

Optimal Dynamic Hotel Pricing

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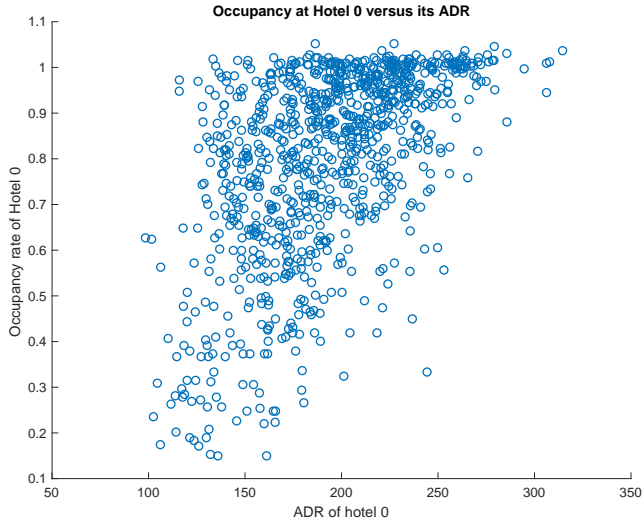
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What we do in this paper

- A new data set on hotel reservations at a luxury hotel market (7 luxury hotels)
- Traditional demand estimation OLS or IV fail to produce reasonable estimates of demand
- Formulate a **dynamic programming model** of optimal dynamic hotel pricing
- Use the **Method of Simulated Moments (MSM)**
- Recover plausible estimates of the demand for hotels despite the lack of valid instruments
- **Optimal price (by model) → Actual price (by hotel)**

Occupancy rates vs ADR at hotel 0



Hotel pricing problem

- A complex, dynamic optimization problem.
- Further, these decisions must be updated very *frequently*.
- “it has become clear that there is **growing interest** in pricing and revenue optimization as a topic of study both within business schools and management science /operations research departments. ” (Phillips, 2005 *Pricing and Revenue Optimization*)
- However it is **not clear** the extent to which **RMS (Revenue Management System) depend on economic tools** such as *dynamic programming* and *demand estimation*
- Can economics, using the tools of **dynamic programming and structural econometrics**, bring new insights and understanding and methodologies to the field of revenue management?

Demand estimation: key to revenue management

- Optimal pricing depends critically on accurate knowledge of customer demand
 - Recognizing the **stochastic nature of demand** and bookings of potential customers in the market
 - Understanding customers' evaluation of the relative desirability of the competing hotels and their degree of **price sensitivity**
- Endogeneity on demand estimation
 - Regressions of hotel occupancy (Q) on hotel prices (P) → spurious **positively sloped demand functions**
 - Few relevant instrumental variables (or Instrument-free demand estimation by MacKay and Miller 2018)
 - Demand is given by a conditional probability distribution which is generally **nonlinear in prices**

Literature of Dynamic pricing

- Theoretical model by Ivanov (2014), Anderson and Xie (2012), Zhang and Lu (2013), and Zhang and Weatherford (2016)
- Optimal selling strategies using mechanism design when buyers are forward looking (**Board and Skrzypacz 2016**)
- Secondary market in MLB ticketing (**Sweeting 2012**) - dynamic auction
- Dynamic structural estimation approach in airline market (**Williams 2018**) - monopoly route
- Dynamic structural models with continuous decisions and endogenous censoring by Merlo, Ortalo-Magne, and Rust (2015) and Hall and Rust (2018)

Several aspects in hotel demand estimation

- **Perishable** inventory—Flight tickets, Concert tickets, Food
- **Stochastic** demand process—Probability/Non-static
- Different types of customers—Leisure, Business, Group
- **Endogeneity** of prices and demand—high positive correlation between them
- Data **censoring** problem—Number of potential customers in the market
- Seasonal effect / Demand shocks

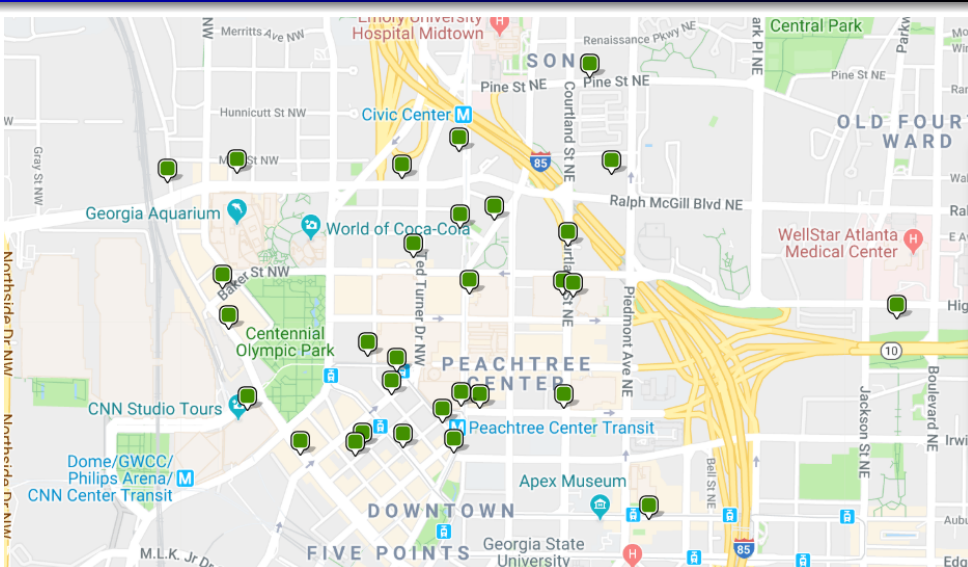
Hotel Reservation Website Example



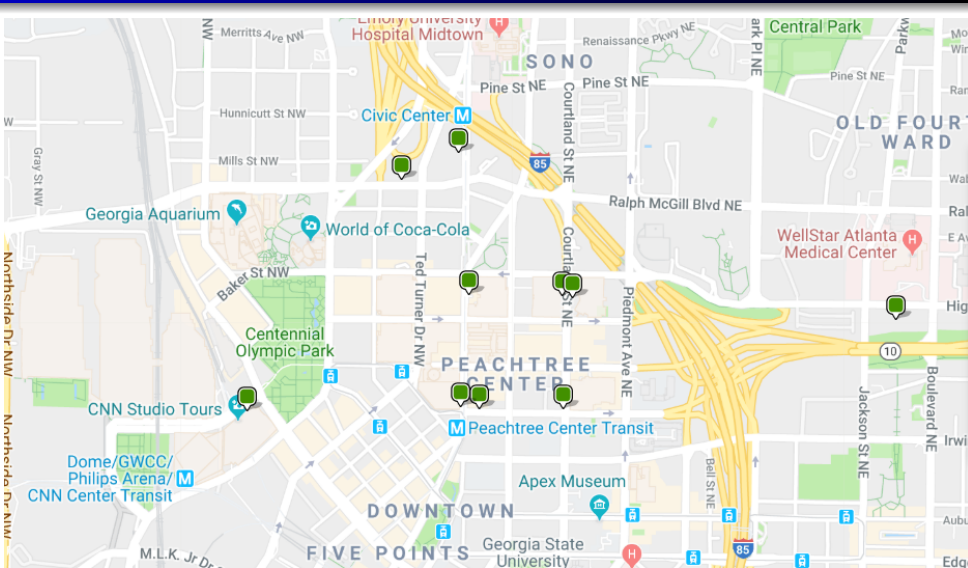
Hotels.com
The Obvious Choice™

- **Best Available Rate (BAR)**
 - **Standard price** with 24-hour advance free cancellation
 - "a rate available to the general public that does not require pre-payment and does not impose cancellation or change penalties and/or fees, other than those imposed as a result of a hotel property's normal cancellation policy." (*Wikipedia*)
- *Across Platform Parity Agreements (APPA)*

All available hotels in downtown Atlanta on March 25



Filters : Luxury Hotels in Atlanta downtown



Hotel Market Data

- We obtained a detailed *computerized reservation database* from a hotel in a major city, which we will refer to as **Hotel 0**.
- We see every reservation and cancellation at this hotel over a **37 month period**: from October 2010 to October 2013.
- In addition, the company purchased daily spot **prices** (Best Available Rate, BAR) of its **6 closest competitors** from *Market Vision* [▶ an example observation](#)
- We also augmented this data on average reservation prices and **occupancy of its competitors** from *Smith Travel Research* (STR)

Stylized Fact of Hotel Data

- 7 hotels are competing in the neighborhood and they are classified as luxury hotels.
- Hotel 0's price is **below average** in the competing hotels.
(Listing price) ▶ Hotel List
- 95% are standard rooms in hotel 0 ▶ Room type
- **Distinguishable customer type** : Business, Leisure and Group ▶ Customer type
- **Business customers** hold key portion of this market.
(Weekday vs. Weekend) ▶ Customer share ▶ graph
- **Seasonality** and co-movement in price ▶ Seasonality graph
▶ comovement
- Reservations and cancellation pattern varies by DBA (Day before arrival) ▶ reservation ▶ cancellation

Key Assumptions

- **No multi-night stay.** We treat multi-night reservation as several single-night reservations.
- Rooms are **homogeneous**.
- Hotel 0 sets *BAR* each day for each future arrival date.
- **Multiple segments of customers**, indexed by $s \in \{1, \dots, S\}$
 - Segments differ only in arrival process, price elasticity and exogenous discount rate σ_s
 - Customer of segment s wishing to book at Hotel 0 t -day ahead pays $\mathbf{p}_t \sigma_s$. (t indicates *Days Before Arrival*, **DBA**)
- Structural parameters are different across types of arrival days, but same within

Stochastic Arrival of Potential customers in the market

- A total number of segment s customers $r_{t,s}$ arrive in the market t days prior to occupancy.
- $r_{t,s}$ follows exogenous distribution: **Zero-inflated Negative Binomial** with parameters $(\gamma_{t,s}, \phi_{t,s}, \mu_{t,s})$

$$\begin{aligned}\pi(r_{t,s} = 0 | \gamma_{t,s}, \phi_{t,s}, \mu_{t,s}) &= \gamma_{t,s} + (1 - \gamma_{t,s}) \times \dots \\ &\quad \dots \binom{r_{t,s} + \phi_{t,s} - 1}{r_{t,s}} q_{t,s}^{\phi_{t,s}} (1 - q_{t,s})^{r_{t,s}} \\ \pi(r_{t,s} > 0 | \gamma_{t,s}, \phi_{t,s}, \mu_{t,s}) &= (1 - \gamma_{t,s}) \times \dots \\ &\quad \dots \binom{r_{t,s} + \phi_{t,s} - 1}{r_{t,s}} q_{t,s}^{\phi_{t,s}} (1 - q_{t,s})^{r_{t,s}}\end{aligned}\tag{1}$$

where $q_{t,s} = \phi_{t,s} / (\mu_{t,s} + \phi_{t,s})$.

Choice Probability of hotel 0

- **Customers** make static discrete choices about which hotel to book at, with **the choice probability** to reserve **at hotel 0** given by

$$P_s(p_t, \rho_t) = \frac{1}{1 + \exp\{\alpha_s + \beta_s(\sigma_s p_t - \sigma_s \rho_t)\}} \quad (2)$$

where p_t is BAR price of hotel 0 at t ,

ρ_t is **the average of competing hotels' BAR** at t ,

α_s and β_s are the choice probability parameters,

σ_s is an average discount rate for each type of consumer.

Demand for hotel 0

- **Conditional on $\mathbf{r}_{t,s}$** , the demand for hotel 0, is

$$\tilde{a}_{t,s} \sim \text{bin}(r_{t,s}, P_s(p_t, \rho_t))$$

$$a_{t,s} = r_{t,s} \cdot P(p_t, \rho_t | \alpha_s, \beta_s) = \frac{r_{t,s}}{1 + \exp[\alpha_s - \beta_s(\sigma_s p_t - \sigma_s \rho_t)]} \quad (3)$$

- The **unconditional demand** for hotel 0 is

$$\begin{aligned} f_t(a_t | p_t, \rho_t) &= \sum_{s \in S} \sum_{r \geq a} \binom{r_{t,s}}{a_{t,s}} P(p_t, \rho_t | \alpha_s, \beta_s)^a \times \dots \\ &\dots [1 - P(p_t, \rho_t | \alpha_s, \beta_s)]^{(r-a)} \pi(r_{t,s} | \phi_{t,s}, \mu_{t,s}) \end{aligned} \quad (4)$$

Random Cancellation

- **Deterministic probability** of cancellation
- Exogenous variables
- Effect by p_t and ρ_t
- The total number of cancellations t -day prior to occupancy, c_t , follows distribution $\mathbf{c}_t \sim \mathbf{e}_t(\mathbf{c}|\mathbf{n}_t)$
- The **potential cancellation dist.** $\tilde{c}_t \sim e_t(c|n_t, \bar{p}_t, p_t, \rho_t)$
 - Strategic cancellation
 - Weak evidence
 - A high computational burden

Enforcing capacity constraint

- We assume Hotel 0 do not overbook, and thus *ration demand* so that $n_t \leq \bar{n}$ with probability 1 for all $t \geq 0$. Mapping η implement this rationing. Number of new reservations (n_{1t}, \dots, n_{St}) is given by

$$(n_{1t}, \dots, n_{St}) = \eta(n_{1t}^d, \dots, n_{St}^d, c_t, n_t, \bar{n}) \quad (5)$$

- Law of motion for n_t is

$$n_{t-1} = n_t - c_t + \sum_s n_{st} \quad (6)$$

By construction $n_{t-1} \leq \bar{n}$ with probability 1

Law of motion

- ADR \bar{p}_t

$$\begin{aligned}\bar{p}_{t-1} &= \frac{(n_t - c_t)\bar{p}_t + \sum_s n_{st}\delta_s p_t}{n_{t-1}} \\ &\equiv \lambda(n_t, n_{1t}, \dots, n_{St}, \bar{p}_t, p_t)\end{aligned}\tag{7}$$

- Competitors' price ρ_t

$$\rho_{t-1} \sim h(\rho | \rho_t)\tag{8}$$

DP model

- Let $V_t(n, \bar{p}, \rho)$ be the maximal expected revenue Hotel 0 expects t days prior to occupancy, if its current occupancy is n , the average price (ADR) of these n reservations is \bar{p} and the average BAR of its competitors is ρ_t .
- On $t = -1$ (day after arrival) there are no further decisions and hotel's realized profit for that day can be calculated:

$$V_{-1}(n, \bar{p}, \rho) = \min[\bar{n}, n](\bar{p} - \omega)$$

where \bar{n} is the hotel's capacity and ω is the marginal cost of servicing a room.

Bellman equation

- At the start of each day $t = 0, 1, \dots, T$ prior to arrival, the hotel observes (n_t, \bar{p}_t, ρ_t) and sets its (BAR) p_t to maximize profit
- For $t = -1$, $V_{-1}(n, \bar{p}, \rho) = n \cdot (\bar{p} - \omega)$
- For $t = 0, 1, 2, \dots, T$

$$\begin{aligned}
 V_t(n, \bar{p}, \rho) = & \\
 \max_{\bar{p}} \int_{\rho'} \sum_{n_1^d} \dots \sum_{n_S^d} \sum_c V_{t-1}(n', \bar{p}', \rho') & \\
 \cdot e_t(c|n, \bar{p}, p, \rho) \cdot f_{1t}(n_1^d|p, \rho) \dots f_{St}(n_S^d|p, \rho) \cdot h_t(\rho'|\rho). & (9)
 \end{aligned}$$

Property of DP

- **Theorem 1** *For each $t \in \{1, \dots, T\}$ the value function V_t has the representation*

$$V_t(n, \bar{p}, \rho) = V_t^f(n, \bar{p}, \rho) + V_t^b(n, \bar{p}, \rho) \quad (10)$$

where V_t^f is the “forward looking component” that equals the expected profits from rooms that are not yet booked, whereas V_t^b is the “backward looking component” that equals expected profits from rooms that are already booked.

Property of DP

- **Assumption 1** *The conditional probability distributions for the number of new transient and group reservation requests, r_t^d and g_t^d are independent of the hotel's ADR \bar{p} .*
- **Assumption 2 (Exogenous cancellations)** *The conditional probability distributions for the number of cancellations, c_t , by existing customers does not depend on the hotel 0's BAR p or ADR \bar{p} .*
- Assumption 2 holds if the conditional probability density $e_t(c|n, p, \rho, \bar{p})$ in the Bellman equation (9) does not depend on (p, \bar{p}) . We do not have strong evidence that cancellation decisions depend on hotel 0's BAR and ADR.

Property of DP

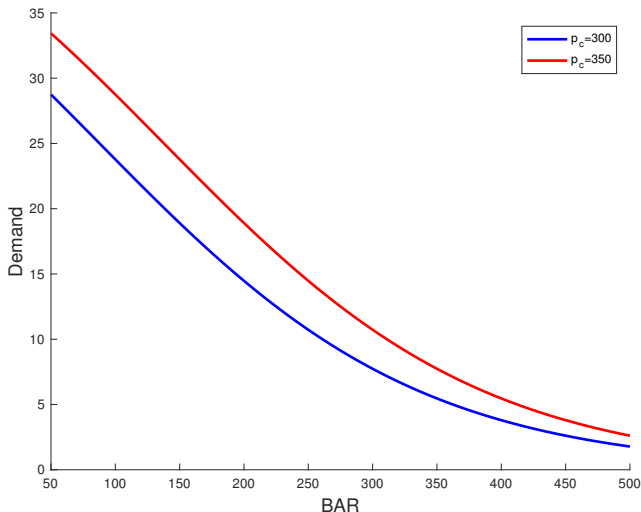
- **Theorem 2** *If Assumption 1 and 2 hold, then for each $t \in \{1, \dots, T\}$ the forward looking component of the value function V_t^f is independent of \bar{p} , i.e. it can be written as $V_t^f(n, \rho)$ and depends on (n, ρ) but not \bar{p} .*
- **Theorem 3** *If Assumptions 1 and 2 hold then for each $t \in \{1, \dots, T\}$ the optimal decision rule for BAR p_t^* is independent of \bar{p} , i.e. it can be written as $p_t^*(n, \rho)$ and depends on (n, ρ) but not \bar{p} .*

Solution to a simple example at $t = 0$

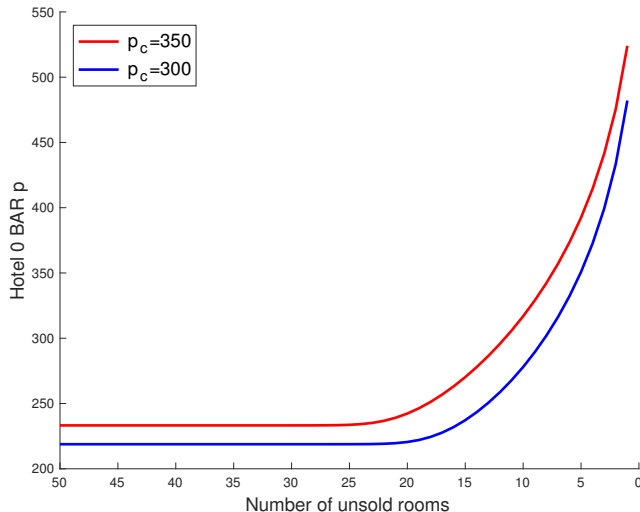
- Suppose hotel 0 knows that on day $t = 0$ that $k_0 = 50$ customers will be arriving in this market and deciding where to stay.
- Thus, $\tilde{r}_0 \sim \text{bin}(50, P_0(p, \rho))$ is the probability distribution for demand for Hotel 0.
- *Expected demand* is $D_0(p, \rho) = 50 * P_0(p, \rho)$, but the hotel must enforce *overbooking constraint* $\tilde{r}_0 \leq \bar{n} - n$ (remaining unsold rooms) with probability 1.
- Hotel's problem is

$$V_0^f(n, \rho) = \max_p E \{ \min[\tilde{r}_0(p, \rho), \bar{n} - n](p - \omega) \}$$

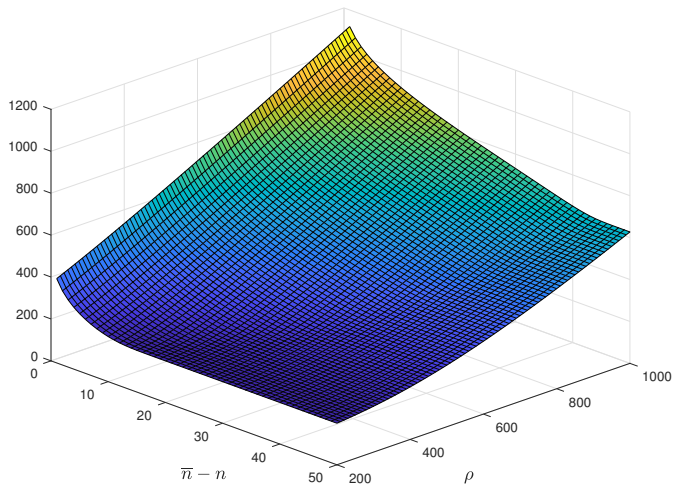
Expected demand at $t = 0$, $\rho = 300$ and $\rho = 350$



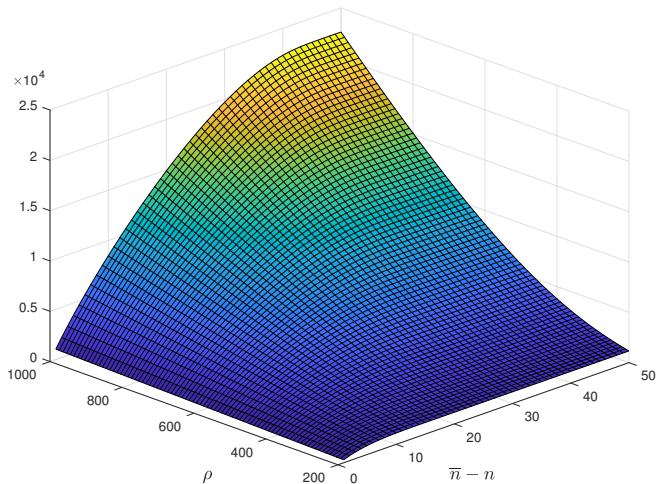
Optimal prices at $t = 0$, $\rho = 300$ and $\rho = 350$



Optimal BAR $t = 0$



Optimal profit $V_0(n, \bar{p}, \rho)$



How we estimated the model

MSM (Method of Simulated Moments)

- 1 Set up the parameters (Including Initial Guess)
 - 2 Find the optimal prices and value function by solving DP
 - 3 Generate simulation data
 - 4 Find the distance between simulation data and actual data
 - 5 Update the parameters which enable the distance shorter
- Repeat 2-5 until convergence

List of moments

Table 8: List of Moments

Hotel	Description of Moment	Number of Moments
Hotel 0	avg. occupancy rate, by t	47
	distribution of occupancy on t=0	28
	avg. Transient reservations (Leisure+Business), by t	47
	variance of Transient reservations, by t	47
	prob. of no Group reservations, by t	47
	avg. Group reservations, by t	47
	prob. of non-zero cancellations, by t	47
	avg. cancellation rate, by t	46
	avg. BAR, by t	47
	avg. ADR on t=0	1
	distribution of ADR on t=0	28
All Hotels	avg. occupancy rate on t=0	1
	distribution of occupancy rate on t=0	48
Total		481

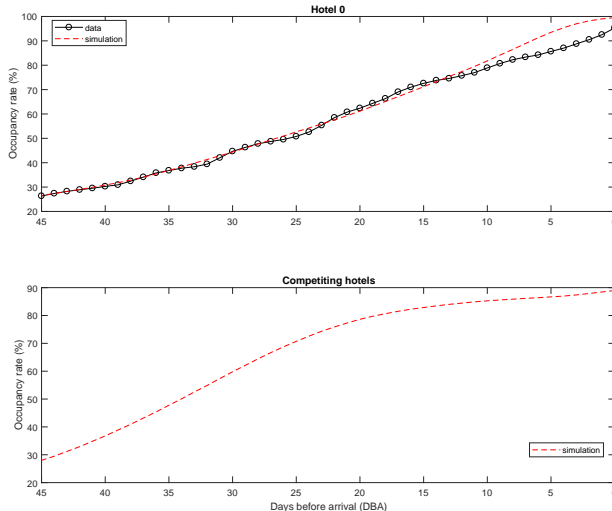
Estimate of elasticity

Table 9: Estimates of Choice Parameters (a_τ, b_τ)

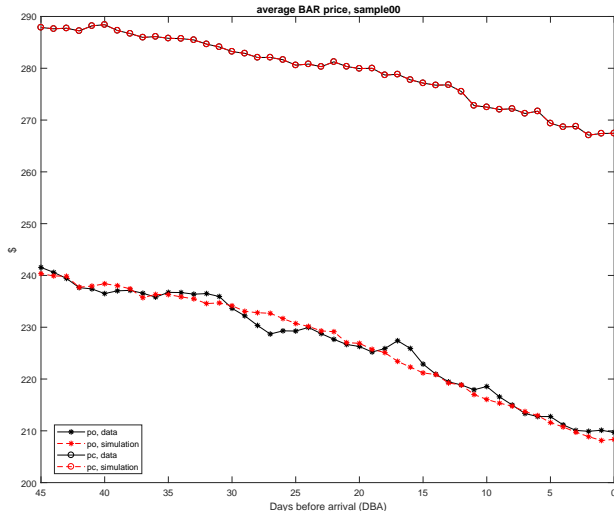
Segment	Parameter	Lowest Demand (0-25%)	Medium-Low Demand (25-50%)	Medium-high Demand (50-75%)	Highest Demand (75-100%)
Weekday	Leisure	a_τ -1.698 (0.384)	-1.546 (0.338)	-1.329 (0.174)	-2.300 (2.798)
		b_τ -0.008 (0.001)	-0.007 (0.001)	-0.010 (0.001)	-0.074 (0.036)
	Business	a_τ -1.618 (1.151)	-1.904 (0.150)	-1.047 (0.134)	-2.564 (0.742)
		b_τ -0.006 (0.002)	-5.8E-3 (2.7E-4)	-0.006 (0.001)	-0.091 (0.091)
	Group	a_τ -0.539 (1.115)	-0.935 (0.152)	-1.167 (0.360)	-1.370 (1.362)
		b_τ -0.012 (0.005)	-0.011 (0.002)	-0.012 (0.002)	-0.094(0.055)
Weekend	Leisure	a_τ -1.580 (0.091)	-1.803 (0.328)	-0.296 (0.515)	-3.821 (15.325)
		b_τ -0.008 (0.001)	-0.009 (0.002)	-0.035 (0.046)	-0.128 (0.980)
	Business	a_τ -1.358 (0.149)	-1.262 (0.314)	-2.203 (2.480)	-3.874 (5.172)
		b_τ -0.007 (0.001)	-0.007 (0.003)	-0.007 (0.010)	-0.076 (0.269)
	Group	a_τ -0.813 (0.076)	-0.913 (0.217)	-0.002 (0.003)	-2.537 (4.421)
		b_τ -0.012 (0.003)	-0.017 (0.002)	-0.015 (0.010)	-0.134 (0.194)

Note: standard errors in parentheses.

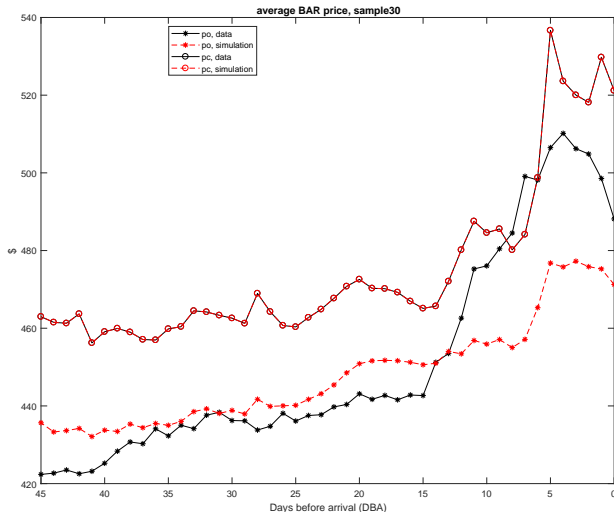
Fit of model: occupancy on busiest weekends



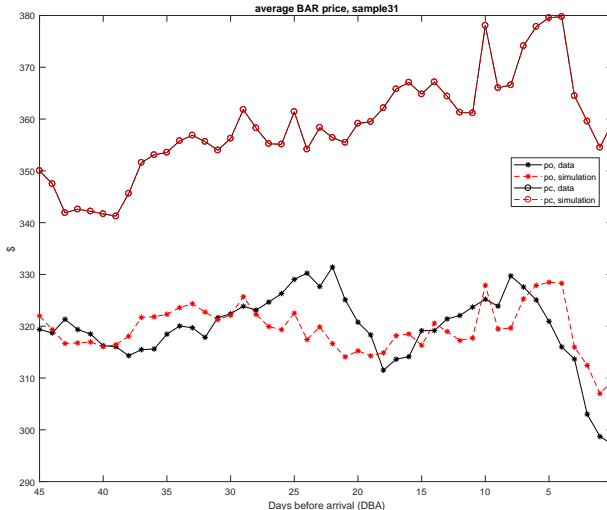
Fit of model: BAR on least busy weekdays



Fit of model: BAR on most busy weekdays

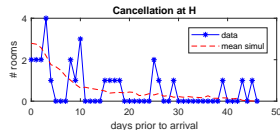
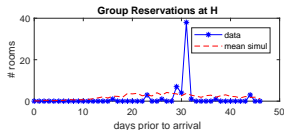
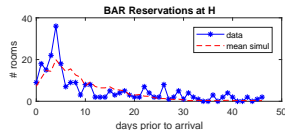
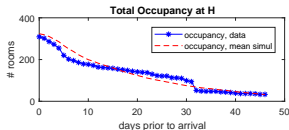
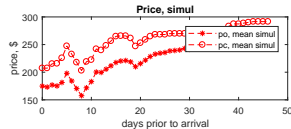
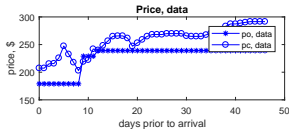


Fit of model: BAR on busiest weekends



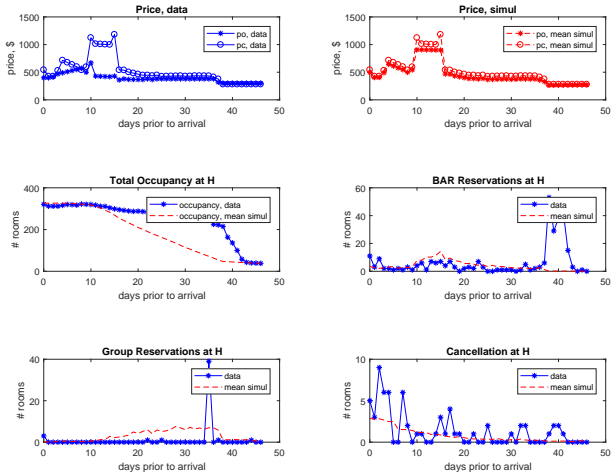
Model vs data on specific busiest weekend Day21

Mean Trajectory for Day = 21, sample31



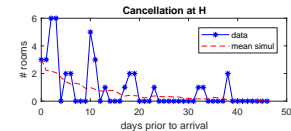
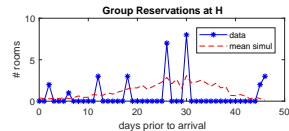
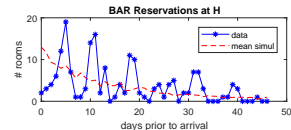
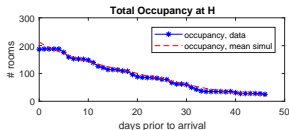
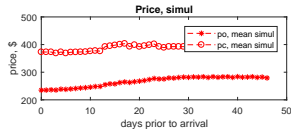
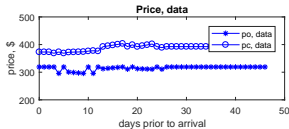
Model vs data on specific busiest weekend Day1

Mean Trajectory for Day = 1, sample31



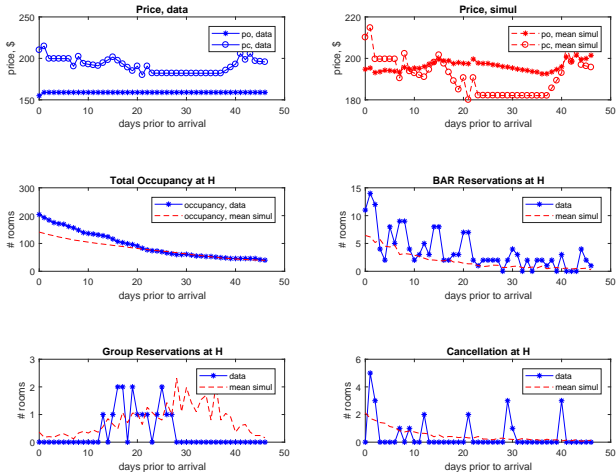
Model vs data on specific least-busy weekday 1

Mean Trajectory for Day = 1, sample00

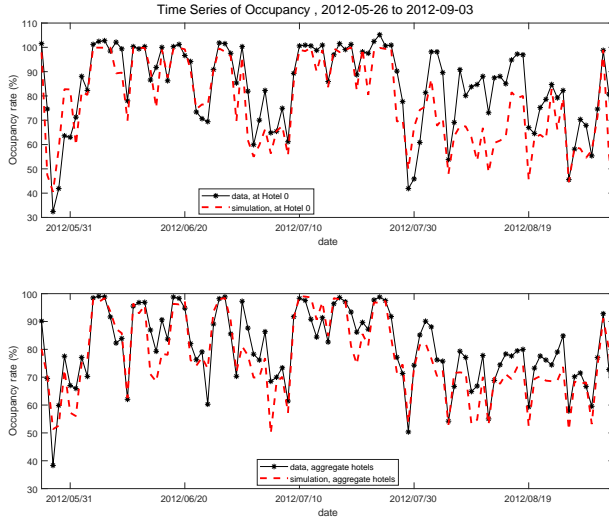


Model vs data on specific least-busy weekday 21

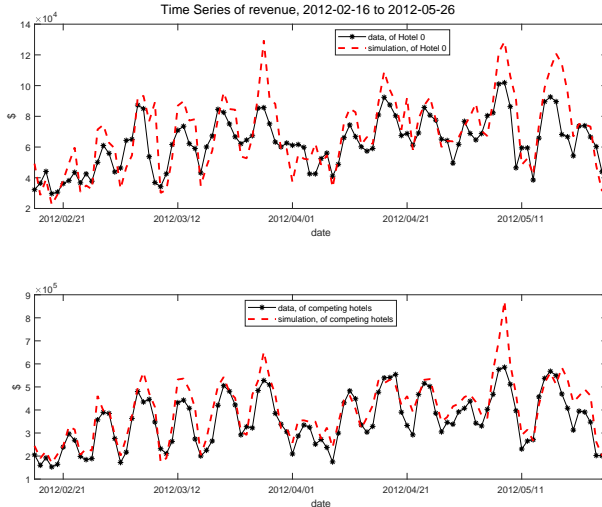
Mean Trajectory for Day = 21, sample00



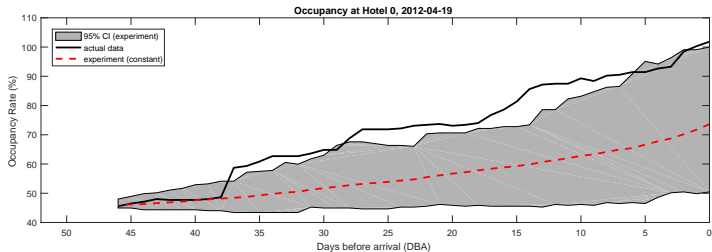
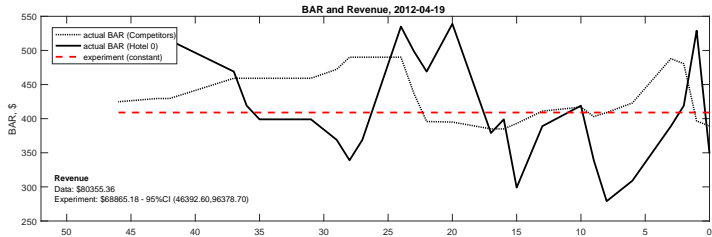
Actual vs predicted Occupancy: 5/26 - 9/3/2012 (final)



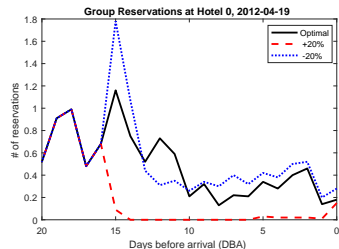
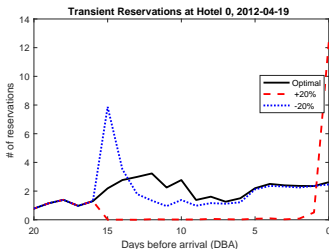
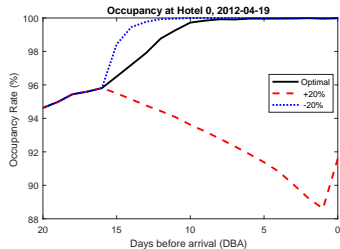
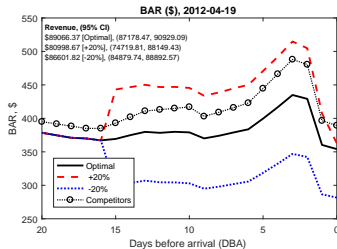
Actual vs predicted Revenues: 5/26 - 9/3/2012 (final)



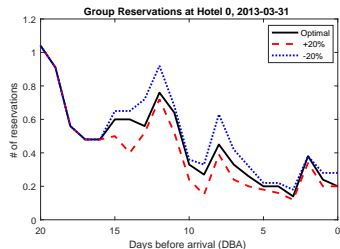
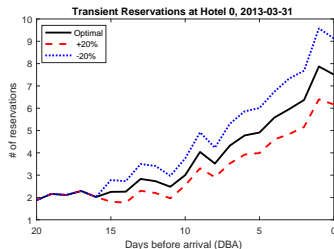
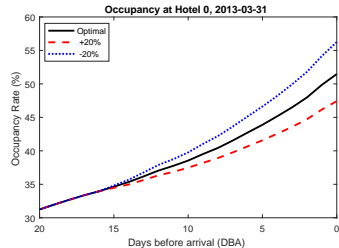
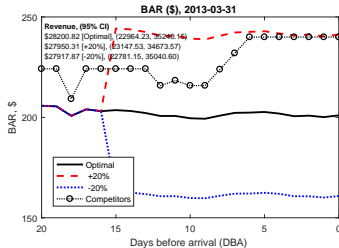
Counter-factual : Constant price (hotel 0), sample31



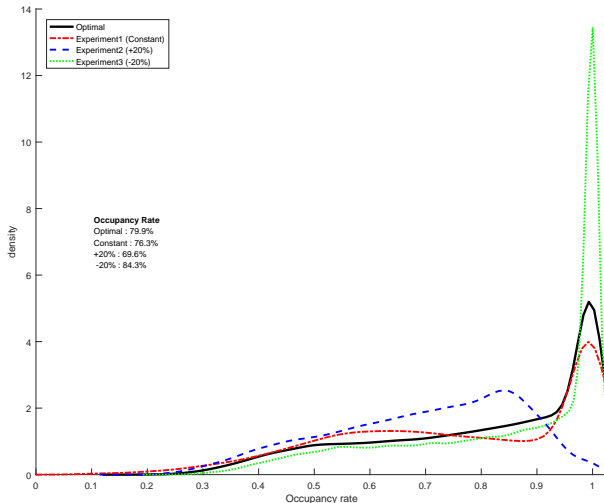
Counter-factual : optimal price $\pm 20\%$, sample31



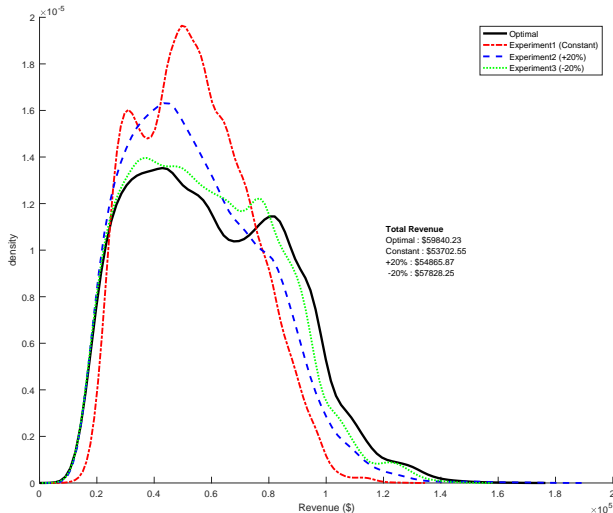
Counter-factual : optimal price $\pm 20\%$, sample00



Counter-factual : occupancy distribution (full sample)



Counter-factual : revenue distribution (full sample)



Conclusion

- Introduced a DP model for hotel pricing, which allows for competition, heterogeneous demand and intertemporal price discrimination
- Sensible estimation of demand
- Accurate prediction of reservation/price dynamics
- Future works
 - allow for full equilibrium
 - relax the assumption of optimality

Firms in the Local Luxury Hotel Market

Table 1: Hotels in the local market in our study

Property	Avg. BAR	Star	Class	Chained Brand	Rate	Capacity Share	Distance to mass transit	Cancel Policy
hotel 0	\$ 293.26	4	Luxury	No	4.4	15%	3 min	1 day before
hotel 1	\$ 338.29	4	Upper Up	Yes	4.2	19%	8 min	2 day before
hotel 2	\$ 253.51	4	Upper Up	No	4.2	9%	8 min	3 day before
hotel 3	\$ 285.16	4	Upper Up	No	4.4	12%	3 min	1 day before
hotel 4	\$ 454.30	5	Luxury	Yes	4.7	10%	10 min	1 day before
hotel 5	\$ 397.09	4	Luxury	No	4.6	19%	10 min	Strict
hotel 6	\$ 282.64	4.5	Upper Up	No	4.4	16%	5 min	3 day before

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List of Data Sets

Table 2: Data description

Data	The first day of occupancy	The last day of occupancy	Observations	Description
market vision	2010-09-21	2014-08-13	609,181	competitors' price
reservation raw ¹	2009-09-01	2013-10-31	201,176	reservations detail information
cancellation raw ²	2009-09-01	2013-10-31	29,241	cancel detail information
daily pick-up report	2010-09-16	2014-05-21	475,187	daily revenue report
STR market data	2010-01-01	2014-12-31	1,731	competitors' occupancy
Data range	2010-10-01	2013-10-31		37 months

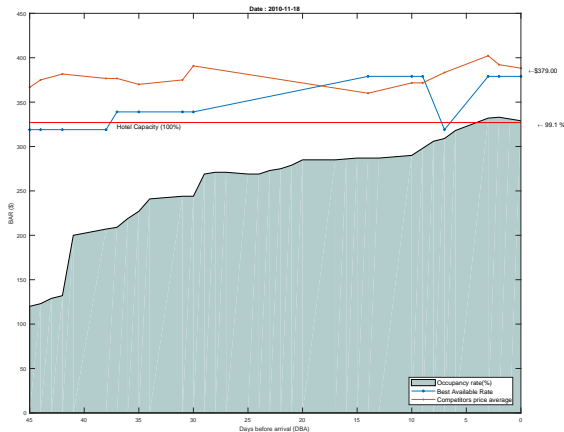
Customer share by type

Table 7: Customer distribution by subsample

Sample (Weekday)	Customer share			Occupancy rate	Sample (Weekend)	Customer share			Occupancy rate
	business	leisure	group			business	leisure	group	
00	0.18	0.61	0.21	51.6 %	01	0.12	0.67	0.21	58.7 %
10	0.20	0.51	0.29	73.9 %	11	0.13	0.65	0.22	81.5 %
20	0.25	0.39	0.36	88.5 %	21	0.14	0.60	0.26	91.0 %
30	0.26	0.30	0.44	99.2 %	31	0.15	0.56	0.30	95.3 %

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Example Arrival Date: Busy Weekend 11/18/2010



Multiple Products: Room Types

Table 3: Room Types

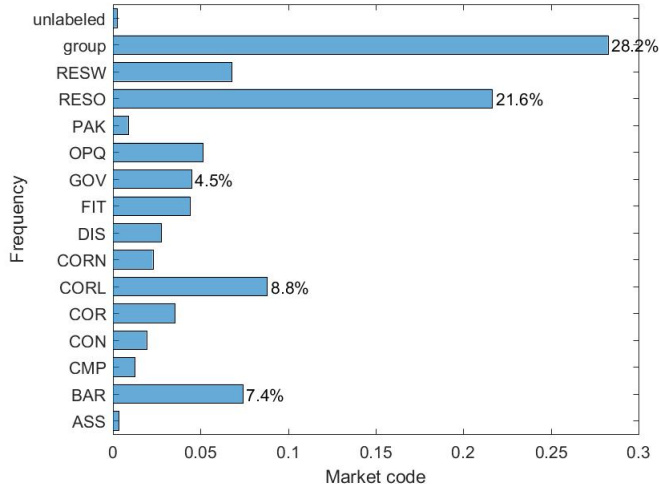
Code	Description	% of rooms (before renovation)	% of rooms (after renovation)	Rack Rate
B1K	Superior, 1 King	57	43	\$203.15
B2D	Superior, 2 double beds	33	19	\$ 203.15
A1K	Deluxe, 1 King	4	14	\$ 253.15
A2D	Deluxe, 2 double beds	1	14	\$ 253.15
GD1K	Grand Deluxe, 1 King	0	3	\$ 303.15
GD2D	Grand Deluxe, 2 double beds	0	1.5	\$ 303.15
<i>others</i>	Suites, etc	5	5.5	>\$600 or negotiated

Multiple Segments: Reservation/Contract Types

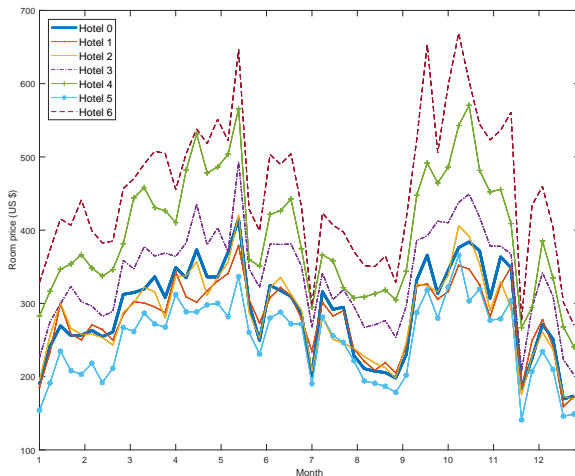
Table 4: Hotel Reservation type

Category	Market Segment	Title	Description	Booking Share
Transient	BAR	Best Available Rate	Best available rates that have hotel house cancellation policy, rate codes BAR only applicable in this segment	68.4%
	CON	Consortia/TMC	Consortia, Travel Management Companies bookings	
	RESW	Restricted-Web	Advance purchase and/or any promotional offers available in Hotel 0 collection web site with restrictions such as pre-paid/non-refundable i.e. 10% off 7 day advance purchase, 2mos at 20% off, or limited time offer	
	CORL	Corporate LRA	Corporate/local negotiated rates with last room availability	
	CORN	Corporate NLRA	Corporate/local negotiated rates with Non-last room availability	
	GOV	Government	Federal or state government per diem and/or accounts with per diem equivalent rates	
	PAK	Package	Room package	
	FIT	Wholesale	Locally negotiated wholesale accounts and Third party vacation package	
	DIS	Qualified Discount	AAA, AARP, Employee rate or any qualified discounted rates	
Group	RESO	Restricted-OTAs	Same rates as restricted segment available in OTA merchant sites	31.6%
	OPQ	Opaque	Hotwire/ Priceline	
	CGP	Corporate	corporate group	
	CGV	Government	government group	
	ASS	Association	convention group	
	TOT	Tour & Travel	tour group	
	group	group	uncategorized group	

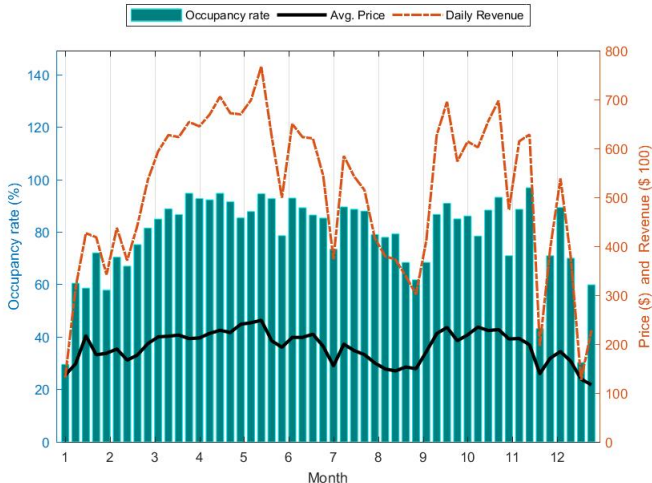
Reservation Frequencies



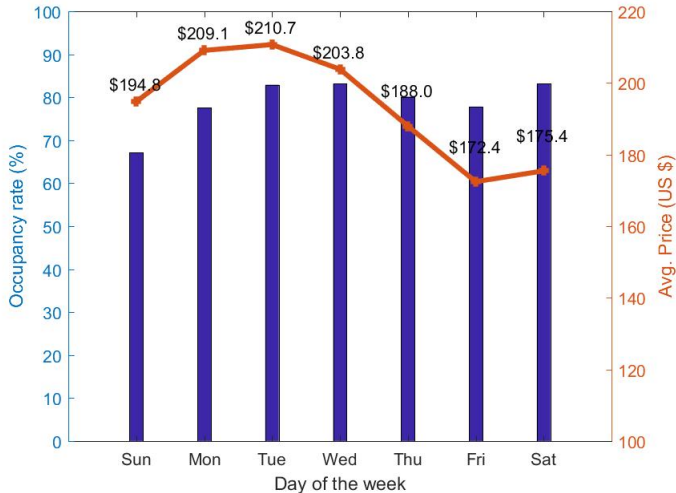
Annual Cycle: BAR



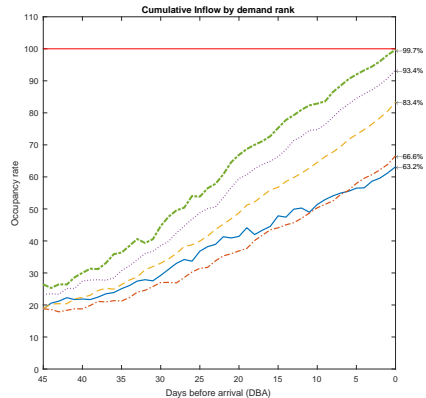
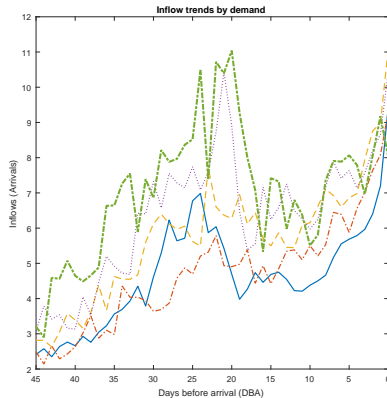
Annual Cycle: Occ, Avg Daily Rate (ADR) and Rev



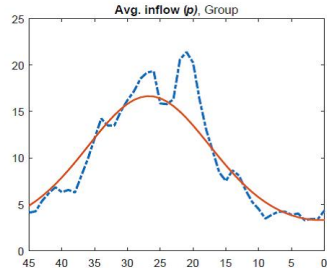
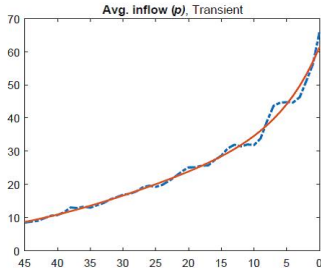
Weekly Cycles: Occ and ADR



Reservation Dynamics: by Type of Day

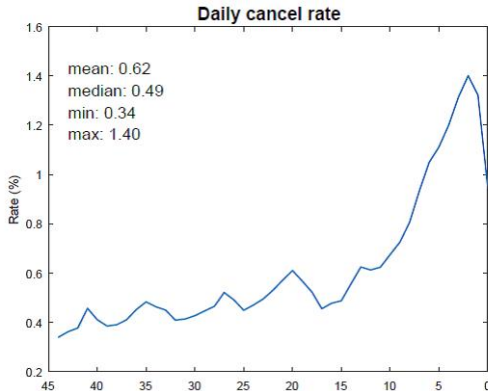


Reservation Dynamics: by Segment

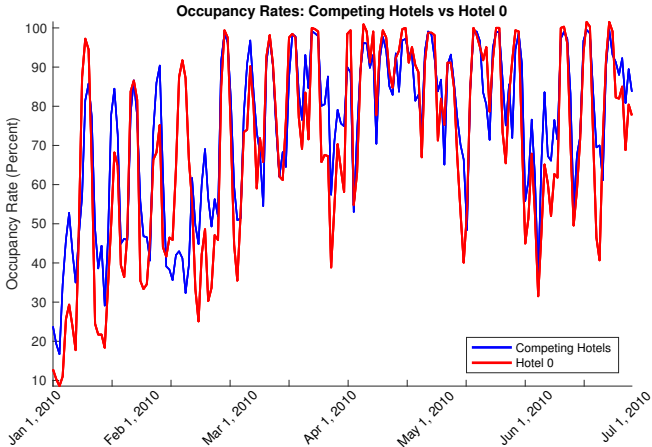


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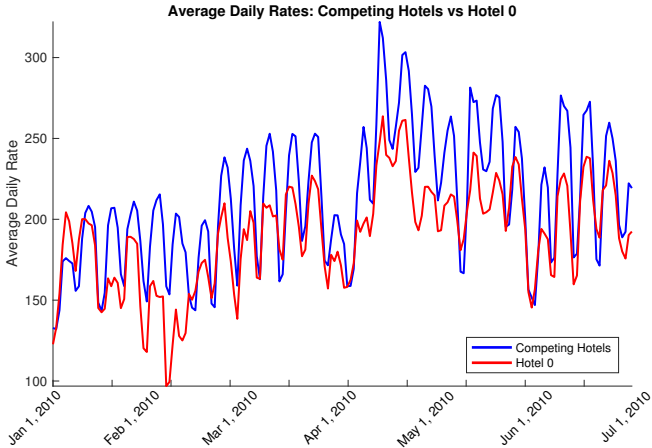
Cancellation Dynamics



Co-movement in Occupancy Rate



Co-movement in ADR



Regression of ADR_0

Variable	Estimate	Standard Error
constant	143.7	3.1
OCC_0	0.68	0.04
$N = 1277, R^2 = 0.17$		

Variable	Estimate	Standard Error
constant	30.07	2.19
ADR_c	0.76	0.011
OCC_0	-0.013	0.021
$N = 1277, R^2 = 0.82$		
(adding monthly and daily dummies raises R^2 to 0.86)		

Downward-sloping Demand?

