Competition, Corruption and Institutional Design*

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Abstract

This paper offers a model to study competition and corruption with a principal-agent framework. We provide two key results on the optimal institutional design. First, in quality-only competition, corruption does no harm to the principal, but in quality-price competition, corruption negatively affects the principal. Second, with no corruption, quality-price competition is a superior institutional setting for the principal compared with quality-only competition when the principal's net benefit is sufficiently large, whereas with corruption, introducing price competition can lead to a worse outcome for the principal given the high price distortion involved.

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1 Introduction

Corruption has a long history of intervening in various resource-allocation problems.[1] It has been repeatedly argued that corruption can disrupt an efficient allocation,[2] and that, in particular, when the allocation problems involve price settlement, corruption can increase the costs of procuring goods and services.

Allocative inefficiency and price distortion are emphasized and more evident in a competitive environment: an agent chooses a "winner" from competing candidates. Many applications in economic situations fit this description. In the public sector, two suppliers may compete to sell a product to a government procurement officer; in the private sector, two upstream firms may compete to sell an output to a downstream firm; and in labor markets, two candidates may compete to be hired by an organization.

Following a pioneering work by Rose-Ackerman (1975), papers study corruption within a competitive environment and the link between corruption and market structure (see Shleifer and Vishny (1993), Ades and Di Tella (1999), Laffont and N'Guessan (1999), Burguet and Che (2004) and Compte, Lambert-Mogiliansky and Verdier (2005)). In all of these papers, the existence of a corruptible agent is an *essential element* for their findings; however, no previous work examines the optimal design problem for the agent. On the other hand, the three-tiered hierarchical structure with principal/agent/firm introduced by Tirole (1986) provides a seminal modelling structure to study the role of an agent (for example, Khalil, Lawarrée and Yun (2010)), but has not been extended to a competitive environment.[3]

[1]See Bardhan (1997) for prescriptions for deterring corruption in the fourth century B.C.

[2]The collection by Elliott (1997) and the book by Rose-Ackerman (1999) provide comprehensive lists of references. Klitgaard (1988), Andvig (1991), Ades and Di Tella (1996), Bardhan (1997), Lambsdorff (2001) and Aidt (2003) are also useful surveys.

[3]There is a large literature on agent problem or delegation (see Mookherjee (2006) for survey), and each paper assumes an environment in which either a standard mechanism approach by a principal cannot be used to elicit information of an agent or the principal prefers delegation to

This paper offers a model to study competition and corruption with a principal-agent framework in order to tackle problems of allocative inefficiency and price distortion. The principal-agent framework embodies *two layers* of an optimal-design problem: an optimal compensation scheme for the agent and an optimal institutional setting. Our aim is to provide the relationship between corruption and market structure for which we consider four different institutional settings depending on the presence of corruption, and on whether the competition is based on quality alone or both quality and price. First, given each institutional setting, we find an optimal compensation scheme for the agent, and later, compare the four institutional settings to find an optimal institutional choice.[4]

The model assumes that a firm's population consists of both high- and low-quality firms, from which two firms are randomly matched with the principal. The principal either might not have the expertise needed to assess each firm's quality or might have the expertise but work in a position managing *many identical* agents.[5] The principal knows that a high-quality firm's product has a higher probability of succeeding at a certain task. Hence, the compensation scheme is structured to depend on whether the task is successful or not. Additionally, if an institutional setting involves price competition, the principal can observe the purchase price; therefore, a cost sharing based on the price is a part of the scheme.

For each institutional setting, we characterize the optimal compensation scheme. If the competition is based on quality and price, a cost-sharing rule between the principal and the agent plays an important role in achieving the optimality with corruption.[6] On the other hand, if the competition is based on quality alone, a

centralization through mechanism design. Otherwise, no agent's decision making can survive; the principal can directly make the agent reveal information.

[4] Of course, often a principal or a government body may not be able to choose the presence of corruption. We may treat it as an exogenous variable.

[5] In this regard, the principal can also be considered a social planner or designer.

[6] The is the reason that some values of the cost-sharing rule that satisfy the optimal solution for quality-price competition without corruption in Proposition 1 do not satisfy the one for quality-price

sufficiently high marginal incentive for the successful task result enables the principal to obtain the optimal payoff regardless of the presence of corruption.

This paper provides two key results on the optimal institutional design. First, we show that in quality-only competition, corruption does no harm to the principal, sharing a similar prediction with the "efficient corruption" argument (see Lui (1985) and Beck and Maher (1986), and also Aidt (2003) for a survey), whereas in quality-price competition, corruption negatively affects the principal. In the latter, the objective of the principal is to buy a better-quality good at a lower price. However, corruption creates a *money flow* between all three participants; that is, money transfers first from the principal to a winning firm, then the firm and the agent divide it. This inevitably involves an increase in the "buying price." On the other hand, in the former, since there is no price competition, the "prize" is fixed. Then, the aim of the principal is to buy a better-quality good. In this case, corruption creates a money flow just between the agent and the firm, which does not affect the principal as long as the high-quality firm is picked.[7]

Second, with no corruption, quality-price competition is a superior institutional setting for the principal compared with quality-only when the principal's net benefit is sufficiently large, which confirms the conventional market outcome: a more competitive environment results in a more efficient allocation. However, with corruption, introducing an additional dimension of competition, price competition, can lead to a worse outcome for the principal given the high price distortion involved. Hence, the conventional view does not hold where there is corruption.

Shleifer and Vishny (1993), Ades and Di Tella (1999) and Laffont and N'Guessan (1999) examine the relationship between corruption and competition with a market structure, but they do not consider a competitive environment such as competitive procurement. Burguet and Che (2004) and Compte, Lambert-Mogiliansky and Verdier (2005) study corruption in a competitive environment, but neither incorpo-

competition with corruption in Proposition 5.

[7]Still, there is a distributive issue between the firm and the agent.

rates a principal-agent problem into it.[8]

This paper is organized as follows. The model is introduced in Section 2, and quality-price competition without corruption and quality-price competition with corruption are analyzed in Sections 3 and 4, respectively. Quality-only competition is discussed in Section 5, and, finally, optimal institutional choice is studied in Section 6. Examples and discussion are in Section 7, Concluding remarks are in Section 8, and all the proofs are collected in an appendix.

2 Model

Consider a situation in which two firms are randomly matched from a population, and they compete to supply a product to a principal. The two firms may have two different technology levels, which yield different product qualities, namely high and low, $q \in \{H, L\}$. The proportion of firms that can produce the high-quality product is $\lambda \in (0, 1)$, and in what follows, a firm with the high- (resp. low-) quality product is denoted by HP (resp. LP). Each firm incurs a regular cost of producing its product, which is normalized as 0. However, production of the high-quality product requires a "qualified worker," whose wage is exogenously given as c.[9]

Each firm knows the other matched firm's technology level and the worker's market wage c, but the principal knows only that the proportion of HP is λ, and c is drawn from an absolutely continuous distribution function F, where its support is given as $[0, \overline{c}]$ satisfying $\overline{c} > 0$. In other words, for the downstream competition between the two firms, we assume a complete information game to make the analysis tractable.[10]

[8] In the former, an agent takes a bribe from a firm in return for manipulating the firm's quality assessment to a buyer, and in the latter, an agent allows a firm that wins bribe bidding to adjust its initial price bid.

[9] One cannot classify HP as a more efficient firm since HP produces a *higher* quality product with a *higher* cost.

[10] Note that in this paper, each firm's quality is assumed to be *given*, i.e., *not bidding quality* in

Since the principal cannot verify the product quality at the moment of making the purchase decision, he or she hires an expert in the industry, called an agent, to delegate the decision to. The two firms bid prices (p_i, p_j) and compete to be chosen by the agent. If the product is of high quality, it succeeds at a certain task with probability r_H, and if the product is of low quality, it succeeds with probability r_L where $1 > r_H > r_L > 0$. The principal gains a return v_S if the task is successful, and v_F if not, with $v_S > v_F \geq 0$.

The principal can only observe the outcome of the task and the price level of the product purchased by the agent, so the principal designs the agent's optimal compensation scheme based on the outcome and the purchase price with three variables $(x_S, x_F, x_p) \in \mathbb{R}^3$. The compensation scheme consists of two parts: incentives (x_S, x_F) and cost sharing x_p.[11] If the product succeeds in the task, the agent's incentive for the outcome is x_S, but if the product fails, it is x_F. In addition, the agent's cost sharing is $x_p p$, where $p \in \mathbb{R}_+$ is the purchase price of the product.

We assume that:

(A1) the principal and the agent are risk-neutral.

(A2) $x_p \in (0, \overline{x}]$ where $\overline{x} \leq 1$.

(A3) the agent's reservation payoff is $U \geq 0$.

For (A2), we impose the limited liability $x_p \leq \overline{x}$, since, for instance, if $x_p > 1$, the principal can obtain a greater revenue from the agent than the purchase price p given $-p + x_p p > 0$. Denote

$$\begin{aligned} x_q &\equiv r_q x_S + (1 - r_q) x_F \text{ for } q \in \{H, L\}, \\ v_q &\equiv r_q v_S + (1 - r_q) v_F \text{ for } q \in \{H, L\}, \\ x &\equiv (x_H, x_L, x_p), \end{aligned}$$

order to make the model widely applicable to competitive environments other than procurement auctions (e.g., job candidates are not able to choose their qualities), so for incomplete information, the downstream competition has the two-dimensional private information (quality and cost).

[11] If the cost sharing depends on the outcomes, success or failure, as well, there does not exist a solution. See Appendix II for the analysis.

where $x \in \Omega \equiv \mathbb{R}^2 \times (0, \overline{x}]$. For each $q \in \{H, L\}$, x_q can be interpreted as the expected incentive for quality q, and v_q as the expected return for quality q.

3 Competition without corruption

We first consider the case without corruption as a benchmark. The principal announces the compensation scheme, and two matched firms bid prices to sell their product. The formal timeline is as follows:

Time 1. Nature determines c.

Time 2. The principal announces the compensation scheme.

Time 3. The two firms are randomly matched, and bid prices.

Time 4. The agent chooses one of them.

The principal makes the move first in this model similar to a "screening" approach, which can be applied to situations where the principal has to manage many identical agents.

The two firms play a simultaneous move game with complete information at time 3. Each firm i's strategy is a bid distribution σ_i over $\mathbb{R}_+$ where p_i is a bid, and after observing their bids and quality levels, the agent chooses one firm, so the agent's strategy is a mapping from $\mathbb{R}_+^2 \times \{H, L\}^2$ to $\Delta(\{i, j\})$ where $\Delta(\{i, j\})$ is the set of probability distributions over the set $\{i, j\}$, and $h(p_i, p_j, q_i, q_j)$ denotes the probability of choosing firm i.

LP's cost 0 can be considered its "type", so a full type space including LP's can be constructed as $[0, \overline{c}]$ in which firm i's type is denoted by $\theta_i \in [0, \overline{c}]$. Firm i's payoff is its bid minus the production cost, $p_i - \theta_i$, if chosen, and 0 otherwise. The agent's payoff from choosing firm i with quality q_i and price-bid p_i is given as the incentive minus the cost sharing, $x_{q_i} - x_p p_i$ for $q_i \in \{H, L\}$.

The agent's sequentially rational strategy h^* at time 4 is to choose firm i with probability 1 if choosing i yields a higher payoff to the agent, $x_{q_i} - x_p p_i > x_{q_j} - x_p p_j$, and firm i with probability $\frac{1}{2}$ if it is indifferent, $x_{q_i} - x_p p_i = x_{q_j} - x_p p_j$.[12] It follows from the agent's optimal strategy that firm i's payoff $\pi : \mathbb{R}^2_+ \times \{H, L\}^2 \to \mathbb{R}_+$ is given as below:

$$\pi(p_i, p_j, q_i, q_j) = \begin{cases} p_i - \theta_i & \text{if} \quad x_{q_i} - x_p p_i > x_{q_j} - x_p p_j, \\ \frac{1}{2}\left[p_i - \theta_i\right] & \text{if} \quad x_{q_i} - x_p p_i = x_{q_j} - x_p p_j, \\ 0 & \text{if} \quad x_{q_i} - x_p p_i < x_{q_j} - x_p p_j. \end{cases}$$

Then, a subgame perfect Nash equilibrium is the agent's sequentially rational strategy h^* with a mixed strategy profile (σ_i^*, σ_j^*) such that for each quality profile $(q_i, q_j) \in \{H, L\}^2$, and for every firm i,

$$u(\sigma_i^*, \sigma_j^*, q_i, q_j) \geq u(\sigma_i, \sigma_j^*, q_i, q_j) \text{ for all } \sigma_i, \text{ and } u(\sigma_i^*, \sigma_j^*, q_i, q_j) \geq 0,$$

where $u(\sigma_i, \sigma_j, q_i, q_j) \equiv \mathbb{E}^\sigma[\pi(p_i, p_j, q_i, q_j)]$ and $\sigma \equiv (\sigma_i, \sigma_j)$ is a mixed strategy profile. In what follows, an equilibrium refers to a subgame perfect Nash equilibrium. Note that the most aggressive bid of LP and HP is 0 and c, respectively. There can be three possible matchings given different quality profiles $(q_i, q_j) \in \{H, L\}^2$:

M1. Two low-quality firms are matched with probability $(1 - \lambda)^2$.

In this case, the agent's payoff is $x_L - x_p p_i$ for all i. Since the two low-quality firms engage in a Bertrand-like competition, a unique equilibrium bid is to choose 0 with probability 1.

M2. Two high-quality firms are matched with probability λ^2.

[12] For Bertrand competition with homogenous products and different marginal costs, in order to resolve the existence problem, it is sometimes "assumed" that the market favors the low-cost firm such that it can charge a price equal to the high-cost firm's marginal cost. Blume (2003) first shows that this non-standard assumption is not necessary, and the conventional outcome holds under the standard rule that both firms split the market when their prices tie. Similarly, in this model, for $x_{q_i} - x_p p_i = x_{q_j} - x_p p_j$, this assumption is not necessary, and the proof works for any strictly mixed strategy $h\left(p_i, p_j, q_i, q_j\right) \in (0, 1)$, but it is clear that choosing each with an equal probability is most reasonable. Furthermore, depending on the parameter c, HP can be a firm like the low-cost firm or LP can be a firm like the low-cost firm, so the assumption favoring the low-cost firm simply does not work in this model.

In this case, the agent's payoff is $x_H - x_p p_i$ for all i. Since the two high-quality firms engage in the competition, a unique equilibrium bid is to choose c with probability 1.

M3. Two different-quality firms are matched with probability $2\lambda(1-\lambda)$.

When two different-quality firms are matched, with slight abuse of the notations, we use (p_H, p_L) instead of (p_i, p_j). In this case, the agent compares the payoff from choosing HP, $x_H - x_p p_H$, with the payoff from choosing LP, $x_L - x_p p_L$, to choose a firm. The agent's net incentive is denoted by $\Delta x \equiv x_H - x_L$ and the net incentive per cost sharing is y such that

$$y \equiv \frac{\Delta x}{x_p}. \tag{1}$$

For $x_H - x_p c = x_L$ or $c = y$, it is straightforward to find that a unique equilibrium bid profile is $(p_H^*, p_L^*) = (c, 0)$.[13] However, for $c \neq y$, there is no pure strategy Nash equilibrium, and the existence of an equilibrium relies on a mixed strategy profile as in the typical Bertrand competition with homogenous products and different marginal costs (see Blume (2003) and Kartik (2011)). If $x_H - x_p c > x_L$ or $c < y$, then there is HP's bid that can beat LP's most aggressive bid 0, so HP becomes a winning firm similar to the "low-cost firm" in the Bertrand competition, whereas if $x_H - x_p c < x_L$ or $c > y$, there is LP's bid that can beat HP's most aggressive bid c, so LP becomes a winning firm. By restricting solutions to weakly undominated strategies, we can have a *unique equilibrium outcome* similar to the one in Kartik (2011).

Lemma 1 *Suppose that two different-quality firms are matched. Then, for any equilibrium in weakly undominated strategies satisfying the reservation payoff U, if $c < y$, HP bids y and wins with probability 1, and if $c > y$, LP bids $c - y$ and wins with probability 1.*

In equilibrium, if HP's cost c is lower than y, then there exists $p_H > c$ such that $x_H - x_p p_H = x_L$, which makes HP win with probability 1 while LP randomizing

[13] This event can realize with measure zero probability.

its bid, but if HP's cost c is greater than y, then there exists $p_L > 0$ such that $x_H - x_p c = x_L - x_p p_L$, which makes LP win with probability 1 while HP randomizing its bid.

The principal's expected payoff is $\mathbb{E}[v_H - c]$, where c is the price the principal pays to one of the HPs, when the two high-quality firms are matched with probability λ^2; and the principal's payoff is v_L, where 0 is the price the principal pays to one of the LPs, when the two low-quality firms are matched with probability $(1-\lambda)^2$. Last, the two different-quality firms are matched with probability $2\lambda(1-\lambda)$, and for each $q \in \{H, L\}$, if $c < y$ (resp. $c > y$), HP (resp. LP) wins, which makes the principal pay y (resp. $c - y$) to the firm. Then, the principal's (gross) payoff is given as

$$\Pi(x) \equiv \lambda^2(v_H - \mathbb{E}c) + (1-\lambda)^2 v_L + 2\lambda(1-\lambda)\left\{\int_0^y [v_H - y]dF(c) + \int_y^{\bar{c}} [v_L + y - c]dF(c)\right\}, \tag{2}$$

and, similarly, the agent's incentive payoff is

$$\Gamma(x) \equiv \lambda^2(x_H - x_p\mathbb{E}c) + (1-\lambda)^2 x_L + 2\lambda(1-\lambda)\left\{\int_0^y x_L dF(c) + \int_y^{\bar{c}} [x_H - x_p c]dF(c)\right\}, \tag{3}$$

and the third term is derived from $\int_0^y [x_H - x_p y]dF(c) + \int_y^{\bar{c}}[x_L - x_p(c-y)]dF(c)$.

The principal's net payoff is the difference between the principal's payoff and the agent's incentive payoff. Then, the principal solves the following maximization problem:

$$\begin{cases} \max_{x\in\Omega}[\Pi(x) - \Gamma(x)] \\ \text{s.t. } \Gamma(x) \geq U \text{ (IR)} \end{cases} \tag{P1}$$

Proposition 1 characterizes the set of solutions to the maximization problem.[14]

Proposition 1 *x^* solves (P1) if and only if x_p^* and Δx^* satisfy $y^* = \Delta x^*/x_p^*$ where*

$$y^* \in \arg\max_{y\in[0,\bar{c}]}\left\{\int_0^y [v_H - y]dF(c) + \int_y^{\bar{c}} [v_L + y - c]dF(c)\right\}. \tag{4}$$

[14] In addition, one can show that if $v_H - v_L \in (0, \bar{c})$, then a solution must be an interior solution with $y^* \in (0, \bar{c})$.

For any solution to (P1), the individual rationality condition (IR) must be binding $\Gamma(x) = U$, so it is sufficient to find x maximizing $\Pi(x)$ given the constraint, which is equivalent to a solution in (4). A solution to (P1) is called the first-best solution, and we next introduce corruption to examine how the agent's payoff can change with bribery.

4 Competition with corruption

Now, two firms bid bribes as well as prices at time 3. We assume that firms bid prices without observing each other's bribe bids.[15] Then, the same analysis applies for both cases: the simultaneous bidding of prices and bribes and the sequential bidding of prices and bribes.

The agent's expected payoff from receiving a bribe is given as $w : \mathbb{R}_+ \to \mathbb{R}$ in order to capture the costs the agent may suffer from corruption.[16] We assume that $w(0) = 0$, for all $b \geq 0$, $w' > 0$ and $w'' < 0$ and that if a bribe is sufficiently large, the marginal expected payoff is close to 0, that is, $\lim_{b \to +\infty} w'(b) = 0$.

Each firm i's strategy is a bid distribution σ_i^b over $\mathbb{R}_+^2$, where p_i is a price bid and b_i is a bribe bid. After observing their bids and quality levels, the agent chooses one firm, so the agent's strategy is a mapping from $\mathbb{R}_+^4 \times \{H, L\}^2$ to $\Delta(\{i, j\})$, and $h^b(p_i, p_j, b_i, b_j, q_i, q_j)$ denotes the probability of choosing firm i.

Firm i's payoff is its price bid minus the sum of its bribe bid and the production cost, $p_i - b_i - \theta_i$ for $\theta_i \in [0, \bar{c}]$, if chosen, and the agent's payoff from choosing firm i with quality $q_i \in \{H, L\}$ and bid (p_i, b_i) is the incentive minus the cost sharing with the expected payoff from receiving a bribe, denoted by

$$U_{q_i}(x, p_i, b_i) \equiv x_{q_i} - x_p p_i + w(b_i). \tag{5}$$

Then, the agent's sequentially rational strategy h^{b*} at time 4 is to choose firm i with probability 1 if choosing i yields a higher payoff to the agent, $U_{q_i}(x, p_i, b_i) >$

[15] This secrecy is widely accepted as a feature of corruption (see Shleifer and Vishny (1993)).

[16] This cost can be the psychological burden of corruption (see Tirole (1992)).

$U_{q_j}(x, p_j, b_j)$, and firm i with probability $\frac{1}{2}$ if it is indifferent, $U_{q_i}(x, p_i, b_i) = U_{q_j}(x, p_j, b_j)$. It follows from the agent's optimal strategy that firm i's payoff $\pi^b : \mathbb{R}_+^4 \times \{H, L\}^2 \to \mathbb{R}_+$ is given as follows:

$$\pi^b(p_i, p_j, b_i, b_j, q_i, q_j) = \begin{cases} p_i - b_i - \theta_i & \text{if} \quad U_{q_i}(x, p_i, b_i) > U_{q_j}(x, p_j, b_j), \\ \frac{1}{2}[p_i - b_i - \theta_i] & \text{if} \quad U_{q_i}(x, p_i, b_i) = U_{q_j}(x, p_j, b_j), \\ 0 & \text{if} \quad U_{q_i}(x, p_i, b_i) < U_{q_j}(x, p_j, b_j). \end{cases}$$

Then, an equilibrium is the agent's sequentially rational strategy h^{b*} with a mixed strategy profile $(\sigma_i^{b*}, \sigma_j^{b*})$ such that for each quality profile $(q_i, q_j) \in \{H, L\}^2$, and for every firm i,

$$u^b(\sigma_i^{b*}, \sigma_j^{b*}, q_i, q_j) \geq u^b(\sigma_i^b, \sigma_j^{b*}, q_i, q_j) \text{ for all } \sigma_i^b, \text{ and } u^b(\sigma_i^{b*}, \sigma_j^{b*}, q_i, q_j) \geq 0,$$

where $u^b(\sigma_i^b, \sigma_j^b, q_i, q_j) \equiv \mathbb{E}^{\sigma^b}[\pi^b(p_i, p_j, b_i, b_j, q_i, q_j)]$ and $\sigma^b \equiv (\sigma_i^b, \sigma_j^b)$ is a mixed strategy profile. The bid of firm i with quality q_i that maximizes the agent's payoff is to solve

$$\max_{(p_i, b_i) \in \mathbb{R}_+^2} U_{q_i}(x, p_i, b_i) \text{ s.t. } p_i - b_i - \theta_i \geq 0. \tag{6}$$

Denote by its solution (p_i^*, b_i^*) and value function $U_{q_i}^*(x, \theta_i)$. It is clear that for each solution, the constraint is binding, that is, $p_i = b_i + \theta_i$, from which the above can be rewritten as

$$\max_{b_i \geq 0} U_{q_i}(x, b_i + \theta_i, b_i) = x_{q_i} - x_p(b_i + \theta_i) + w(b_i). \tag{7}$$

Then, if the cost sharing satisfies $x_p \geq w'(0)$, the agent's payoff maximizing bribe solution is $b_i^* = 0$, and if the cost sharing is given as $x_p < w'(0)$, there exists a unique interior solution $b_i^* > 0$ satisfying $-x_p + w'(b_i^*) = 0$, which is denoted by $\widehat{b}(x_p)$, so

$$\widehat{b}(x_p) = \begin{cases} 0 & \text{if} \quad x_p \geq w'(0), \\ w'^{-1}(x_p) & \text{if} \quad x_p < w'(0). \end{cases} \tag{8}$$

Hence, the agent's payoff maximizing bid is

$$(p_i^*(x_p, \theta_i), b_i^*(x_p)) = (\widehat{b}(x_p) + \theta_i, \widehat{b}(x_p)). \tag{9}$$

As in the benchmark, there can be three possible matchings for $(q_i, q_j) \in \{H, L\}^2$:

M1. In this case, the agent's payoff is $x_L - x_p p_i + w(b_i)$ for all i. Since the two low-quality firms engage in a Bertrand-like competition, a unique equilibrium bid is to choose the agent's payoff maximizing bid with $\theta_i = 0$ such as $(p_i^*(x_p, 0), b_i^*(x_p))$ with probability 1.

M2. In this case, the agent's payoff is $x_H - x_p p_i + w(b_i - c)$ for all i. Since the two high-quality firms engage in the competition, a unique equilibrium bid is to choose the agent's payoff maximizing bid with $\theta_i = c$ such as $(p_i^*(x_p, c), b_i^*(x_p))$ with probability 1.

M3. For the above two cases, each firm will also bid a bribe as well as a price in order to beat the other, identical-quality firm, but when two different-quality firms are matched, it is not clear whether paying a bribe is always an *optimal* decision. First, we assume the situation in which both firms bid bribes as well for the analysis below, and later we show that it is indeed optimal for either quality firm to pay a bribe.

When two different-quality firms are matched, in addition to the notations (p_H, p_L), we use (b_H, b_L) instead of (b_i, b_j). In this case, the agent compares the payoff from choosing HP $U_H(x, p_H, b_H)$ with the payoff from choosing LP $U_L(x, p_L, b_L)$ to choose a firm. The most aggressive bid of firm $q \in \{H, L\}$ is to choose $(p_q^*(x_p, \theta_i), b_q^*(x_p))$ given $\theta_q \in [0, \bar{c}]$. It is routine to verify that for $c \in [0, \bar{c}]$ satisfying $U_H^*(x, c) = U_L^*(x, 0)$, where $U_H^*(x, c)$ and $U_L^*(x, 0)$ are the value functions given H and L, a unique equilibrium bid profile is $((p_q^*(x_p, c), b_q^*(x_p)))_{q \in \{H,L\}}$. However, for $c \in [0, \bar{c}]$ satisfying $U_H^*(x, c) \neq U_L^*(x, 0)$, there is no pure strategy Nash equilibrium, and the existence of an equilibrium relies on a mixed strategy profile. Denote

$$A_H(x) \equiv \{c \in [0, \bar{c}] : U_H^*(x, c) > U_L^*(x, 0)\};\ A_L(x) \equiv \{c \in [0, \bar{c}] : U_H^*(x, c) < U_L^*(x, 0)\}.$$

If $c \in A_H(x)$, then there is HP's bid that can make the agent's payoff higher than LP's maximum payoff $U_L^*(x, 0)$, so HP becomes a winning firm, whereas if $c \in A_L(x)$, then there is LP's bid that can make the agent's payoff higher than HP's maximum payoff $U_H^*(x, c)$, so LP becomes a winning firm. In addition, from the agent's payoff maximizing bid in (9), $U_H^*(x,c) > U_L^*(x,0)$ is equivalent to $c < y$, and $U_H^*(x,c) < U_L^*(x,0)$ is equivalent to $c > y$. The formal statement is presented as follows.

Proposition 2 *Suppose that two different-quality firms are matched to bid prices and bribes. Then, for any equilibrium in weakly undominated strategies satisfying the reservation payoff U, if $c < y$, HP bids $p_H(x) = y + \widehat{b}(x_p)$ and wins with probability* 1*, and if $c > y$, LP bids $p_L(x) = c - y + \widehat{b}(x_p)$ and wins with probability* 1*.*

In equilibrium, if HP's cost c is sufficiently low, then there is HP's bid such that $U_H(x, p_H, b_H) = U_L^*(x,0)$, which makes HP a winning firm with probability 1, but if HP's cost c is sufficiently high, then there is LP's bid such that $U_H^*(x,c) = U_L(x, p_L, b_L)$, which makes LP a winning firm with probability 1.

Proposition 2's result was derived provided that both quality firms always bid bribes even if the option of not paying a bribe is available to them. The following Proposition, however, shows that if $x_p < w'(0)$, paying a bribe is optimal for each quality firm, even when either one can win without paying a bribe. If $x_p \geq w'(0)$, the bribe must be 0 in the bid that maximizes the agent payoff in (7), so the competition with bribe bidding simply reduces to the benchmark case without corruption.

Proposition 3 *If $x_p \geq w'(0)$, the analysis reduces to the competition without corruption, and if $x_p < w'(0)$, each quality firm optimally chooses to pay a bribe.*

The results of Propositions 2 and 3 enable us to derive the principal and the agent's payoffs with corruption, in which $\Pi(x)$ and $\Gamma(x)$ are from the principal's payoff (2) and the agent's payoff (3) in the previous section without corruption. If the same-quality firms are matched, compared with the benchmark case, the equilibrium price bid increases by $\widehat{b}(x_p)$ from (9). If two different-quality firms are matched, compared with the benchmark case, the equilibrium price bid increases by $\widehat{b}(x_p)$, regardless of which quality firm wins such that $p_H(x) = y + \widehat{b}(x_p)$ and $p_L(x) = c - y + \widehat{b}(x_p)$, respectively, which is provided in Propositions 2 and 3.

Proposition 4 *With corruption, the principal's payoff is given as* $\Pi^b(x) = \Pi(x) - \widehat{b}(x_p)$*, and the agent's incentive payoff is* $\Gamma^b(x) = \Gamma(x) - x_p\widehat{b}(x_p)$*.*

The principal's net payoff is the difference between the principal's payoff and the agent's incentive payoff. Then, the principal solves the following maximization problem:

$$\begin{cases} \max_{x\in\Omega}[\Pi^b(x) - \Gamma^b(x)] \\ \text{s.t. } \Gamma^b(x) \geq U \text{ (IR)} \end{cases} \tag{P2}$$

Proposition 4 results in the solution of (P2). The optimal value of $y = \Delta x/x_p$ is the same for (P1) and (P2), but only specific values of x_p solve (P2), whereas for each $x_p \in (0, \overline{x}]$, there exists $\Delta x \in \mathbb{R}$ that solves (P1).

Proposition 5 x^* *solves (P2) if and only if* x^* *satisfies* y^* *from (4), and* $x_p^* = \arg\min_{(0,\overline{x}]} \widehat{b}(x_p)$*.*

For any solution to (P2), (IR) must be binding $\Gamma^b(x) = U$, as in (P1), but the optimal cost-sharing rule plays an important role in (P2), whereas any cost-sharing rule satisfying y^* solves (P1). One can note that with corruption, each quality firm's equilibrium bid increases by $\widehat{b}(x_p)$, so the HP's optimal winning probability with bribery is the same as the HP's optimal winning probability without bribery in Proposition 1.

5 Quality-only competition

In this section, we consider the case in which there is no price competition. The two matched firms compete based on quality alone, and let $\overline{p}$ be a fixed "prize," and $x_p = 0$. Since the fixed prize can be lower than a firm's production cost, in particular, HP's cost c, we allow each firm to choose whether or not to participate in the competition at time 3, instead of bidding prices in the timeline from section 3.[17]

[17] This problem does not arise with price competition in which each firm can bid a price greater than its production cost.

Hence, a winning firm obtains a price as the fixed amount, and the principal always pays it to the winner. The principal's expected payoff is v_H for the two high-quality firms matched with probability λ^2, and the principal's payoff is v_L for the two low-quality firms matched with probability $(1-\lambda)^2$. When the two different-quality firms are matched with probability $2\lambda(1-\lambda)$, the agent's sequentially rational strategy is to choose HP with probability 1 if choosing HP yields a higher payoff to the agent, $x_H > x_L$, and HP with probability $\frac{1}{2}$ if it is indifferent, $x_H = x_L$. Denote by $\Pr(H)$ the probability that HP wins, which depends on (x_H, x_L) and $\overline{p}$. Note that HP's payoff must satisfy $\overline{p} - c \geq 0$ to participate in the competition. Hence, the principal's payoff is given as

$$\Pi^q(x_H, x_L, \overline{p}) \equiv \lambda^2 v_H + (1-\lambda)^2 v_L + 2\lambda(1-\lambda)[\Pr(H) v_H + (1-\Pr(H)) v_L] - \overline{p},$$

and similarly, the agent's incentive payoff is

$$\Gamma^q(x_H, x_L, \overline{p}) \equiv \lambda^2 x_H + (1-\lambda)^2 x_L + 2\lambda(1-\lambda)\left[\Pr(H) x_H + (1-\Pr(H)) x_L\right].$$

Then, the principal solves the following maximization problem:

$$\begin{cases} \max_{(x_H, x_L, \overline{p}) \in \mathbb{R}^2 \times \mathbb{R}_+} [\Pi^q(x_H, x_L, \overline{p}) - \Gamma^q(x_H, x_L, \overline{p})] \\ \text{s.t. } \Gamma^q(x_H, x_L, \overline{p}) \geq U \text{ (IR)} \end{cases} \tag{P3}$$

Since for any solution, the (IR) condition must be binding, as in the previous sections, the above problem can be rewritten as

$$\max_{(x_H, x_L, \overline{p}) \in \mathbb{R}^2 \times \mathbb{R}_+} \Pi^q(x_H, x_L, \overline{p}) \text{ s.t. } \Gamma^q(x_H, x_L, \overline{p}) = U.$$

It follows from $v_H > v_L$ that for any fixed $\overline{p}$, the principal chooses the incentives (x_H, x_L) such that HP wins if HP participates, so any solution must satisfy the condition $x_H > x_L$. Hence, solving (P3) boils down to finding an optimal $\overline{p}^*$ that maximizes $2\lambda(1-\lambda)[F(\overline{p}) v_H + (1 - F(\overline{p})) v_L] - \overline{p}$, and we state it in the following lemma.

Lemma 2 *$(x_H^*, x_L^*, \overline{p}^*)$ solves (P3) if and only if $x_H^* > x_L^*$ and $\overline{p}^*$ satisfies*

$$\overline{p}^* \in \arg\max_{\overline{p} \in \mathbb{R}_+} \{2\lambda(1-\lambda)[F(\overline{p}) v_H + (1 - F(\overline{p})) v_L] - \overline{p}\}. \tag{10}$$

Now, we let firms bid bribes for the competition. Firm i's payoff is the prize minus the sum of its bribe bid and the production cost, $\overline{p} - b_i - \theta_i$ for $\theta_i \in [0, \overline{c}]$, if chosen, and the agent's payoff from choosing firm i with quality $q_i \in \{H, L\}$ and bid b_i is the incentive with the expected payoff from receiving a bribe, $x_{q_i} + w(b_i)$. We can use a similar procedure to (6) for the optimal design problem. The bid of firm i with quality q_i that maximizes the agent's payoff is to solve

$$\max_{b_i \in \mathbb{R}_+} [x_{q_i} + w(b_i)] \text{ s.t. } \overline{p} - b_i - \theta_i \geq 0. \quad (11)$$

Then, HP's bid maximizing the agent's payoff is $\overline{p} - c$, and LP's bid maximizing the agent's payoff is $\overline{p}$. As in the case without corruption, given $v_H > v_L$, for any fixed $\overline{p}$, the principal's problem is to choose (x_H, x_L) such that HP wins if HP participates. Hence, the principal's problem with corruption is to find an optimal $\overline{p}$ that maximizes $2\lambda(1-\lambda)[F(\overline{p}) v_H + (1 - F(\overline{p})) v_L] - \overline{p}$ as above. However, with bribery, we need to provide a *stronger* condition for (x_H, x_L) to make HP win once it participates. For each $c < \overline{p}^*$, (x_H^*, x_L^*) must satisfy $x_H^* + w(\overline{p}^* - c) > x_L^* + w(\overline{p}^*)$. This condition holds if and only if $x_H^* + w(\overline{p}^* - \overline{p}^*) \geq x_L^* + w(\overline{p}^*)$, which can be summarized as below.

Proposition 6 *Suppose that firms bid bribes for the competition. Then $(x_H^*, x_L^*, \overline{p}^*)$ solves (P3) with bribery if and only if $x_H^* - x_L^* \geq w(\overline{p}^*)$ and $\overline{p}^*$ satisfies (10).*

The result establishes that when firms compete on quality alone, regardless of whether they also bid bribes, each solution must satisfy that HP wins given that it participates, and the fixed prize is $\overline{p}^*$ maximizing (10). However, without corruption, any $x_H^* > x_L^*$ yields a participating HP's winning, whereas with corruption, the condition changes to $x_H^* - x_L^* \geq w(\overline{p}^*)$ since the principal must provide a higher incentive to the agent with bribery.

6 Institutional choice

We provide the two key results on the institutional choice of this paper by comparing the principal's optimal payoffs given different institutions. Denote the principal's net payoff $\Delta v \equiv v_H - v_L$, and define

$$\begin{aligned} M(y, \Delta v) &\equiv \left\{ \int_0^y [v_H - y] dF(c) + \int_y^{\overline{c}} [v_L + y - c] dF(c) \right\} \\ &= (\Delta v - y) F(y) + \int_y^{\overline{c}} F(c)\, dc + v_L + y - \overline{c}, \\ N(\overline{p}, \Delta v) &\equiv \{ 2\lambda(1-\lambda)[F(\overline{p}) v_H + (1 - F(\overline{p})) v_L] - \overline{p} \} \\ &= 2\lambda(1-\lambda)[\Delta v F(\overline{p}) + v_L] - \overline{p}. \end{aligned}$$

In addition, denote by their maximums $M^*(\Delta v) \equiv \max_{y \in [0, \overline{c}]} M(y, \Delta v)$ and $N^*(\Delta v) \equiv \max_{\overline{p} \in \mathbb{R}_+} N(\overline{p}, \Delta v)$, respectively. Then, from the result of Proposition 1, if the principal chooses competition based on quality and price without bribery, the maximum payoff is

$$\Phi(Q, P) \equiv \lambda^2 (v_H - \mathbb{E}c) + (1-\lambda)^2 v_L + 2\lambda(1-\lambda) M^*(\Delta v) - U. \tag{12}$$

Proposition 5 implies that if the principal chooses quality-price competition with corruption, the maximum payoff is

$$\Phi(Q, P|B) \equiv \lambda^2 (v_H - \mathbb{E}c) + (1-\lambda)^2 v_L + 2\lambda(1-\lambda) M^*(\Delta v) - \min_{x_p \in (0, \overline{x}]} \widehat{b}(x_p) - U. \tag{13}$$

Last, by Proposition 6, if the principal chooses quality-only competition, regardless of the existence of corruption, the maximum payoff is

$$\Phi(Q) = \Phi(Q|B) \equiv \lambda^2 v_H + (1-\lambda)^2 v_L + N^*(\Delta v) - U. \tag{14}$$

The four cases can be summarized in the following table:

	Quality Only	Quality and Price
No Corruption	$\Phi(Q)$	$\Phi(Q, P)$
Corruption	$\Phi(Q\|B)$	$\Phi(Q, P\|B)$

(15)

Given each row in the above table, corruption or no corruption, we can show which institution is superior, and given each column, quality-only or quality-price competition, we can examine corruption's effect on the principal's payoff.

The first main result of the institutional choice establishes the comparison between corruption and no corruption given each type of competition, and the result immediately follows from (12)-(14).

Proposition 7 *(i)* $\Phi(Q) = \Phi(Q|B)$.

(ii) $\Phi(Q,P) \geq \Phi(Q,P|B)$ *(= only if* $\min_{x_p \in (0,\overline{x}]} \widehat{b}(x_p) = 0$*).*

In quality-only competition, corruption does no harm to the principal, whereas in quality-price competition, corruption negatively affects the principal. In the latter, the objective of the principal is to buy a better-quality good at a lower price. However, with corruption, the money flow from the principal to the corrupt agent through a winning firm creates an increase in the "buying price." On the other hand, in the former, since there is no price competition, the "prize" is fixed, so corruption does not affect the principal as long as the high-quality firm is picked.

With no corruption, the comparison between quality-price and quality-only competition is based on values of Δv given a fixed v_L as follows:

$$\Phi(Q,P) - \Phi(Q) = -\lambda^2 \mathbb{E}c + 2\lambda(1-\lambda)M^*(\Delta v) - N^*(\Delta v).$$

Now, we report that with no corruption, quality-price competition is a superior institutional setting when Δv is sufficiently large, but quality-only is a superior institutional setting when Δv is sufficiently small.

Proposition 8 *For a sufficiently large* $\Delta v > 0$, $\overline{p}^* = \overline{c}$ *and* $\Phi(Q,P) > \Phi(Q)$*, but for a sufficiently small* $\Delta v > 0$, $\overline{p}^* = 0$ *and* $\Phi(Q,P) < \Phi(Q)$.

The second main result of the institutional choice follows from the existence of Δv which makes quality-price competition a superior institution with no corruption in Proposition 7, and shows that the *opposite case* is possible given (13) and (14). If quality-only is a superior institution with no corruption, then the result is trivial; quality-only is a superior institution with corruption as well.

Proposition 9 *(i) If $\Phi(Q) < \Phi(Q, P)$, then $\Phi(Q|B) > \Phi(Q, P|B)$ for a sufficiently large* $\min_{x_p \in (0,\overline{x}]} \widehat{b}(x_p)$.

(ii) If $\Phi(Q) \geq \Phi(Q, P)$, then $\Phi(Q|B) \geq \Phi(Q, P|B)$.

With no corruption, quality-price competition is a superior institutional setting for the principal compared with quality-only when the principal's net benefit is sufficiently large, but with corruption, introducing price competition can lead to a worse outcome for the principal given the high price distortion involved.

7 Examples and discussion

Proposition 8 implies that there exists principal's net payoff $\Delta v > 0$ that makes quality-price competition indifferent to quality-only, $\Phi(Q, P) = \Phi(Q)$. Consider a uniform distribution F with support $[0, 1]$. Then, $M(y, \Delta v) = -\frac{3}{2}y^2 + (\Delta v + 1) y + v_L - \frac{1}{2}$ for $y \in [0, 1]$ and $N(\overline{p}, \Delta v) = 2\lambda(1-\lambda)[\Delta v \overline{p} + v_L] - \overline{p}$ for $\overline{p} \in [0, 1]$, which yields the solutions as below:

$$y^* = \begin{cases} \frac{\Delta v + 1}{3} & \text{if} \quad \Delta v \leq 2, \\ 1 & \text{if} \quad \Delta v \geq 2, \end{cases} \quad \text{and } \overline{p}^* = \begin{cases} 0 & \text{if} \quad \Delta v < \frac{1}{2\lambda(1-\lambda)}, \\ [0, 1] & \text{if} \quad \Delta v = \frac{1}{2\lambda(1-\lambda)}, \\ 1 & \text{if} \quad \Delta v > \frac{1}{2\lambda(1-\lambda)}. \end{cases}$$

It follows from Lemma 1 and Proposition 1 that HP wins with probability 1 when its production cost c is smaller than y^* in quality-price competition without corruption, and from Lemma 2, HP wins with probability 1 when its production cost c is smaller than $\overline{p}^*$ in quality-only without corruption. Note that since $\frac{1}{2\lambda(1-\lambda)} \geq 2$, we have $y^* > \overline{p}^*$ for $\Delta v < \frac{1}{2\lambda(1-\lambda)}$ and $y^* \geq \overline{p}^*$ for $\Delta v \geq \frac{1}{2\lambda(1-\lambda)}$. This shows that HP's winning probability is higher in quality-price competition than in quality-only for each $\Delta v < \frac{1}{2\lambda(1-\lambda)}$. Figure 1 describes the two optimal compensation schemes.

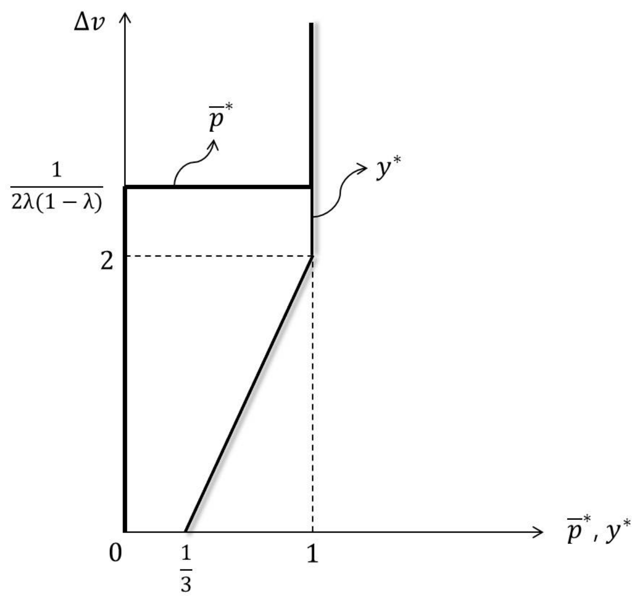

Figure 1: Optimal shemes for the uniform case

Next, we find their corresponding maximum payoffs as

$$M^*(\Delta v) = \begin{cases} \frac{(\Delta v+1)^2}{6} + v_L - \frac{1}{2} & \text{if} \quad \Delta v \leq 2, \\ \Delta v + v_L - 1 & \text{if} \quad \Delta v \geq 2, \end{cases} \quad \text{and}$$

$$N^*(\Delta v) = \begin{cases} 2\lambda(1-\lambda)v_L & \text{if} \quad \Delta v \leq \frac{1}{2\lambda(1-\lambda)}, \\ 2\lambda(1-\lambda)(\Delta v + v_L) - 1 & \text{if} \quad \Delta v > \frac{1}{2\lambda(1-\lambda)}. \end{cases}$$

If $\Delta v > \frac{1}{2\lambda(1-\lambda)}$, then $\overline{p}^* = 1$. Hence, Proposition 8 implies that quality-price competition is better than quality-only, $\Phi(Q,P) > \Phi(Q)$ for $\Delta v > \frac{1}{2\lambda(1-\lambda)}$, and quality-only competition is better than quality-price, $\Phi(Q,P) < \Phi(Q)$ for a sufficiently small $\Delta v > 0$. In addition, $M^*(\Delta v)$ is a strictly increasing function of Δv for $\Delta v \leq \frac{1}{2\lambda(1-\lambda)}$, there exists a *unique* $\Delta\widehat{v} > 0$ that makes $\Phi(Q,P) = \Phi(Q)$ such that for all $\Delta v > \Delta\widehat{v}$, $\Phi(Q,P) > \Phi(Q)$, and for all $\Delta v < \Delta\widehat{v}$, $\Phi(Q,P) < \Phi(Q)$.

However, given a general CDF F, with no further restrictions, it cannot be guaranteed whether there exists such a unique threshold level $\Delta\widehat{v} > 0$ that yields $\Phi(Q,P) = \Phi(Q)$. We introduce the following conditions for $M(y,\Delta v)$ and $N(\overline{p},\Delta v)$, and show that there exists a unique threshold level $\Delta\widehat{v} > 0$ given a general CDF F.

Lemma 3 *Suppose $M(y, \Delta v)$ and $N(\overline{p}, \Delta v)$ are strictly concave functions of y and $\overline{p}$, respectively. If $xf(x) \leq 1$ for all $x \in [0, \overline{c}]$, then $y^* > \overline{p}^*$ for $\Delta v < \frac{1}{2\lambda(1-\lambda)f(\overline{c})}$ and $y^* = \overline{p}^* = \overline{c}$ for $\Delta v \geq \frac{1}{2\lambda(1-\lambda)f(\overline{c})}$.*

Consider an exponential distribution F with support $[0, 1]$. Then, the condition $xf(x) \leq 1$ is satisfied for all $x \in [0, 1]$.[18] Now, one can find $y^* > \overline{p}^*$ for $y^*, \overline{p}^* \in [0, \overline{c}]$ if and only if

$$\frac{d[\Phi(Q, P) - \Phi(Q)]}{d\Delta v} = 2\lambda(1-\lambda)[F(y^*) - F(\overline{p}^*)] > 0,$$

which in turn entails that HP's winning probability is higher in quality-price competition, $y^* > \overline{p}^*$ for $y^*, \overline{p}^* \in [0, \overline{c}]$ if and only if $\Phi(Q, P) - \Phi(Q)$ is a strictly increasing function of Δv. This implies the existence of a unique threshold level $\Delta\widehat{v} > 0$ for $\Phi(Q, P) = \Phi(Q)$.

8 Concluding remarks

This paper models two firms who compete to sell a product based on quality, price and bribe, and examines how a simple institutional design can influence an agent's corrupt behavior.

We showed that the consequence of quality-only competition can be quite different from that of quality-price competition depending on the presence of corruption. Thus a designer for an allocation problem must be cautious when incorporating price competition into resource-allocation mechanisms.

It is often reported that countries show different levels of corruption (see Mauro (1995), Ades and Di Tella (1997), Ades and Di Tella (1999) and Treisman (2000) among others). In underdeveloped countries, there are few legitimate or legal active markets, so it happens that bribe systems replace market systems. This paper shows

[18] $F(c) = \frac{1-e^{-\alpha c}}{1-e^{-\alpha}}$ for $c \in [0, 1]$ and a parameter $0 < \alpha \leq 1$. We have $f(c) = \frac{\alpha e^{-\alpha c}}{1-e^{-\alpha}}$ and $cf(c) = \frac{\alpha c e^{-\alpha c}}{1-e^{-\alpha}}$. Given $\alpha \leq 1$, $cf(c)$ has a unique maximum at $c^* = 1$, and $f(1) - 1 = \frac{\alpha e^{-\alpha}}{1-e^{-\alpha}} - 1 = \frac{\alpha e^{-\alpha} - 1 + e^{-\alpha}}{1-e^{-\alpha}} \leq 0$, where $\alpha e^{-\alpha} + e^{-\alpha} \leq 1$ can be rewritten as $1 + \alpha \leq e^{\alpha}$ for any $\alpha \in (0, 1]$.

that with corruption, introducing price competition can lead to adverse effects on the principal's payoff.

The penalty for getting caught by the government is not considered in the model, because that will affect both a high-quality firm and a low-quality firm.

Appendix I: Proofs

Proof of Lemma 1. **Step 1**. Consider any undominated Nash equilibrium with strategies (σ_H, σ_L). For $q \in \{H, L\}$, denote $\overline{p}_q \equiv \sup[Supp[\sigma_q]]$ and $\underline{p}_q \equiv \inf[Supp[\sigma_q]]$. Each firm's payoff is $p_q - \theta_q$ when it wins, and it is 0 when it loses, given $\theta_q \in [0, \overline{c}]$. Hence, it is weakly dominated for firm q to bid any price $p_q \leq \theta_q$ since there exists $p'_q > \theta_q$ such that firm q's payoff can be positive for some bids by the other firm. Hence, $\underline{p}_q \geq \theta_q$ and σ_q must assign zero probability to θ_q, so $\overline{p}_q > \theta_q$.

Step 2. We only show the proof for $c < y$, and one can apply similar steps to the case with $c > y$. Since $x_H - x_p p_H > x_L - x_p p_L \Leftrightarrow p_L > p_H - y$, given LP's weakly undominated strategy (from step 1, $\underline{p}_L \geq 0$ and σ_L must assign zero probability to 0, so $\overline{p}_L > 0$), $p_H < y$ is never a best response for HP, so $\underline{p}_H \geq y$. It remains to show that $\overline{p}_H = y$. Suppose $\overline{p}_H > y$. We first show $\overline{p}_L = \overline{p}_H - y$. Consider $\overline{p}_L \neq \overline{p}_H - y$ and WLOG, let $\overline{p}_L > \overline{p}_H - y$. Then, for $p_L > \overline{p}_H - y$, LP's expected payoff is 0, but by switching the probability on $p_L > \overline{p}_H - y$ to $p_L < \overline{p}_H - y$, LP obtains a positive expected payoff. Hence, we must have $\overline{p}_L = \overline{p}_H - y$. Note that $\overline{p}_q > \theta_q$ yields $\overline{p}_q - \theta_q > 0$ for $q \in \{H, L\}$ and to satisfy the reservation payoff U, $\overline{p}_L$ and $\overline{p}_H$ must be bounded. We consider two cases: (i) Suppose that σ_L assigns positive probability to $\overline{p}_L$. Then, given $\overline{p}_L$, LP must obtain a positive expected payoff, which implies that σ_H must also assign positive probability to $\overline{p}_H$. Then, the tie-breaking rule entails that either firm does not play a best response. (ii) Suppose that σ_L assigns zero probability to $\overline{p}_L$. Then, $\Pr^{\sigma_L}(p_L \geq \overline{p}_L - \varepsilon) \to 0$ as $\varepsilon \to 0$, which implies that HP's payoff becomes arbitrarily small as $p_H \to \overline{p}_H$, but by choosing $p_H = y$, HP obtains a positive expected payoff, so σ_H is not a best response for HP.

Step 3. Furthermore, there exists an undominated Nash equilibrium such that

HP bids y with probability 1, and LP mixes with a bid drawn from a uniform distribution on $[y, y+\delta]$ for a sufficiently small $\delta > 0$. ■

Proof of Proposition 1. First, the individual rationality condition (IR) is binding for any solution to (P1). Then, we solve

$$\begin{cases} \max_{x\in\Omega} \left\{\int_0^y [v_H - y] dF(c) + \int_y^{\bar{c}} [v_L + y - c] dF(c)\right\} \\ \text{s.t. } \Gamma(x) = U \text{ (IR)} \end{cases}$$

In addition, for each $y \in \mathbb{R}_+$, $x_H = x_p y + x_L$ as defined in (1), so for each $y \in \mathbb{R}_+$, there exists $x \in \Omega$ such that $\Gamma(x) = U$ as follows:

$$\lambda^2 (x_p y + x_L - x_p \mathbb{E}c) + (1-\lambda)^2 x_L + 2\lambda(1-\lambda) \left\{ \int_0^y x_L dF(c) + \int_y^{\bar{c}} [x_p y + x_L - x_p c] dF(c) \right\} = U.$$

Hence, the above maximization problem can be simply replaced by

$$\max_{y\in\mathbb{R}_+} \left\{ \int_0^y [v_H - y] dF(c) + \int_y^{\bar{c}} [v_L + y - c] dF(c) \right\}.$$

Any $x \in \Omega$ such that $y > \bar{c}$ cannot be a solution since its maximum value is given as $v_H - y$, but there exists $x' \in \Omega$ such that x' yields y' with $y' < y$ and $\Gamma(x') = U$. This enables us to restrict our attention to x satisfying $y \in [0, \bar{c}]$. There exists a solution x^* to (P1) such that $\Gamma(x^*) = U$ and y^* where

$$y^* \in \arg\max_{y\in[0,\bar{c}]} \left\{ \int_0^y [v_H - y] dF(c) + \int_y^{\bar{c}} [v_L + y - c] dF(c) \right\}.$$

■

Proof of Proposition 2. Since $U_H^*(x, c) = x_H - x_p(\widehat{b}(x_p) + c) + w(\widehat{b}(x_p))$, and $U_L^*(x, 0) = x_L - x_p \widehat{b}(x_p) + w(\widehat{b}(x_p))$, we have $A_H(x) = \{x \in \Omega : c > y\}$ and $A_L(x) = \{x \in \Omega : c < y\}$.

Step 1. Show that for each $b'_q \neq \widehat{b}(x_p)$, (p'_q, b'_q) is weakly dominated. Consider

$$\max_{(p_q,b_q)\in\mathbb{R}_+^2} [p_q - b_q] \text{ s.t. } U_q(x, p_q, b_q) = U_q(x, p'_q, b'_q), \tag{16}$$

which can be rewritten as

$$\max_{b_q\geq 0} \left[\frac{x_q + w(b_q) - U_q(x, p'_q, b'_q)}{x_p} - b_q \right]. \tag{17}$$

Since we pick (p_q, b_q) such that $U_q(x, p_q, b_q) = U_q(x, p'_q, b'_q)$ in (16), the probability that firm q wins (firm q loses) given (p_q, b_q) is the same as the probability given (p'_q, b'_q), and furthermore, firm q wins with positive probability for some bids by the other firm $\widetilde{q}$ with $\widetilde{q} \neq q$. A unique solution to the problem (17) is the solution $\widehat{b}(x_p)$ from (8), which implies that for each $b'_q \neq \widehat{b}(x_p)$, (p'_q, b'_q) is weakly dominated.

Step 2. Find the winning bids. From step 1, we can only examine p for a fixed $\widehat{b}(x_p)$, and the similar proof of Lemma 1 can be applied. By substituting $\widehat{b}(x_p)$ into the constraint below:

$$U_q(x, p_q, b_q) = U^*_{\widetilde{q}}(x, \theta_i) \text{ for } \widetilde{q} \neq q, \tag{18}$$

we can find each firm's winning bid

$$p_H(x) = y + \widehat{b}(x_p);\ p_L(x) = c - y + \widehat{b}(x_p),$$

where y is defined as $y = \Delta x / x_p$ in (1). ■

Proof of Proposition 3. If $x_p \geq w'(0)$, the result follows from (8). Let $x_p < w'(0)$ and we divide the proof for HP into two cases. The same argument applies to the case in which LP wins, so the proof for LP is omitted.

Case 1. LP chooses to pay a bribe, and HP can win even without paying a bribe. If HP does not pay a bribe, HP obtains p'_H from $x_H - x_p p_H = U^*_L(x, 0)$ such that

$$p'_H = \frac{\Delta x + x_p \widehat{b}(x_p) - w(\widehat{b}(x_p))}{x_p} = y - \frac{-x_p \widehat{b}(x_p) + w(\widehat{b}(x_p))}{x_p}.$$

If HP pays a bribe, then HP wins and obtains $p_H(x) - \widehat{b}(x_p) = y$ from Proposition 2. For $x_p < w'(0)$, $-x_p \widehat{b}(x_p) + w(\widehat{b}(x_p)) > 0$, so $[p_H(x) - \widehat{b}(x_p)] - p'_H > 0$, and paying a bribe is optimal.

Case 2. LP chooses not to pay a bribe, and HP can win even without paying a bribe.

If HP does not pay a bribe, HP obtains y from $x_H - x_p p_H = x_L$. If HP pays a bribe, then HP wins and obtains $p^\dagger_H$ such that

$$p^\dagger_H = \frac{\Delta x - w(\widehat{b}(x_p))}{x_p}, \text{ so } p^\dagger_H - \widehat{b}(x_p) = y - \frac{-x_p \widehat{b}(x_p) + w(\widehat{b}(x_p))}{x_p}.$$

Note that $p_H^{\dagger}$ is derived from Proposition 2 where $U_L^*(x, \theta_i)$ in (18) changes to x_L. For $x_p < w'(0)$, $-x_p\widehat{b}(x_p) + w(\widehat{b}(x_p)) > 0$, so $[p_H^{\dagger} - \widehat{b}(x_p)] - y > 0$, and paying a bribe is optimal. ■

Proof of Proposition 5. The individual rationality condition (IR) is binding for any solution to (P1). Then, we solve

$$\begin{cases} \max_{x \in \Omega} \left\{ \int_0^y [v_H - y] dF(c) + \int_y^{\overline{c}} [v_L + y - c] dF(c) - \widehat{b}(x_p) \right\} \\ \text{s.t. } \Gamma(x) - x_p \widehat{b}(x_p) = U \text{ (IR)} \end{cases}$$

As in Proposition 1, for each $y \in \mathbb{R}_+$, $x_H = x_p y + x_L$, so for each $y \in \mathbb{R}_+$, there exists $x \in \Omega$ such that $\Gamma(x) - x_p \widehat{b}(x_p) = U$. First, from Proposition 1, y^* maximizes $\left\{ \int_0^y [v_H - y] dF(c) + \int_y^{\overline{c}} [v_L + y - c] dF(c) \right\}$. There exists $x_p^* \in (0, \overline{x}]$ that minimizes $\widehat{b}(x_p)$ since as $x_p \to 0$, $\widehat{b}(x_p) \to +\infty$. Hence, y^* and x_p^* maximizes:

$$\left\{ \int_0^y [v_H - y] dF(c) + \int_y^{\overline{c}} [v_L + y - c] dF(c) - \widehat{b}(x_p) \right\},$$

and we can choose $(x_H^* - x_L^*)$ such that $y^* = \Delta x^* / x_p^* = (x_H^* - x_L^*) / x_p^*$. ■

Proof of Proposition 8. If Δv is sufficiently large, $\overline{p}^* = \overline{c}$. Since $M^*(\Delta v) \geq (v_H - \overline{c})$ and $N^*(\Delta v) = 2\lambda(1-\lambda)v_H - \overline{c}$,

$$\begin{aligned} \Phi(Q, P) - \Phi(Q) &\geq -\lambda^2 \mathbb{E}c + 2\lambda(1-\lambda)(v_H - \overline{c}) - 2\lambda(1-\lambda)v_H + \overline{c} \\ &= -\lambda^2 \mathbb{E}c - 2\lambda(1-\lambda)\overline{c} + \overline{c} \\ &= (2\overline{c} - \mathbb{E}c)\lambda^2 - 2\overline{c}\lambda + \overline{c} > 0 \text{ for all } \lambda \in (0,1), \end{aligned}$$

where the strict inequality follows from $(2\overline{c} - \mathbb{E}c) > 0$ and

$$4\overline{c}^2 - 4\overline{c}(2\overline{c} - \mathbb{E}c) = -4\overline{c}^2 + 4\overline{c}\mathbb{E}c = 4\overline{c}(\mathbb{E}c - \overline{c}) < 0.$$

If Δv is sufficiently small, $\overline{p}^* = 0$. Since $M^*(\Delta v) = (\Delta v - y^*) F(y^*) + \int_{y^*}^{\overline{c}} F(c)\, dc + v_L + y^* - \overline{c}$ and $N^*(\Delta v) = 2\lambda(1-\lambda)v_L$,

$$\Phi(Q, P) - \Phi(Q) = 2\lambda(1-\lambda) \left[(\Delta v - y^*) F(y^*) + \int_{y^*}^{\overline{c}} F(c)\, dc + y^* - \overline{c} \right] - \lambda^2 \mathbb{E}c$$

Note that

$$\left[-y^{*}F\left(y^{*}\right)+\int_{y^{*}}^{\overline{c}}F\left(c\right)dc+y^{*}-\overline{c}\right]<0 \text{ for any } y^{*}\in\left[0,\overline{c}\right].$$

Hence, $\Phi\left(Q,P\right)-\Phi\left(Q\right)<0$ for a sufficiently small Δv. ■

Proof of Lemma 3. $M\left(y,\Delta v\right)$ and $N\left(\overline{p},\Delta v\right)$ have the solutions given as:

$$y^{*}=\begin{cases}\widehat{y} & \text{if} \quad \Delta v\leq\frac{1}{f(\overline{c})}+\overline{c},\\ \overline{c} & \text{if} \quad \Delta v\geq\frac{1}{f(\overline{c})}+\overline{c}.\end{cases} \quad \text{and } \overline{p}^{*}=\begin{cases}0 & \text{if} \quad \Delta v\leq\frac{1}{2\lambda(1-\lambda)f(0)},\\ \widehat{p} & \text{if} \quad \Delta v\in[\frac{1}{2\lambda(1-\lambda)f(0)},\frac{1}{2\lambda(1-\lambda)f(\overline{c})}],\\ \overline{c} & \text{if} \quad \Delta v\geq\frac{1}{2\lambda(1-\lambda)f(\overline{c})}.\end{cases}$$

We divide the case into two.

Case 1. $\frac{1}{f(\overline{c})}+\overline{c}\leq\frac{1}{2\lambda(1-\lambda)f(0)}$

Then, it is clear that $y^{*}>\overline{p}^{*}$ for $\Delta v<\frac{1}{2\lambda(1-\lambda)f(\overline{c})}$ and $y^{*}=\overline{p}^{*}=\overline{c}$ for $\Delta v\geq\frac{1}{2\lambda(1-\lambda)f(\overline{c})}$

Case 2. $\frac{1}{f(\overline{c})}+\overline{c}>\frac{1}{2\lambda(1-\lambda)f(0)}$

(i) $\Delta v\leq\frac{1}{2\lambda(1-\lambda)f(0)}$: Since $y^{*}=\widehat{y}>0$ and $\overline{p}^{*}=0$, $y^{*}>\overline{p}^{*}$.

(ii) $\Delta v\in(\frac{1}{2\lambda(1-\lambda)f(0)},\frac{1}{f(\overline{c})}+\overline{c})$: $\widehat{y}$ and $\widehat{p}$ satisfy their first order conditions:

$$\left(\Delta v-\widehat{y}\right)f\left(\widehat{y}\right)+1-2F\left(\widehat{y}\right)=0;\ 2\lambda(1-\lambda)\Delta vf\left(\widehat{p}\right)-1=0.$$

From the first or der condition of $N\left(\overline{p},\Delta v\right)$,

$$\Delta vf\left(\widehat{p}\right)=\frac{1}{2\lambda(1-\lambda)}\geq 2.$$

Consider the first order condition of $M\left(y,\Delta v\right)$ given $\widehat{p}$:

$$\begin{aligned}\left(\Delta v-\widehat{p}\right)f\left(\widehat{p}\right)+1-2F\left(\widehat{p}\right) &= \frac{1}{2\lambda(1-\lambda)}-\widehat{p}f\left(\widehat{p}\right)+1-2F\left(\widehat{p}\right)\\ &\geq 2-\widehat{p}f\left(\widehat{p}\right)+1-2F\left(\widehat{p}\right)\\ &= 1-\widehat{p}f\left(\widehat{p}\right)+2\left[1-F\left(\widehat{p}\right)\right]>0.\end{aligned}$$

Since $M\left(y,\Delta v\right)$ is a strictly concave function of y, we have $\widehat{y}>\widehat{p}$.

(iii) $\Delta v\in[\frac{1}{f(\overline{c})}+\overline{c},\frac{1}{2\lambda(1-\lambda)f(\overline{c})})$: Since $y^{*}=\overline{c}$ and $\overline{p}^{*}<\overline{c}$, $y^{*}>\overline{p}^{*}$.

(iv) $\Delta v\geq\frac{1}{2\lambda(1-\lambda)f(\overline{c})}$: Note $y^{*}=\overline{p}^{*}=\overline{c}$. ■

Appendix II: Non-existence problem

Let the agent's cost sharing be $z_S p$ for success, and $z_F p$ for failure, where $p \in \mathbb{R}_+$ is the purchase price of the product. We assume that $z_S, z_F \in (0, \overline{z}]$ where $\overline{z} \leq 1$. The agent compares $x_H - z_H p_H$ with $x_L - z_L p_L$ to choose a firm. Denote $\Delta x \equiv x_H - x_L$ and

$$y \equiv \frac{\Delta x}{z_H}.$$

Then, if $x_H - z_H c > x_L$ or $c < y$, there exists $p_H > c$ such that $x_H - z_H p_H = x_L$. If HP's cost c is sufficiently low, then there is HP's bid that ties with LP's most aggressive bid 0. Similarly, if $x_H - z_H c < x_L$ or $c > y$, there exists $p_L > 0$ such that $x_H - z_H c = x_L - z_L p_L$, i.e.,

$$p_L = \frac{x_L - x_H + z_H c}{z_L} = \frac{z_H}{z_L}\left(c - \frac{x_H - x_L}{z_H}\right) = \frac{z_H}{z_L}\left(c - y\right).$$

Now, if HP's cost c is sufficiently high, then there is LP's bid that ties with HP's most aggressive bid c. For $c = y$, it is straightforward to find that a unique equilibrium bid profile is $(p_H^*, p_L^*) = (c, 0)$. However, for $c \neq y$, there is no pure strategy Nash equilibrium, and the existence of an equilibrium relies on a mixed strategy profile. By restricting solutions to weakly undominated strategies, we can have a unique equilibrium outcome such that for any equilibrium in weakly undominated strategies satisfying the reservation payoff U, if $c < y$, HP bids y and wins with probability 1, and if $c > y$, LP bids $\frac{z_H}{z_L}\left(c - y\right)$ and wins with probability 1.

Then, the principal's gross payoff is given as

$$\Pi\left(x, z\right) \equiv \lambda^2(v_H - \mathbb{E}c) + \left(1 - \lambda\right)^2 v_L + 2\lambda(1-\lambda)\left\{\int_0^y [v_H - y]dF(c) + \int_y^{\overline{c}} [v_L - \frac{z_H}{z_L}\left(c - y\right)]dF(c)\right\},$$

and similarly, the agent's incentive payoff is

$$\Gamma\left(x, z\right) \equiv \lambda^2(x_H - z_H \mathbb{E}c) + \left(1 - \lambda\right)^2 x_L + 2\lambda(1-\lambda)\left\{\int_0^y x_L dF(c) + \int_y^{\overline{c}} [x_H - z_H c]dF(c)\right\},$$

and the third term is derived from $\int_0^y [x_H - z_H y]dF(c) + \int_y^{\overline{c}} [x_L - z_L \frac{z_H}{z_L}\left(c - y\right))]dF(c)$.

The principal's net payoff is the difference between the principal's payoff and the agent's incentive payoff. Then, the principal solves the following maximization problem:

$$\begin{cases} \max_{(x,z)\in\Omega}[\Pi(x,z)-\Gamma(x,z)] \\ \text{s.t. } \Gamma(x,z)\geq U \text{ (IR)} \end{cases}$$

First, the individual rationality condition (IR) is binding for any solution to (P1). Then, we solve

$$\begin{cases} \max_{(x,z)\in\Omega}\left\{\int_0^y [v_H-y]dF(c)+\int_y^{\overline{c}}[v_L-\frac{z_H}{z_L}(c-y)]dF(c)\right\} \\ \text{s.t. } \Gamma(x,z)=U \text{ (IR)} \end{cases}$$

Since z_L disappears in the constraint $\Gamma(x,z)$, for any solution, $z_L=\overline{z}$. Suppose there exists a solution with Δx^*, z_H^* and $y^*=\Delta x^*/z_H^*$. Choose $z_H'\in(0,z_H^*)$ and $\Delta x'$ such that $y^*=\Delta x'/\,z_H'$. Then, the principal's net payoff strictly increases while (IR) is still satisfied.

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