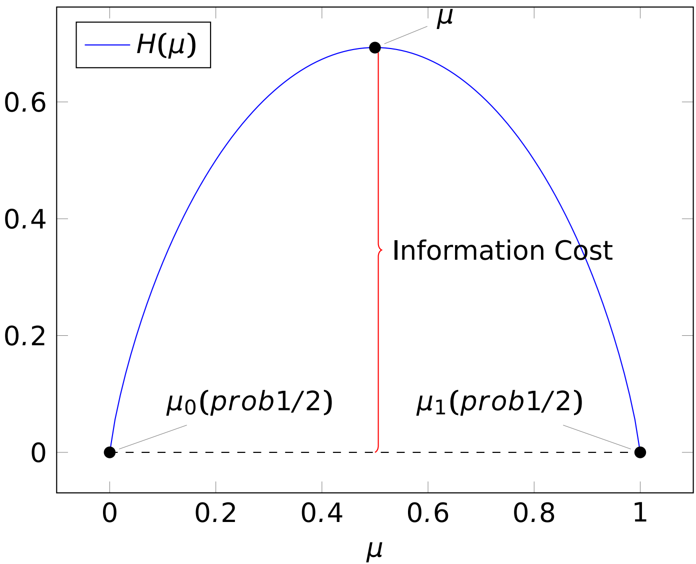
subject to $G(0|L) + G(1|L) = 1$ and $G(0|H) + G(1|H) = 1$.

The solution satisfies

$$\frac{G(1|H)}{G(1|L)} = \frac{\overline{\mu}}{1-\overline{\mu}} \frac{1-\mu}{\mu} > 1 \quad \frac{G(0|H)}{G(0|L)} = \frac{\underline{\mu}}{1-\underline{\mu}} \frac{1-\mu}{\mu} < 1$$

- Choosing interview = Choosing posterior

Information cost



Firm's value functions

The firm's value function after interview

$$\hat{J}(w, \mu_s) = \max\{0, [\mu_s y_H + (1-\mu_s) y_L] - w\}$$

Thereby, the firm hires the worker if and only if

$$\mu_s \geq \mu^0(w) \equiv \frac{w - y_L}{y_H - y_L}$$

The firm's value function before conducting interview

$$J(w, \mu) = \max_G E_G \left[\hat{J}(w, \hat{\mu}_s) - C(G, \mu)\right]$$

where $G \in \Delta S \times \{H, L\}$ and $E_G(\mu_s) = \mu$

Hiring probability

- The optimal information strategy determines the hiring probability for each type $\pi_j(w, \mu)$ for $j = L, H$

$$\pi_j(w, \mu) = P_{G^*(w,\mu)}(\mu_s \geq \mu^0(w) | \text{type} = j)$$

- The unconditional hiring probability for the firm is $\mu\pi_H(w, \mu) + (1 - \mu)\pi_L(w, \mu)$
- Everyone shares the rational belief $\mu(w, t)$ for all (w, t)
  - Given the fact that a worker with t has applied to w, what is the probability of being H-type?

Free entry and workers' value function

Free entry condition determines $\theta(w,t)$ given $\mu(w,t)$

$$r^{-1} \cdot q(\theta(w,t))J(w,\mu(w,t)) \leq k, \quad \theta(w,t) \geq 0$$

Given $\theta(w,t)$ and $\mu(w,t)$, H-type workers' value function is

$$U_H(t) = b + r^{-1} \cdot \left(\max_{(w,t)} \left\{ p(\theta(\cdot))\pi_H(w,\mu(\cdot))[w - U_H(t)] \right\} + \frac{\partial}{\partial t} U_H(t) \right)$$

- Belief affects how many vacancies are created
- Job-finding rate = Meeting rate × Hiring prob
- $\mu(\cdot)$ matters for workers' search
- The max term is the return to search $R_j(t)$

Equilibrium

An equilibrium consists of

- Firms' optimal information strategy $G(w,t)$ given $\mu(w,t)$, and firm value function
- Workers' optimal search decisions and value function given $\pi_j(w,t)$, which is determined by $\mu(w,t)$
- Free entry of firms $\theta(w,t)$ given $\mu(w,t)$
- Labor market clearing: for some $l(w,t) \geq 0$, $\sum l(w,t)\theta(w,t)\mu(w,t) = f(t)$ and $\sum l(w,t)\theta(w,t)(1-\mu(w,t)) = 1-f(t)$
- Belief restriction on $\mu(w,t)$

Belief restriction

$\mu(w, t)$ must coincide with the actual fraction if workers apply to (w, t)

- Requirement for Perfect Bayesian Equilibrium

Arbitrary belief can support many equilibria, for example

- All workers apply to $w = x$ because firms do not post any vacancy except it as $\mu = 0$ for all $w \neq x$

Necessary to determine the off-the-path belief $\mu(w, t)$

Belief restriction

Applying the divinity argument (Banks and Sobel, 1987; Gale, 1996; Guerrieri et al., 2010)

- Given a deviation, assigning the weight to the type that is 'more likely to' choose that deviation

$\mu(w,t) > 0$ if H-type chooses (w,t) at lower θ given $\mu(w,t)$

$$\exists\theta > 0, \qquad \begin{aligned} &p(\theta)\pi_H(w,\mu(w,t))[w - U_H(t)] \geq R_H(t) \\ &p(\theta)\pi_L(w,\mu(w,t))[w - U_L(t)] \leq R_L(t) \end{aligned}$$

of if H-type chooses (w,t) at lower θ for any μ

$$\exists\theta > 0, \qquad \begin{aligned} &p(\theta)[w - U_H(t)] \geq R_H(t) \\ &p(\theta)\sup_{\mu'}\left\{\frac{\pi_L(w,\mu')}{\pi_H(w,\mu')}\right\}[w - U_L(t)] \leq R_L(t) \end{aligned}$$

(Constrained) Efficient allocation

When type is observable, given $x \equiv (E_H, E_L, H, L)$,

$$
\begin{aligned}
rW(x,t) &= \max_{\theta_H,\theta_L} \Big\{ r(y_H E_H + y_L E_L + bH) - r(\theta_H H + \theta_L L)k \\
&\quad + (W_1 - W_3)\dot{H} + (W_2 - W_4)\dot{L} + \frac{\partial}{\partial t} W(x,t) \Big\} \\
\dot{H} &= -p'(\theta_H) H \\
\dot{L} &= -p'(\theta_L) L
\end{aligned}
$$

By guessing $W(x,t) = y_H E_H + y_L E_L + U_H^* H + U_L^* L$,

$$
p'(\theta_H^*) = \frac{rk}{y_H - U_H^*}, \quad p'(\theta_L^*) = \frac{rk}{y_L - U_L^*}
$$

$$
U_H^* = \frac{rb + p(\theta_H^*) y_H}{r + p(\theta_H^*)}, \quad U_L^* = \frac{p(\theta_L^*) y_L}{r + p(\theta_L^*)}
$$

Separating equilibrium

To prevent mimicking, (no screening when $\mu \in \{0, 1\}$)

$$\begin{cases} p(\theta_H)(w_H - U_H) \geq p(\theta_L)(w_L - U_H) \\ p(\theta_L)(w_L - U_L) \geq p(\theta_H)(w_H - U_L) \end{cases} \Rightarrow p(\theta_H) < p(\theta_L), w_H > w_L$$

- In any separating equilibrium, H-type workers remain unemployed longer → positive duration dependence
- Financial market: better assets staying the market longer (DeMarzo and Duffie, 1999; Guerrieri and Shimer, 2014)

When unconstrained (= efficient allocation),

- Higher outside option for H-type → $w_H > w_L$
- Job-finding probability may be higher because $y_H > y_L$

Separating equilibrium

The social planner chooses $\theta_H < \theta_L$ iff $y_H - b < y_L$

- Allocate more firms when net productivity is higher

An equilibrium is not efficient provided that $y_H - y_L > b$

- IC condition for L-type must bind

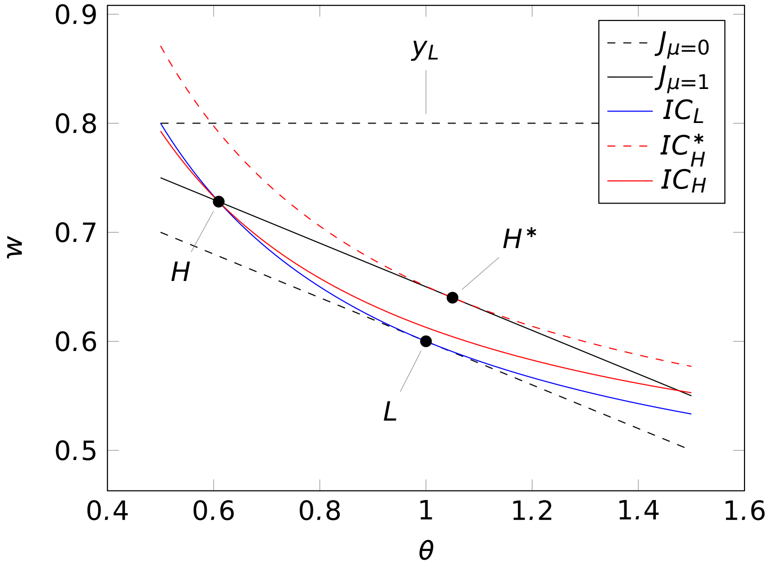
$$p(\theta_H)(w_H - U_L) = p(\theta_L)(w_L - U_L), \quad w_H > w_L$$

- Necessary condition: $w_H < y_L$, which requires

$$p(\theta(y_L, 1))(y_L - U_L) < p(\theta_L)(w_L - U_L)$$

  $\theta(y_L, 1)$: market tightness when $w = y_L$ and $\mu = 1$
- Necessary condition is satisfied if $y_H \approx y_L$
- No screening occurs in the equilibrium

Separating equilibrium



Optimal information acquisition

What if the constrained $w_H > y_L$?

- Need to consider the optimal screening

The firm's problem

$$\begin{aligned} J(w,\mu) &= \max_G E_G\left[\hat{J}(w,\mu_s)+\lambda H(\mu_s)\right]-\lambda H(\mu) \\ & \quad \text{s.t} \quad E_G(\mu_s)=\mu \end{aligned}$$

- Concavification of $\hat{J}(w,\cdot)+\lambda H(\cdot)$ (Kamenica and Gentzkow, 2011)

Optimal information acquisition

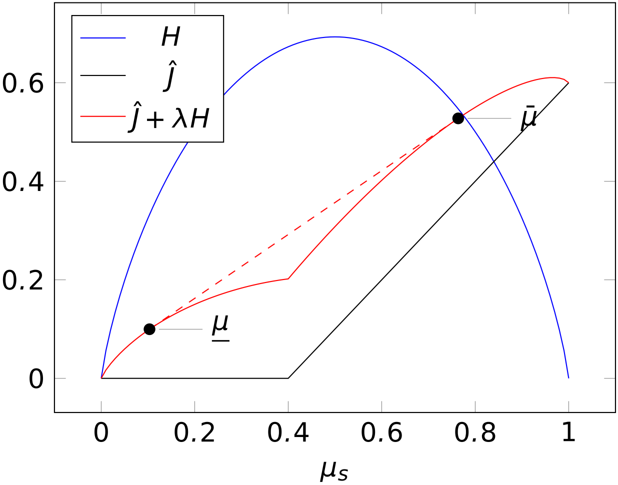


Figure: Entropy and firm value given w

Characterization of the optimal information acquisition

If $w \in (y_L, y_H)$, then

- $\mu \leq \underline{\mu}(w)$: No screening/No hiring
- $\mu \in (\underline{\mu}(w), \overline{\mu}(w))$: Screening with $\mu_s \in \{\underline{\mu}, \overline{\mu}\}$ / Hiring only when $\mu_s = \overline{\mu}$
- $\mu \geq \overline{\mu}(w)$: No screening/Hiring

where $\underline{\mu}(w)$ and $\overline{\mu}(w)$ are

$$\underline{\mu}(w) = \frac{\exp\left(\frac{w-y_L}{\lambda}\right) - 1}{\exp\left(\frac{y_H-y_L}{\lambda}\right) - 1}, \quad \overline{\mu}(w) = \frac{\exp\left(\frac{y_H-y_L}{\lambda}\right) - \exp\left(\frac{y_H-w}{\lambda}\right)}{\exp\left(\frac{y_H-y_L}{\lambda}\right) - 1}$$

Hiring strategy for (w, μ)

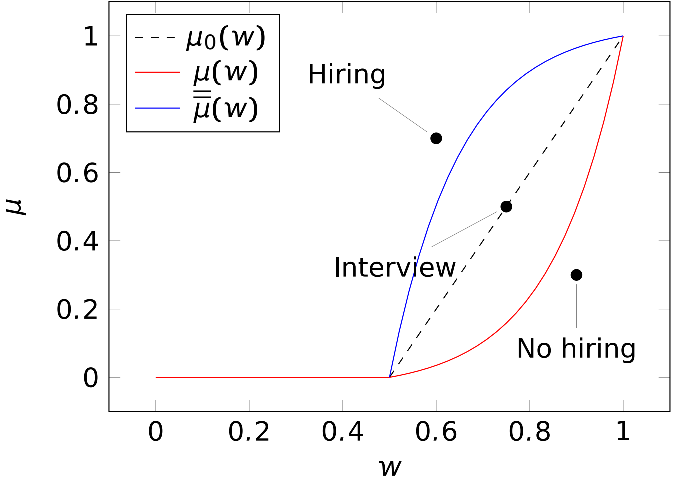
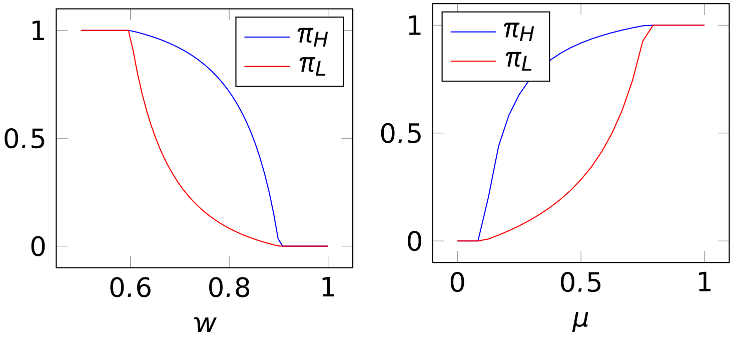


Figure: $\underline{\mu}$ and $\overline{\mu}$

Properties of the optimal information strategy



- $\pi_H > \pi_L$, both are decreasing in w and increasing in μ
- π_H / π_L is increasing in w and decreasing in μ
  - H-type is relatively more hired when screening is tougher

Properties of the equilibrium belief

"Required job-finding rate" of j-type for w

$$\phi_j(w) = \frac{R_j}{w - U_j} \tag{1}$$

The equilibrium belief is determined by the ratio

$$\begin{aligned}
\frac{\phi_H(w)}{\phi_L(w)} &\geq \sup\{\pi_H/\pi_L\} = \exp\left(\frac{y_H - y_L}{\lambda}\right) \Rightarrow \mu(w) = 0 \\
\frac{\phi_H(w)}{\phi_L(w)} &\leq \inf\{\pi_H/\pi_L\} = 1 \Rightarrow \mu(w) = 1 \\
\frac{\phi_H(w)}{\phi_L(w)} &\in \left(1, \exp\left(\frac{y_H - y_L}{\lambda}\right)\right) \Rightarrow \frac{\pi_H(w, \mu(w))}{\pi_L(w, \mu(w))} = \frac{\phi_H(w)}{\phi_L(w)}
\end{aligned}$$

Properties of the equilibrium belief

The required job-finding rate ratio is decreasing in w

- H-type prefers higher wages due to $U_H > U_L$

The equilibrium belief is increasing (because π_H/π_L is decreasing in μ)

$$\mu(w) = 0 \quad \text{if} \quad w \leq w_0$$
$$\mu(w) \uparrow 1 \quad \text{as} \quad w \to y_H$$

When $\mu(w) \in (0,1)$, actual job-finding rate ratio = required job-finding rate ratio

- If one type finds it optimal, the other type does so

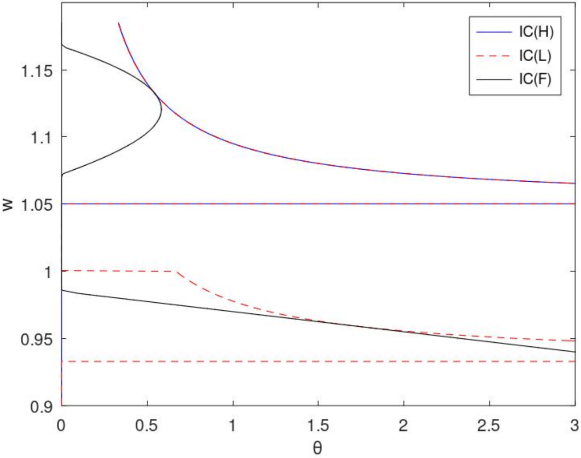
Upward-pooling equilibrium

When $f(0)$ is not too high, there exists an equilibrium such that

- Time-invariant payoffs and belief: $U_H(t) = U_H, U_L(t) = U_L, \mu(w,t) = \mu(w)$
- Two wage levels $w_H > w_L$ appear in the equilibrium
  - $\mu(w_H) \equiv \mu^* \in (0,1)$ and $\pi_H(w_H, \mu^*) > \pi_L(w_H, \mu^*)$
  - $\mu(w_L) = 0$ and no-screening

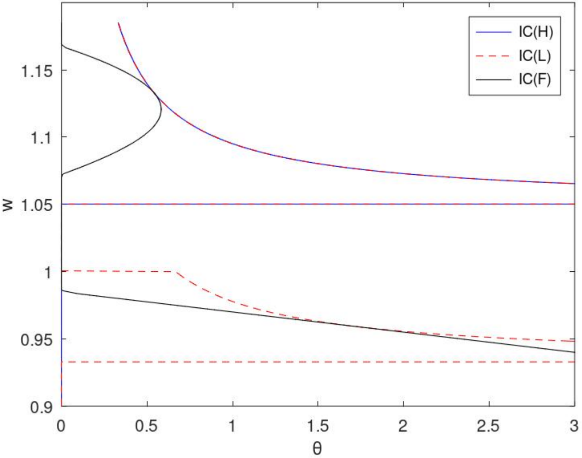
In this type of equilibrium, all H-type workers apply to w_H, while L-type workers apply to both w_H and w_L

Upward-pooling equilibrium



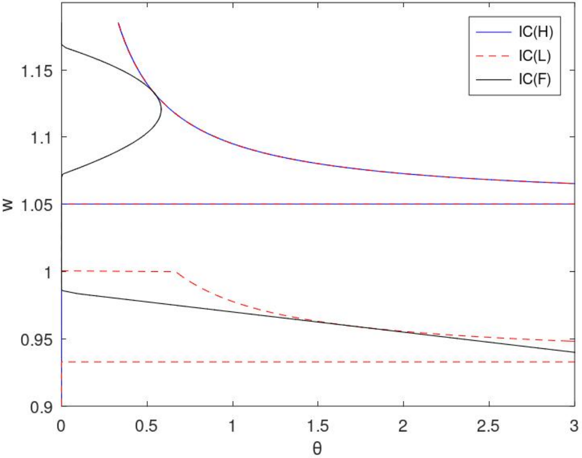
- Two distinct free entry condition (low- and high-wage regions)

Upward-pooling equilibrium



- $IC(H) = IC(L)$ in the high-wage region

Upward-pooling equilibrium



- This equilibrium exists only if $f(t) \leq \mu^*$ for all t

Duration dependence

The average wage conditional upon t

$$\begin{aligned}\bar{w}(t) &= f(t)w_H + \left[\frac{1-\mu^*}{\mu^*}f(t)w_H + \left(1 - \frac{f(t)}{\mu^*}\right)w_L\right] \\ &= \frac{f(t)}{\mu^*}w_H + \left(1 - \frac{f(t)}{\mu^*}\right)w_L\end{aligned}$$

Negative duration dependence exists if and only if

$$\bar{w}'(t) < 0 \iff f'(t) < 0 \iff \left|\frac{H'(t)}{H(t)}\right| > \left|\frac{L'(t)}{L(t)}\right|$$

$H(t), L(t)$: number of H and L type unemployed workers

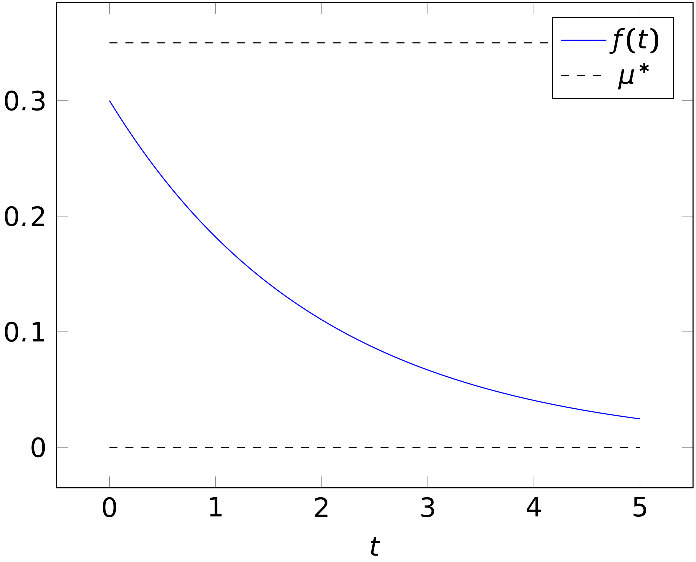
Duration dependence

The rate at which each type leaves the unemployment pool
($p_j \equiv p(\theta(w_j, \mu^*))$)

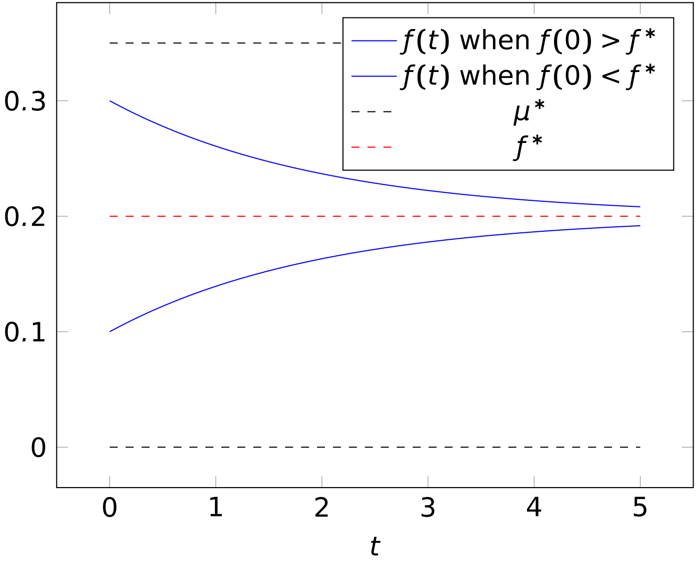
$$
\begin{aligned}
\left|\frac{H'(t)}{H(t)}\right| &= \pi_H p_H \\
\left|\frac{L'(t)}{L(t)}\right| &= \pi_L p_H \frac{(1-\mu^*)f(t)}{\mu^*(1-f(t))} + p_L \frac{\mu^* - f(t)}{\mu^*(1-f(t))}
\end{aligned}
$$

- $\pi_H p_H > \pi_L p_H$, but $\pi_H p_H$ and p_L depend on parameters
- When $f(t) \approx \mu^*$, then $|H'/H| > |L'/L|$ holds
- $\left|L'/L\right|$ is monotone in $f(t)$
  - The upward-pooling equilibrium exists if $f(0) < \mu^*$
  - If $\pi_H p_H < p_L$, there exists $f^* < (0, \mu^*)$ such that $|H'/H| = |L'/L|$

Case1: $\pi_H p_H > p_L$



Case2: $\pi_H p_H < p_L$



What if $f(0) > \mu^*$?

Not enough L-type workers to sustain the upward-pooling

U_L in the upward-pooling equilibrium only depends on y_L

- Because they are indifferent between w_H and w_L

When $f(0)$ is high enough, L-type must receive a higher payoff from the pooling wage $w^P(t)$ than w_L

- Everyone applies to the pooling wage $\rightarrow \mu(w^P(t)) = f(t)$

H-type is more likely to be hired $\rightarrow f(t)$ falls over t

- $(U_H(t), U_L(t)) \downarrow$, $f(t) \downarrow$
- Once $f(t) \leq \mu^*$, the equilibrium becomes the upward-pooling (time-invariant)
- Negative duration dependence

Conclusion

This paper revisits the duration dependence in a model with

- Adverse selection & Endogenous information acquisition

I characterize the type of equilibrium

- Separating/Upward-pooling/Pooling and upward-pooling

Duration dependence can be positive or negative

- Positive in separating and upward-pooling with $f(0) \approx 0$
- Negative in upward-pooling with $f(0) >> 0$