Inequality and Optimal Monetary Policy*

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Abstract

In this paper, I investigate the effect of income inequality on optimal monetary policy design. To this end, I introduce heterogeneity by incorporating segmented labor markets with a Limited Asset Market Participation (LAMP) into the standard New Keynesian DSGE model used in Erceg et al. (2000). Because of the difference in real rigidity across sectors, an economic shock, especially a monetary policy shock, causes a variation in the wage premium. This variation in wage premium encourages firms to substitute workers across sectors, which induces stickier aggregate nominal wage and more volatile macroeconomic variables. At the same time, however, a change in wage premium leads to a greater change in the employment gap across sectors (given a plausible value of elasticity of substitution), and thus income inequality is negatively related to the wage premium. In addition, income inequality poses a trade-off with traditional policy objectives such as the output gap even under flexible wages. Therefore, it is desirable to include inequality as a separate goal for a Central Bank. Welfare analysis of various scenarios shows that when a Central Bank ignores inequality, focusing only on aggregate variables, there are significant welfare losses.

Keywords: Inequality, Optimal monetary policy, Limited asset market participation, Sticky wages

JEL Classifications: E24, E32, E58, E61

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"*Because monetary policy is transmitted through many channels, direct and indirect and because households differ in many respects (with regard to socio-demographic factors, such as age and education, as well as economic variables, such as income, wealth, employment status and housing status) monetary policy does not affect all households in the same way. ...it is not only the extent of income and wealth shocks that affects consumers welfare, but also the fluctuation in their consumption expenditure. All households are not equal in this respect. Some of them are able to insure against wealth shocks and can thus mitigate the adverse consequences of such shocks for their well-being. But poorer households have limited or no access to the financial system (let alone to financial markets) and do not have adequate buffers in the form of precautionary savings. Consequently, their consumption and welfare are particularly vulnerable to adverse shocks. Even if all households were hit by negative shocks to the same extent, poorer, less-insured households would suffer from more volatile consumption and lower welfare.*"

Benoît Coeuré in "What can monetary policy do about inequality?" Oct. 17. 2012

1 Introduction

Since the recent financial crisis, inequality has become a hotly debated issue once again not only for economists but also for the public at large. Data indicate that earning inequality has grown rapidly over the past three decades and it tends to become even more pronounced in a recession.[1] This upward trending and counter-cyclical inequality has already been recognized and studied by many economists. The empirical and theoretical literature have shown that inequality is a cause and a consequence of macroeconomic volatility at the same time.[2]

The literature, however, has mainly focused on the relationship between inequality and macroeconomic volatility itself rather than discussing policy implications. Even though some papers consider policy measures, they focuses either on fiscal policy or on the level of inequality, rather than the dynamics of inequality.[3] However, it is well known that monetary policy has a dispropor-

[1]See Krueger et al. (2010) and Heathcote et al. (2010)

[2]See Breen and Garca-Pealosa (2005), Fitoussi and Saraceno (2010), Ghiglino and Venditti (2011), Stiglitz (2012), Kumhof et al. (2013), and Dosi et al. (2013)

[3]For example, in Aghion et al. (1999) and Dosi et al. (2013), the authors argue that counter-cyclical fiscal policy

tionate effect and thus it induces variations in inequality. Therefore, inequality variations caused by monetary policy might matter at a business cycle frequency as well. If monetary policy leads to a variation in inequality, and the change in inequality amplifies macroeconomic volatility, central bankers who are seeking economic stability may need to take into account such inequality variations. In fact, many papers show that a contractionary monetary policy shock raises income inequality as I discuss later in section 2. Yet, in spite of such clear evidence of monetary policy effect, researchers have not reached a consensus as to the appropriate policy response to this inequality; In some studies, such as Romer and Romer (1999) and Haltom (2012), the authors assert that monetary policy focusing on price stability tends to minimize the re-distributional effects; On the other hand, Stiglitz (2012) among others points out that policy makers need to pay serious attention to the inequality when they construct monetary policy.

In the present paper, I try to propose a new mechanism through which a monetary policy influences aggregate dynamics via inequality variation in a context of optimal monetary policy. Thus, the two objectives of this paper are to investigate the following: 1) how inequality variation affects aggregate dynamics and consequently social welfare; and 2) whether such effect of inequality variation matters for optimal monetary policy design.

Before constructing a model to meet those objectives, I consider some necessary prerequisites to an appropriate model: the model should be able to account for, at least, three salient features of data associated with income inequality:[4] The first is the higher volatility in unemployment of less-educated (or lower income) households than in high-educated (or higher income) households; The second is that, as Pourpourides (2011) and Champagne and Kurmann (2013) note, wages for high-educated workers are more volatile than those for less-educated workers, while employment is less volatile for high-educated workers than less-educated workers;[5] The third is that a contractionary monetary policy shock increases income and consumption inequality (Coibion et al. (2012) and Gornemann et al. (2012)).

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dampens business cycle fluctuations and the fiscal policy becomes more effective relative to monetary policy as the degree of inequality rises respectively

[4]Note that although income inequality itself arises from various income sources, the Consumer Expenditure Survey (CEX) data shows labor income is the largest contributor to total income for most households. Therefore, for the sake of simplicity, I focus only on employment and wages which consist of labor income and derive some stylized facts on those variables from the literature.

[5]Similarly, Heathcote et al. (2010) finds that earning dynamics of households in the upper end of the income distribution are driven by changes in wages while changes in hours play a central role in the earning dynamics at the lower end of the income distribution.

Obviously, a representative agent model or single labor market model cannot explain such differentials in labor market variables and the dynamics of inequality. Therefore, in an attempt to account for these stylized facts, I introduce heterogeneity into a standard New Keynesian DSGE model used in Erceg et al. (2000) by assuming segmented labor markets and the Limited Asset Market Participation (hereafter LAMP). These two features of the model are well supported by empirical micro-evidences but also introduce heterogeneity in a relatively simple and tractable way.

In the baseline model, there are two types of households, one supplying high-skilled labor and the other supplying low-skilled labor. In the model, labor markets are segmented for different skilled workers.[6] Even though I classify the households into high-skilled and low-skilled, they are different only in regards to labor demand and supply elasticities. I assume that the low-skilled workers are more substitutable than the high-skilled workers in production, and hence, the demand for low-skilled workers is more susceptible to a change in low-skilled wages. This assumption is consistent with empirical findings in the literature such as Lichter et al. (2014), in which the authors show that there is a significant heterogeneity in labor demand elasticity and, in particular, labor demand for unskilled workers and workers with atypical contracts is more responsive to wage rate changes.

I also introduce heterogeneity in the labor supply side by assuming LAMP following Galí et al. (2007) and Furlanetto (2011). For the sake of simplicity and tractability, I assume that the low-skilled households are also the households that have limited access to the financial market. These households are therefore not able to smooth their consumption through financial assets. Because of this restriction, their consumption and labor supply are more susceptible to income changes. Intuitively, low-skilled workers with a lower wage have to spend a greater fraction of their earnings on living costs. If financial transactions require some cost or financial intermediaries require a high standard for their financial services, it is relatively more difficult to gain access to financial markets for the low-skilled households. Therefore, low-skilled households' labor supply becomes more sensitive to changes in their wages. On the other hand, since high-skilled workers with higher wages have better opportunities to access financial markets, high-skilled workers have a much superior capacity to offset the fluctuation in their wages. Thus, their consumption and labor supply are relatively stable compared to low-skilled workers.

[6] I use high-skilled workers, Ricardian agents, and financially included agents inter-changeably.

These assumptions about labor markets structure enable the model to generate variations in income and consumption inequality across sectors after an economic shock and to show how the variations in inequality magnify a macroeconomic volatility. In particular, the greater elasticity in both labor supply and demand for low-skilled workers leads to a flatter low-skilled wage Phillips curve even if nominal rigidities are the same across sectors.[7] The effectively stickier low-skilled wages, in turn, induce a more volatile employment and unemployment rates for low-skilled workers. Furthermore, the difference in such real rigidities generates a variation in the wage premium defined as the gap between average high-skilled wages and average low-skilled wages, which, in turn, brings about strategic complementarities in wage setting resulting in stickier adjustment of aggregate nominal wages and greater fluctuation of real variables such as output, employment and unemployment.[8] This is because changes in the wage premium force firms to substitute relatively cheap workers for expensive workers, and that raises (lowers) marginal rates of substitution between consumption and leisure and thus wages for relatively cheap (expensive) workers. However, these two competing forces on aggregate nominal wage are dominated by high-skilled wages because low-skilled wages are effectively stickier than high-skilled wages. As a consequence, both high-skilled and low-skilled wages initially decrease after negative demand shock but high-skilled wages bounce back somewhat in response to the fall in the wage premium whereas low-skilled wages decrease further. Thus, aggregate wages cannot decrease as much as they do under the single labor market model, which causes more volatile real variables. An endogenous shift term in the aggregate wage Phillips curve captures this indirect effect of shocks on the aggregate wage. In addition, given that the elasticity of substitution across sectors is greater than 1, as found in the literature, a variation in the wage premium results in more variation in employment gap between two different skilled households, and hence, labor income inequality changes in the opposite direction of the wage premium.

The paper then studies optimal monetary policy based on the Central Bank's welfare loss function which is obtained from the second order approximation to the weighted average of households'

[7]The greater elasticities of labor demand and supply for low-skilled workers imply that low-skilled unemployment fluctuates more given a change in wages; conversely, low-skilled wages are relatively stable given a change in unemployment. Thus, the slope of the wage Phillips curve that describes the relationship between the wage inflation rate and unemployment rate becomes flatter as the elasticities increase.

[8]A difference in the size of change in wages across sectors gives rise to sectoral wages mutually reinforcing one another.

life-time utility function as in Bilbiie (2008). A Central Bank's target variables include consumption and income inequality as well as standard objectives (price and wage inflation, and the output gap from its efficient level). Galí (2011)'s specification of unemployment is adopted so that the output gap can be transformed into unemployment. This specification helps to express the Central Bank's loss function in terms of observable variables only. As is well-known in the literature, when an economy features both price and wage stickiness, the Central Bank cannot achieve efficient equilibrium and thus suffers from a substantive welfare loss. In this paper, I find that inequality poses an additional policy trade-off with output gap after idiosyncratic productivity shocks even with flexible wages. Accordingly, the first best allocation is not attainable when sectoral productivity shocks hit the economy. In other words, inflation targeting is not an optimal policy even under the flexible wages in contrast to Erceg et al. (2000). I therefore conclude a Central Bank may need to take into account consumption and income inequality when constructing monetary policy.

Finally, I conduct counter-factual experiments in which the Central Bank sets its optimal monetary policy as if the true economy is different from the baseline model. I consider three different scenarios: 1) the central bank recognizes sticky wages but not segmented labor market; 2) the central bank recognizes segmented labor markets but not sticky wages; and 3) the central bank recognizes neither sticky wages nor segmented labor market. The results indicate that when the Central Bank ignores labor market segmentation and, consequently, inequality, the welfare losses are significantly larger than those of the baseline model, even if the Central Bank recognizes wage stickiness.

The remaining sections are organized as follows: In Section 2, I review the literature on the disproportionate effect of monetary policy and LAMP. Section 3 describes the model in detail and Section 4 gives equilibrium and market clearing conditions, In Section 5, I define consumption and income inequality and discuss the aggregate dynamics of the model. I conduct numerical simulations in Section 6 to show the disproportionate effect of monetary policy and heterogeneity in labor market dynamics. In Section 7, I contemplate an optimal monetary policy design that takes into account inequality with a welfare analysis, and provide concluding remarks in Section 8.

2 Related Literature

As noted above, inequality has been mostly ignored in monetary policy design in spite of its disproportionate effects of monetary policy. Rather, the literature has focused on the relationship itself between monetary policy and inequality. In particular, Carpenter and Rodgers III (2004) shows that a contractionary monetary policy lowers the employment-population ratios of minorities and less-skilled households and raises their unemployment rates than those of skilled households. Breen and Garca-Pealosa (2005) show that high volatility of monetary policy has been shown to result in high-output volatility and assert that, as a regression tax, higher inflation causes a greater inequality. Recently, Coibion et al. (2012) have also studied the effects of monetary policy shock using micro-level data on income and consumption, and found that contractionary monetary policy actions have systematically increased inequality in the U.S. since 1980. Romer and Romer (1999) also empirically analyzes the influence of monetary policy on inequality and show that an expansionary monetary policy lowers inequality temporarily by boosting the economy. However, they also argue that higher inflation after the expansionary monetary policy shock would lead to a tight monetary policy resulting in a rise in unemployment, which would, in turn, offset the temporary positive effect on inequality.

The theoretical literature has focused on the relationship between inflation and wealth distribution (and hence inequality) over the long-run. Among others Albanesi (2007) demonstrates that inflation is positively related to income inequality due to the relative vulnerability to inflation of low income households. Williamson (2008) addresses the monetary policy effect on an economy with segmented financial and goods markets. He argues that contractionary monetary policy shocks reallocate wealth from those connected to the financial market toward the unconnected agents, and that, therefore, consumption and income inequality fall after the shocks. However, there are very few papers in which the authors study the disproportionate effect of monetary policy and inequality at a business cycle frequency. Dosi et al. (2013) discuss the relationship between income inequality and monetary policy using an agent-based Keynesian model. In particular, they find a non-linearity of monetary policy impact and argue that a contractionary monetary policy lead a more "unequal" economies. Gornemann et al. (2012) build a structural model in a New Keynesian framework with search and matching friction and find that contractionary monetary policy shocks lead to a pronounced increase in earnings, income, wealth, and consumption heterogeneity. However, even though their model features a richer environment considering various income sources, they do not discuss the effect of inequality on optimal monetary policy design.

In addition, Aghion et al. (1999) shows that unequal access to investment opportunity leads to greater fluctuations in real variables and argues that counter-cyclical fiscal policy is quite effective in stabilizing economy. Similarly, the LAMP framework has been used mostly for analyzing the effect of fiscal policy on aggregate output. Among others, Furlanetto (2011) extends the basic LAMP model by considering segmented labor markets. He argues that a common wage and employment is suboptimal for both Ricardian and Rule-of-Thumb agents. That is because their consumption behaviors are different in response to an exogenous shock, which creates a variation in the relative marginal rate of substitution between consumption and labor, and thus, a gap in the desired wage across the sectors. Therefore, the common wage (and hours) assumption is to exclude both agents from a mutually beneficial trade. In contrast, there are a couple of papers that use LAMP to investigate optimal monetary policy; Ascari et al. (2011) and Areosa and Areosa (2006). However, neither paper is able to model income inequality and the dynamics of labor market variables due to their own assumptions: the single labor market in the earlier paper and the Cobb-Douglas production function with flexible wages in the later one. The LAMP framework with segmented labor market and staggered wages allows me to discuss consumption and income inequality in two ways: the effect of inequality on optimal monetary policy and monetary policy's impact on inequality.

3 Model

There are two different types of skilled households with each type of household consisting of a continuum of workers supplying labor to the corresponding skilled labor market. Wages are set by representative unions for each type of workers in segmented markets. There is a continuum of monopolistic competitive firms producing differentiated goods and they determine the price given the wages and aggregate demand. In the benchmark model, a Central Bank sets a nominal interest rate following a "Taylor-type" rule.

To simplify the model, I make three assumptions. The first is sectoral immobility; Workers are prohibited from crossing from the low-skilled labor market to the high-skilled labor market and vice versa. The second is a constant population share of a sector. Thus the relative size of each labor sector remains constant over time. Third, I also assume that there is only one good producing sector in which a monopolistic firm hires both high-skilled and low-skilled workers and both skilled workers are aggregated into one homogeneous effective labor input and used to produce differentiated goods.

3.1 Household

I assume that there are two levels of skill, $j \in \{H, L\}$, by which the households are categorized as high skilled or low skilled households. For j-skilled households, there are a large number of identical households which are comprised of a continuum of members represented by the unit indexed by $i \in [0,1]$. The index $i \in [0,1]$ indicates the type of labor service in which a given household member is specialized in a sector. I also assume that a constant fraction, s, of the total population are high-skilled workers and $1-s$ fraction of population are the low-skilled workers in every period. These two types of households are heterogeneous in two dimensions: first, low-skilled workers are more substitutable than high-skilled workers so that the labor elasticity of substitution between the low-skilled workers is greater than that between the high-skilled workers. In other words, demand for the low-skilled workers is more responsive to changes in wage; second, low-skilled workers are limited in their access to financial markets so that they cannot smooth their consumption using financial assets. That is, they use up all the disposable income in every period.

Representative households of j-skilled households maximize their discounted lifetime utility (3.1) subject to budget constraint (3.2) for $j \in \{H, L\}$. While labor demand, $N_t^j(i)$, is determined by the aggregation of firm's labor demand decisions and allocated uniformly across j-skilled households, workers choose their optimal wages, $W_t^j(i)$. Therefore, both $W_t^j(i)$ and $N_t^j(i)$ are taken as given by each household. Each household's discounted lifetime utility in j sector is given by:

$$\sum_{t=0}^{\infty} \beta^t U(C_t^j, N_t^j(i); \chi_t) = \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(C_t^j\right)^{1-\sigma}}{1-\sigma} - \chi_t \int_0^1 \frac{\left(N_t^j(i)\right)^{1+\varphi}}{1+\varphi} di \right] \tag{3.1}$$

where the variable χ_t is a aggregate labor supply shock following AR(1) process in log ($\log \chi \equiv \xi$),

$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_t^\xi$, and $\varepsilon_t^\xi \sim \mathcal{N}(0, \sigma_\xi^2)$. The parameter σ is the inverse of intertemporal elasticity of substitution, and the parameter φ denotes the inverse of the Frisch labor supply elasticity of workers which is common for all types of workers. Aggregate consumption of a representative household with j-skill is given by

$$C_t^j \equiv \left(\int_0^1 C_t^j(z)^{\frac{\epsilon_p - 1}{\epsilon_p}} dz \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

where $C_t^j(z)$ is the quantity consumed of good z by a j-skilled household, ε_p is the elasticity of substitution between two differentiated goods, and $N_t^j(i)$ for $i \in [0,1]$ is the fraction of members specialized in type i labor in each j-skilled household who are employed in period t.[9] and the parameter ε_p is the elasticity of substitution over differentiated goods. The high-skilled households budget constraint is given by:

$$\int_0^1 P_t(z) C_t^H(z) dz + Q_t B_t \leq B_{t-1} + \int_0^1 W_t^H(i) N_t^H(i) di + \Pi_t \tag{3.2}$$

where $P_t(z)$ is the price of good z, $W_t^H(i)$ is the nominal wage for type i high-skilled labor, B_t represents purchases of nominally riskless one-period discount bonds paying one monetary unit, Q_t is the price of that bond, and Π_t is a lump-sum component of income at time t.

The first order conditions for the maximization problem subject to the budget constraint give the high-skilled consumption Euler equation:

$$Q_t = \beta E_t \left(\frac{C_{t+1}^H}{C_t^H} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \tag{3.3}$$

Low-skilled households have the same utility function as high-skilled households, (3.1), but they do not face an intertemporal consumption decision because they are not able to hold bond in this simple model. Rather, they consume all the disposable income in each period:

$$\int_0^1 P_t(z) C_t^L(z) dz = \int_0^1 W_t^L(i) N_t^L(i) di \tag{3.4}$$

[9]when all variables are measured in per capita term, $\frac{N_t^H}{Pop_t} = \frac{Pop_t^H}{Pop_t} \frac{N_t^H}{Pop_t^H} = s \frac{N_t^H}{Pop_t^H}$. Therefore, N_t^j where $j \in \{H, L\}$ can be interpreted as Employment to Population Ratio and participation rate respectively as explained in later.

In addition, optimal demand for each good resulting from utility maximization takes the familiar form:

$$C_t^j(z) = \left(\frac{P_t(z)}{P_t}\right)^{-\epsilon_p} C_t^j$$

for $j \in \{H, L\}$ where $P_t \equiv \left(\int_0^1 P_t(z)^{1-\epsilon_p} dz\right)^{\frac{1}{1-\epsilon_p}}$ denotes the price index for final goods.

3.2 Wage Determination

The labor markets are monopolistically competitive, and wages are determined by the representative unions. Nominal rigidities in wages are introduced through Calvo (1983) pricing; For each labor market, only $1 - \theta_w$ fraction of workers can re-optimize their wage. When re-optimizing their wage in period t, workers choose a wage W^{j*}, where again $j \in \{H, L\}$, in order to maximize their households' utility taking all aggregate variables including the aggregate wage index as given.[10] I assume that the Calvo parameters are the same, $\theta_w = \theta_w^H = \theta_w^L$, following Barattieri et al. (2014). In addition, I assume that high-skilled workers are not easily substituted by others relative to low-skilled workers; $\varepsilon_w^H < \varepsilon_w^L$. This assumption implies that the markup of high-skilled workers in wage setting is greater than that of low-skilled workers, and it also assure that the high-skilled wage is larger than the low-skilled wage on average for a given Frisch elasticity. As will be explained in section 3.4, these two elasticities are closely related to divergent unemployment rates. The optimal wage setting rule for j-skilled workers for $j \in \{H, L\}$ can be obtained from the maximization problem subject to the budget constraint and the corresponding labor demand schedule determined by firms:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ N_{t+k|t}^j \left(C_{t+k}^j\right)^{-\sigma} \left(\frac{W^{j*}}{P_{t+k}} - \mathcal{M}_t^j MRS_{t+k|t}^j\right)\right\} = 0 \tag{3.5}$$

where $N_{t+k|t}^j$ denotes the aggregate quantity demanded in period $t+k$ of j-skilled workers whose wage was last reset in period t. Here, $MRS_{t+k|t}^j \equiv \chi_t \left(C_{t+k}^j\right)^{\sigma} \left(N_{t+k|t}^j\right)^{\varphi}$ is the period $t+k$ marginal rate of substitution between consumption and labor for a high-skilled worker whose wage is reset in period t, and $\varepsilon_{w,t}^j$ is the elasticity of substitution between two different types of workers in j sector, $\mathcal{M}_t^j \left(\equiv \frac{\varepsilon_{w,t}^j}{\varepsilon_{w,t}^j - 1}\right)$ is the desired or frictionless wage markup and $\mu_t^{nj} \equiv \log \mathcal{M}_t^j$. The first order log

[10] Aggregate wage for j-skilled workers is given by $W_t^j \equiv \left(\int_0^1 W_t^j(i)^{1-\epsilon_w^j} di\right)^{\frac{1}{1-\epsilon_w^j}}$ and aggregate nominal wage is defined as $W_t \equiv \left(\gamma_H \left(W_t^H\right)^{1-\eta} + \gamma_L \left(W_t^L\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}$.

approximation of (3.5) around the zero inflation steady states gives the optimal wage equation as following:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ w_t^{j^*} - p_{t+k} - \mu_{t+k}^{nj} - mrs_{t+k|t}^{j} \right\} = 0 \tag{3.6}$$

Defining the j-skilled sector's average marginal rate of substitution as $MRS_t^j \equiv \chi_t \left(C_t^j\right)^{\sigma} \left(N_t^j\right)^{\varphi}$, the marginal rate of substitution of each individual in the sector can be written in terms of the relationship between the average marginal rate of substitution and the relative wage.[11]

$$\begin{aligned} mrs_{t+k|t}^{j} &= mrs_{t+k}^{j} + \varphi \left(n_{t+k|t}^{j} - n_{t+k}^{j}\right) \\ &= mrs_{t+k}^{j} - \epsilon_w^j \varphi \left(w_t^{j*} - w_{t+k}^{j}\right) \end{aligned} \tag{3.7}$$

Finally, combining (3.6), (3.7) and the log-linearized form of the aggregate wage index, I obtain the j-skilled wage Phillips curve as:[12]

$$\pi_t^j = \beta E_t \left\{\pi_{t+1}^j\right\} - \kappa_w^j \left(\mu_t^j - \mu_t^{nj}\right) \tag{3.8}$$

where $\pi^j \equiv w_t^j - w_{t-1}^j$ is the j-skilled wage inflation, μ_t^{nj} is the wage markup shock of j-skilled workers, and $\mu_t^j \equiv w_t^j - p_t - mrs_t^j$ denotes the log average j-skilled wage markup and $\kappa_w^j \equiv \frac{\Theta}{1+\epsilon_w^j \varphi} > 0$ where $\Theta \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w} > 0$.[13]

3.3 Firms and Price Determination

Monopolistically competitive firms hire workers from both labor markets and then aggregate these workers with CES technology into homogeneous effective labor input. Each firm produces a differ-

[11] where $N_t^j \equiv \int_0^1 N_t^j(i)di$ is the sector j aggregate employment rate.
[12] $w_t^H = \theta_w w_{t-1}^H + (1-\theta_w) w_t^{H*}$.
[13] See Galí (2011) for the detailed derivation.

entiated good $z \in [0,1]$ using a production function which is given by:

$$Y_t(z) = A_t H_t(z)$$
$$\text{where } H_t(z) = \left[\gamma_H^{\frac{1}{\eta}} \left(N_t^H(z)\right)^{\frac{\eta-1}{\eta}} + \gamma_L^{\frac{1}{\eta}} \left(N_t^L(z)\right)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}$$
$$\text{and } N_t^j(z) \equiv \left(\int_0^1 N_t^j(i,z)^{\frac{\varepsilon_{w,t}^j-1}{\varepsilon_{w,t}^j}} di\right)^{\frac{\varepsilon_{w,t}^j}{\varepsilon_{w,t}^j-1}}$$

where $H_t(z)$ is the homogeneous effective labor input of firm z obtained by labor aggregation technology; η is elasticity of substitution between high-skilled ($N_t^H(z)$) and low-skilled labor ($N_t^L(z)$); γ_j is a parameter governing the relative income share of j-skilled of labor; $\varepsilon_{w,t}^j$ is the labor elasticity of substitution with in the corresponding sector $j \in \{H, L\}$ as I mentioned above. The variable A_t is an exogenous technology process which is assumed that $a_t \equiv \log A_t$ and $a_t = \rho_a a_{t-1} + \varepsilon_t^a$ where $\rho_a \in (0,1)$ and ε_t^a is a white noise process with a zero mean and variance σ_a^2. The firm's cost minimization problem, taking wages and aggregate demand as given, implies the following set of labor demand schedules:

$$N_t^j(i,z) = \left(\frac{W_t^j(i)}{W_t^j}\right)^{-\varepsilon_w^j} N_t^j(z) \text{ and } N_t^j(z) = \gamma_j \left(\frac{W_t^j}{W_t}\right)^{-\eta} H_t(z) \text{ where } j \in \{H, L\}$$

for all $i \in [0,1]$ and for all $z \in [0,1]$.[14] I introduce nominal rigidities in price through the Calvo (1983) pricing. Firms' profit maximization problem subject to the sequence of demand schedule constraint $Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}}\right)^{-\varepsilon_p} C_{t+k}$, for $k = 0, 1, 2, \ldots$ leads to the optimality condition for the firm:

$$\sum_{k=0}^{\infty} \theta_p^k E_t \left\{Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \mathcal{M}^p \Psi_{t+k|t}\right)\right\} = 0$$

where $Y_{t+k|t}$ denotes output at time $t+k$ of a firm that last reset its price in period t, $Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t}\right)^{-\sigma} \frac{P_t}{P_{t+k}}$ is the relevant stochastic discount factor for nominal payoffs in period $t+k$, $\Psi_{t+k|t} \equiv \frac{W_{t+k}}{A_{t+k}}$ is the nominal marginal cost in period $t+k$ of producing quantity $Y_{t+k|t}$ and $\mathcal{M}^p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$ is the desired or frictionless price markup over the marginal cost. Log-linearization of

[14]Log-linearized employment rate of i-type of j-skilled labor is given by $n_t^j(i) = -\varepsilon_w^j \left(\omega_t^j(i) - \omega_t^j\right) + n_t^j$ and average j-skilled employment rate is given by $n_t^j = -\eta \left(\omega_t^j - \omega_t\right) + y_t - a_t$

the optimality condition around the zero inflation steady state yields

$$\sum_{k=0}^{\infty} (\beta\theta_p)^k E_t \left\{ p_t^* - \psi_{t+k|t} \right\} = 0$$

Note that lower case variables denote the log-deviation of the variables from the steady state. The price inflation equation can be derived using the log-linearized price index, $p_t = (1-\theta_p)p_t^* + \theta_p p_{t-1}$:

$$\pi_t^p = \beta E_t \left\{ \pi_{t+1}^p \right\} + \kappa_p mc_t \tag{3.9}$$

where $\pi_t^p \equiv p_t - p_{t-1}$ is wage inflation, mc_t denotes average real marginal cost, $mc_t = \omega_t - a_t \; (\equiv \widetilde{\omega}_t)$, and $\kappa_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} > 0$.

3.4 Unemployment

I define sectoral unemployment rates following Galí (2011). An individual will be willing to work in period t if and only if the real wage for his labor type exceeds his disutility of labor. Thus the marginal j-skilled supplier of type i labor, $L_t^j(i)$, is given by

$$\frac{W_t^j(i)}{P_t} = \chi_t \left(C_t^j \right)^{\sigma} \left(L_t^j(i) \right)^{\varphi}$$

Define the aggregate labor force (or participation rate) as $L_t^j \equiv \int_0^1 L_t^j(i)di$, then the first order approximations gives the log-linearized estimate relation:

$$w_t^j - p_t = \sigma c_t^j + \varphi l_t^j + \xi_t$$

The unemployment rate u_t^j can be written as the log difference between the labor force and employment:[15]

$$u_t^j \equiv l_t^j - n_t^j$$

[15] $u_t^j = 1 - \frac{N_t^j}{L_t^j} \Rightarrow -u_t^j \approx \log(1-u_t^j) = n_t^j - l_t^j$. Note, in efficient steady state, all labor force has to be hired ($l = n$, that is, $u = 0$); and government subsidies impose symmetric labor market ($l^H = l^L = n^H = n^L$).

Noting that real wage is the markup over the marginal rate of substitution, $\mu_t^j \equiv \left(w_t^j - p_t\right) - mrs_t^j = \left(w_t^j - p_t\right) - \left(\sigma c_t^j + \varphi n_t^j + \xi_t\right)$, the unemployment rate can be written as:

$$\mu_t^j = \varphi u_t^j \tag{3.10}$$

Therefore, as Galí (2011) noted, (3.10) implies that unemployment fluctuations are a consequence of variations in the wage markup. Finally, combining (3.8) with (3.10), I derived the sectoral New Keynesian wage Phillips Curve:

$$\pi_t^j = \beta E_t \left\{\pi_{t+1}^j\right\} - \kappa_j \varphi \left(u_t^j - u_t^{nj}\right) \tag{3.11}$$

The slope becomes flatter as labor demand elasticity increases. Therefore, high-skilled workers face a steeper wage Phillips curve, and accordingly, high-skilled nominal wages are more volatile than low-skilled nominal wages in response to unemployment fluctuation. Conversely, low-skilled unemployment is more volatile given changes in wage, which is consistent with the empirical findings shown in Pourpourides (2011) and Champagne and Kurmann (2013).

3.5 Government

The government budget constraint is:

$$P_t G_t + B_{t-1} = Q_t B_t + T_t$$

where T_t is the lump-sum tax from high-skilled household after subsidies which are used to eliminate desired markups on price and wages. I assume that government spending, G_t, is zero at any period.

4 Equilibrium and Market Clearing

4.1 Steady States

I consider the zero inflation efficient steady states. I assume that government can eliminate markups in both goods and labor markets by giving appropriate subsidies.[16] I also assume that the govern-

[16] That is $\mathcal{M}^j\left(1-\tau^j\right) = 1$ for $j \in \{P, H, L\}$ where τ^j is the subsidies for j market.

ment does not issue government bonds in the steady states. This guarantees that the wages and the consumptions are the same for any type of workers in the steady states.[17] Note that aggregate consumption is now given by

$$C_t = sC_t^H + (1-s)C_t^L$$

and log-linearized as $c_t = sc_t^H + (1-s)c_t^L$. Thus, in the steady state, I obtain

$$C^H = C^L = C = Y = H = N^H = N^L$$

4.2 Labor Market Equilibrium

Since the labor markets are segmented, aggregate j-skilled labor supply must be equal to the firm's aggregate labor demand for j-skilled labor in equilibrium for $j \in \{H, L\}$. Accordingly, the j-skilled labor market clearing condition is following:

$$N_t^j = \int_0^1 N_t^j(z)dz = \int_0^1 \int_0^1 N_t^j(i,z)didz \quad = \gamma_j \Delta_t^j \Delta_t^P \left(\frac{W_t^j}{W_t}\right)^{-\eta} \frac{Y_t}{A_t}$$

Note that Δ_t^P, Δ_t^H and Δ^L are measures for the price, high skilled, and low skilled wage dispersion respectively, and can be approximated to 1 up to the first order.[18] Log-linearization of the employment in each sector around steady state can be written:

$$n_t^j = -\eta\left(\omega_t^j - \omega_t\right) + (y_t - a_t) \tag{4.1}$$

Note that the sectoral employment rates are affected by not only aggregate demand but also by the relative wage, and thus, the wage premium. This is important because the effect of a variation in the wage premium affects sectoral employments in opposite way, and therefore, generates differentials in employment rates across sectors. For instance, although both employment rates decrease initially in response to a positive technology shock, high-skilled employment decreases less than low-skilled

[17] Log-linearization of aggregate wage index is given by $w_t = \frac{W^H N^H}{WH} w_t^H + \frac{W^L N^L}{WH} w_t^L = sw_t^H + (1-s)w_t^L$. Since steady state wages are the same, the relative labor income of each sector equals its population share. In addition, given the same wages with zero bond-holding, all the workers enjoy the same level of consumption.

[18] $\Delta^P \equiv \int_0^1 \left(\frac{P_t(z)}{P_t}\right)^{-\varepsilon_p} dz \approx 1 + \frac{\varepsilon_p}{2} Var_z\{p_t(z)\}$, $\Delta_t^j \equiv \int_0^1 \left(\frac{W_t^j(i)}{W_t^j}\right)^{-\varepsilon_w^j} di \approx 1 + \frac{\varepsilon_w^j}{2} Var_i\left\{w_t^j(i)\right\}$ for $j \in \{H, L\}$ Details for the second order log approximations see Galí (2011).

employment due to the decrease in wage premium; the greater elasticity of substitution between low-skilled workers than between high-skilled workers induces more volatile low-skilled employment and effectively stickier wages for low-skilled workers, and thus, the shock results in a decline in the wage premium. Moreover, when the two different skilled workers are highly substitutable, a change in the wage premium leads to a greater gap between sectoral employment rates. Therefore, aggregate productivity shock that lowers the wage premium causes greater inequality.

4.3 Resource Constraint and Consumption Euler Equation

Because of the absence of investment and government spending in a closed economy, all outputs produced by each firms are consumed. Therefore, the market clearing condition is $C_t(z) = Y_t(z)$ for all $z \in [0, 1]$, and hence, $C_t = Y_t$. From (3.3), the log-linearized high-skilled consumption Euler equation is given by:

$$c_t^H = E_t c_{t+1}^H - \frac{1}{\sigma}\left\{i_t - E_t \pi_{t+1}^p\right\} \tag{4.2}$$

where $i_t (= -q_t)$ is the nominal interest rate on a risk-free bond.[19] and low-skilled consumption is just equal to low-skilled worker's labor income:

$$c_t^L = \omega_t^L + n_t^L \tag{4.3}$$

where ω_t^L is the average real wage for the low-skilled workers. Noting that $c_t^H = \frac{c_t - (1-s)c_t^L}{s}$, good market clearing condition, and (4.1), I derive aggregate consumption Euler equation as:

$$\begin{aligned} c_t &= E_t c_{t+1} - \frac{s}{\sigma}\left\{i_t - E_t \pi_{t+1}^p\right\} - (1-s)\left\{\Delta E_t c_{t+1}^L\right\} \\ y_t &= E_t y_{t+1} - \frac{1}{\sigma}\left\{i_t - E_t \pi_{t+1}^p\right\} - \frac{1-s}{s}\left\{(1-\eta s)\Delta E_t \omega_{t+1}^L + \eta s \Delta E_t \omega_{t+1}^H - \Delta E_t a_{t+1}\right\} \end{aligned} \tag{4.4}$$

5 Inequality and aggregate dynamics

When an economy is efficient, the wages are determined at the level at which the marginal rate of substitution equals the marginal product of labor in any given period. Using this condition, I

[19] $Q_t = \frac{1}{R_t} = \frac{1}{1+i_t}$.

obtained the efficient level of output and the interest rate that makes output gap equal to zero in equilibrium.[20] I now define a variable $\widetilde{X}_t \equiv X_t - X_t^E$ as the difference from its efficient level.

5.1 Consumption and Income Inequality

In this subsection, I study the relationship between inequality and wages. I measure the consumption inequality by the Gini coefficient. The households are divided into two skilled groups and I assumed perfect risk sharing within a household. As all the members of a household therefore enjoy the same level of consumption, the economy has only two types of consumption level and so the Gini coefficient is given by $\mathcal{G}_t^c = (1-s)\left\{1 - \frac{C_t^L}{C_t}\right\}$ and is approximated as:[21]

$$\mathcal{G}_t^c \approx -(1-s)\left(c_t^L - c_t\right) = -(1-s)\left(\widetilde{\omega}_t + (\eta - 1)s\widetilde{\omega}_t^R\right) \tag{5.1}$$

Similarly, I define labor income inequality by the Gini coefficient, $\mathcal{G}_t^I = (1-s)\left(1 - \frac{\mathcal{X}_t^L}{\mathcal{X}_t}\right)$ where $\mathcal{X}_t$ is the economy's average labor income or total payment of the economy.[22]

$$\mathcal{G}_t^I \approx -(1-s)\left(\widehat{\mathcal{X}}_t^L - \widehat{\mathcal{X}}_t\right) = -s(1-s)(\eta-1)\widetilde{\omega}_t^R \tag{5.2}$$

If the production function is Cobb-Douglas, that is $\eta = 1$, the relative income share is constant and hence income inequality is fixed over time. If two different skilled workers are close to complementary inputs ($\eta < 1$), then income inequality moves along with wage premium because relative employment does not change as much as wage premium. However, if two inputs are highly substitutable ($\eta > 1$), relative employment variation dominates a change in wage premium, and hence, income inequality moves in the opposite direction of the wage premium. Moreover, the greater η implies the stronger effect of wage premium on income inequality.[23]

[20] See Appendix A for the details.

[21] $c_t^L - c_t = \omega_t^L + n_t^L - y_t = \widetilde{\omega}_t^L + a_t + a_t^L + \left(\eta s\widetilde{\omega}_t^R + y_t - a_t - a_t^L\right) - y_t = \widetilde{\omega}_t^L + \eta s\widetilde{\omega}_t^R = \widetilde{\omega} + (\eta-1)s\widetilde{\omega}_t^R$.

[22] See Appendix **??** for details.

[23] In an extreme case in which $\eta = 0$, production function becomes Leontief production function, that is two different skilled workers are perfect complements, income inequality only depends on the wage premium. In other extreme case in which $\eta = \infty$, the two different workers are perfect substitute, and even a very tiny deviation of the wage premium from its efficient level makes one sector takes all.

5.2 IS Curve

Combining (4.4) and (5.1), I derive the economy's IS curve as

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}\left\{i_t - E_t \pi^p_{t+1} - r^E_t\right\} + \frac{1}{s}\Delta \mathcal{G}^c_{t+1} \tag{5.3}$$

where $y^E_t = \frac{1+\varphi}{\sigma+\varphi}a_t - \frac{1}{\sigma+\varphi}\xi_t$, $x_t = y_t - y^E_t$, and $r^E_t = \sigma \Delta y^E_{t+1}$. The IS curve differs from that of standard LAMP model due to the extra term associated with consumption inequality, which comes from the imperfect risk-sharing across households.[24] When all the agents are not financially excluded, that is when $s = 1$, the IS curve becomes the one that is in a standard NK model. Since labor markets are segmented and households do not perfectly share the risk (the labor income shock), consumption responses are different after a real interest rate change. As will be explained later, low-skilled workers who are prohibited from holding bonds only respond to their labor income change rather than an interest rate change. Therefore, the impact of monetary policy on output is weakened by the presence of Non-Ricardian households.[25] The last term captures this channel.

5.3 Wage Phillips Curves

Note that wage markups can be expressed as the difference between the real wage and the marginal rate of substitution. Solving for markups in terms of wages, I obtain the aggregate wage Phillips curve which is a convex combination of two sectoral wage Phillips curves as in (C.5).[26] In doing this, I assume log utility function, $\sigma = 1$, to simplify the equation, which allows me to focus only on the relationship between inequality and the macroeconomic volatility.The aggregate wage Phillips curve is then given by:

$$\pi^w_t = \beta E_t \pi^w_{t+1} + \kappa_w (1+\varphi) x_t - \kappa^H_w \widetilde{\omega}_t + \Upsilon \mathcal{G}^I_t + \epsilon_t \tag{5.4}$$

where $\Upsilon \equiv \Theta \frac{\eta(1+\varphi)}{\eta-1}\left\{\frac{1}{1+\varepsilon^H_w \varphi} - \frac{1}{1+\varepsilon^L_w \varphi}\right\}$ and ϵ_t is the composition of wage markup shocks.[27] Since the (absolute) slope of the curve is decreasing in labor demand elasticity, it can clearly be seen that the

[24] It is actually very similar to Ascari et al. (2011) but the extra term is now associated with consumption inequality rather than a real wage gap.

[25] In present paper, I mean low skilled workers (households) by both financially excluded agents and Non-Ricardian agents.

[26] See the Appendix C for details.

[27] $\epsilon_t \equiv s\kappa^H_w \mu^{nH}_t + (1-s)\kappa^L_w \mu^{nL}_t$.

slope will be steeper as the population share of high-skilled workers increases. The aggregate wage Phillips curve differs from the standard one due to the presence of the endogenous shift term related to the wage-premium, and hence income inequality. This endogenous shift term brings about more sluggish aggregate nominal wage, and thus, more volatile macroeconomic variables. For instance, suppose that when an economy is hit by a negative demand shock, then output decreases and unemployment rates increase, thereby pushing the nominal wage to fall. However, the high-skilled nominal wage falls more than the low-skilled nominal wage because the latter is effectively stickier. This causes a decrease in the wage-premium and strategic complementarities in wage setting. Once the wage premium decreases, firms start to substitute high-skilled workers for low-skilled workers that raises high-skilled employment and lowers low-skilled employment, which, in turn, causes a higher marginal rate of substitution between consumption and labor of high-skilled workers and lower that of low-skilled workers. As a result, high-skilled wages bounce back while low-skilled wages decrease further. Thus, the strategic complementarities dampen the decrease in high-skilled wages and amplify the decrease in low-skilled wages.[28] However, since the magnitude of high-skilled wages adjustment is greater than that of low-skilled wages, the aggregate wage is influenced by the changes in high-skilled wages. Therefore, the net effect of the cross-sector income effect is to generate slower aggregate wage adjustments. On the other hand, if the elasticity of substitution across sectors is sufficiently high, $\eta > 1$, income inequality widens because a change in employment gap is larger than that of the wage premium. Similarly, in response to a positive technology shock, unemployment rates increase (due to the more efficient labor aggregation technology) putting downward pressure on nominal wages. Again, high-skilled wages decrease more than those of low-skilled workers, thereby reducing the wage premium. Note that when $\varepsilon_w^H = \varepsilon_w^L = \varepsilon_w$, the aggregate wage Phillips curve coincides with Galí (2011) in which neither wage premium nor income inequality has a significant role in aggregate wage dynamics. Therefore, inequality does not require any policy intervention when all workers face the same labor market condition even if the markets are segmented.

[28]Lee (2011) builds a NK model based on firm specific labors and heterogeneity in firms' price setting frequency and discusses the aggregate effect of the heterogeneity. He argues that heterogeneity in price rigidities across sectors creates cross-sector income effect and hence strategic complementarities. As a result, the aggregate Phillips curve has endogenous shift terms arising from the heterogeneity and this term causes stickier aggregate price adjustment than the homogeneous model due to changes in relative price in response to aggregate shocks.

6 Quantitative Analysis

To complete the model, I additionally define equations for the wage dynamics as:

$$\begin{aligned}\widetilde{\omega}_t^L &= \widetilde{\omega}_{t-1}^L + \pi_t^L - \pi_t^p - \Delta\omega_t^E \\ \widetilde{\omega}_t^H &= \widetilde{\omega}_{t-1}^H + \pi_t^H - \pi_t^p - \Delta\omega_t^E \\ \widetilde{\omega}_t &= s\widetilde{\omega}_t^H + (1-s)\widetilde{\omega}_t^L\end{aligned} \tag{6.1}$$

where $\omega_t^E = a_t$. In addition, for the benchmark model, I assume the Central Bank sets a nominal interest rate following a Taylor rule type of monetary policy responding to inflation and output gap.

$$i_t = \phi_\pi \pi_t + \phi_x x_t + \nu_t \tag{6.2}$$

where ν_t is a monetary policy shock following AR(1) process, $\nu_t = \rho_\nu \nu_{t-1} + \varepsilon_t^\nu$ and $\varepsilon_t^\nu \sim \mathcal{N}(0, \sigma_\nu^2)$.

6.1 Calibration

I set discount factor β to 0.99 for a 4% annual nominal interest rate. I assume relative risk averse agents by setting intertemporal elasticity of consumption parameter, σ, to 2. I compute the historical average high-skilled labor income share, s, from CPS data to be around 0.53. The price rigidity parameter, θ_p is set to 0.75 which implies that the average duration of price is one year. I adopt the wage rigidity parameter from Barattieri et al. (2014). Their estimate for the parameter is 0.822 implying that only 17.8% of hourly workers experienced a wage change in a quarter. Moreover, as they find little evidence of heterogeneity in wage adjustment frequency across the sectors, I set the same Calvo parameter for both labor markets. Elasticity of substitution between differentiated goods (ε_p) and labor parameters (ε_w^H and ε_w^L) are set to, 9, 3.8, and 6.2 implying 12.5%, 36% and 19% markups, respectively. This imply that the average (or aggregate) wage markup is about 25% which is the value estimated by Galí et al. (2012). I adopt the value for the elasticity of substitution between different skilled workers (η) from Mollick (2011) who estimate the elasticity of labor substitution across education levels and argues that the plausible value varies over 2.00 to 3.21.[29] The inverse of Frisch elasticity is set to 5 to be consistent with 5% average unemployment (natural rate of unemployment). Finally, following Christiano et al. (2010), the standard deviation of technology shock, labor supply shock, and monetary policy are set to 0.62, 0.24, and 0.13 respectively.

Table 1: Baseline Calibration

Parameter	Description	Value
β	Discount factor	0.99
σ	Intertemporal elasticity of consumption	2
s	High-skilled income share	0.53
θ_p	Calvo parameter for price adjustment	0.75
θ_w	Calvo parameter for wage adjustment	0.822
ε_p	Elasticity of substitution between differentiated goods	9
η	Elasticity of substitution between different skilled workers	2.43
ε_w^H	Elasticity of substitution between high-skilled workers	3.8
ε_w^L	Elasticity of substitution between low-skilled workers	6.2
φ	Inverse of the Frisch labor supply elasticity	5
ϕ_π	Inflation reaction coefficient of monetary policy	1.5
ϕ_y	output gap reaction coefficient of monetary policy	0.2
σ_a	Standard deviation of aggregate technology shock	0.62
σ_ξ	Standard deviation of labor supply shock	0.24
σ_ν	Standard deviation of monetary policy shock	0.13

6.2 Dynamic Responses

6.2.1 Monetary policy shock

Figure 1 shows the dynamic responses of sectoral variables and inequality measures to an increase in nominal interest rate by one standard deviation. This rise in the interest rate initially lowers only high-skilled consumption because low-skilled consumption is not affected by nominal interest rate but by their labor income. The decline in high-skilled consumption induces weaker aggregate demand (and thus lower output gap) as well as lower demand for both high-skilled and low-skilled labor. Consequently, the shock reduces employment and raises unemployment rates pushing aggregate nominal wage down. Real wages decline as well even though price inflation moves procyclically. "Talyor type" rule of monetary policy responds to this disinflation (and drop in output gap) low-

[29] Previous studies in the literature such as Katz and Murphy (1992) and Krusell et al. (2000) estimate elasticity of substitution between skilled workers and unskilled workers as 1.67 and 1.41 respectively. The estimates are somewhat lower than Mollick (2011)'s estimate but still greater than 1.

ering nominal interest rate. Therefore, the real interest rate declines thereafter, and high-skilled consumption is recovered gradually. On the other hand, low-skilled consumption fall as well because of drop in both employment and real wages. Labor force participation rates rise due to the negative wealth effect, and thus, unemployment rates rise more than the decrease in employment to population ratio. However, about two third of the increases in unemployment can be attributed to the decrease in employment which is in line with Erceg and Levin (2013)'s findings that unemployment rates mostly influenced by employment-to-population ratio as labor force participation rate is acyclical.

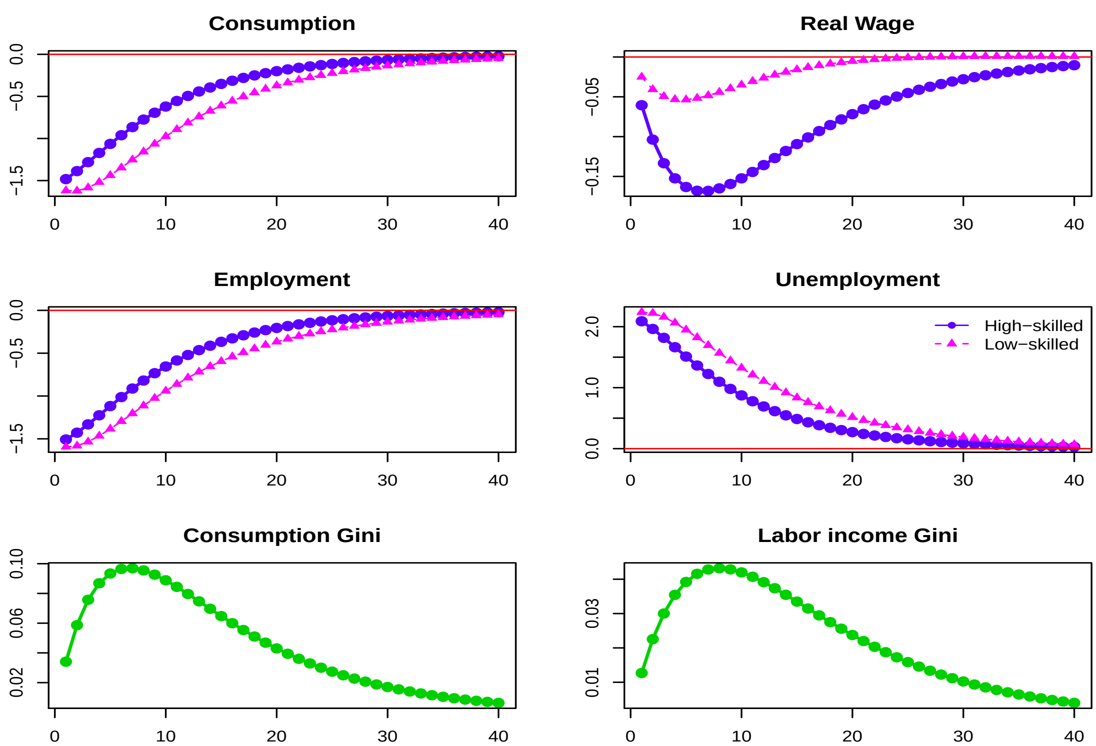
The monetary policy shock also has a disproportionate effect on labor market variables. Note that when nominal wages are under downward pressure, high-skilled wages drop more because of low-skilled wages are stickier than high-skilled wages due the greater labor demand elasticity. In other words, since the high-skilled wage Phillips curve is steeper, high-skilled nominal wages respond more sensitively to a change in unemployment, and this leads to a decrease in the wage premium. Even though low-skilled real wages decrease much less than high-skilled real wages, labor income for low-skilled workers actually decreases more than that for high-skilled workers due to greater drops in employment. This is because firms substitute high-skilled workers for low-skilled workers in response to the drop in the wage premium making even further decrease in low skilled workers. Moreover, the size of the rise in the employment gap is larger than that of drop in the wage premium because the two different skilled workers are highly substitutable given the parameterization, ($\eta > 1$). As a consequence, labor income inequality rises after the tightening of monetary policy. This is consistent with the findings in Pourpourides (2011) in which the author takes U.S. data from 1979 to 2003 and shows that high-skilled wages are more volatile than those of low-skilled workers, while high-skilled employment is relatively more stable than that of low-skilled workers.[30]

In addition, high-skilled consumption decreases less than low-skilled consumption as high-skilled workers smooth their consumption, consequently, consumption inequality increases after the contractionary monetary policy shock. This result is consistent with empirical evidence obtained by

[30] More recently, Champagne and Kurmann (2013) analyze the wage data of CPS in various dimension and find "substantial heterogeneity in how the absolute volatility of hourly wages of different worker groups changes over time. The largest increases in volatility occur for skilled workers (with a college degree) that are either male and young or middle-aged or salaried." As a byproduct, they also show that volatility of skilled wages is greater than that of unskilled wages in any given period and at any decomposition except for older workers (aged $60 - 70$).

Coibion et al. (2012). They intensively studied the effect of monetary policy on various measure of inequality and argue that "contractionary monetary policy shocks are associated with higher levels of economic inequality". However, according to their simulation, earning inequality decreases initially for about 2 years and the volatility of earning inequality is greater than consumption inequality for all measure of inequality. This discrepancy might arise from the absence of other sources of income because consumption relies on total income rather than on labor earnings only. For instance, as Coibion et al. (2012) noted, the labor income share of total income is larger for the higher quantiles in income distribution. This finding implies that the lower quantiles would reduce consumption relatively less after a negative labor income shock than they would when labor income is a unique source of income. More importantly, monetary policy impact on inequality is quite persistent as it depends on wage variation.

Figure 1: Dynamic responses to the positive monetary policy shock



6.2.2 High-skilled productivity shock

Figure 5 in Appendix displays impulse responses of the same variables to a positive high-skilled productivity shock. The shock widens marginal productivity gap between high-skilled and low-skilled workers, and therefore, firms increase high-skilled workers and reduce low-skilled employment. This puts upward pressure on high-skilled nominal wage and downward pressure on low-skilled nominal wages and hence raises both the wage premium and income inequality. However, as the magnitude of the changes in high-skilled wages is much larger, high-skilled employment increases less relative to the decrease in low-skilled employment. Consequently, high-skilled unemployment falls somewhat while low-skilled unemployment increase substantially. The greater high-skilled productivity leads to a decline in real marginal cost and lower inflation. The inflation falls enough to push up even the low-skilled real wages. A Taylor type rule of monetary policy forces the nominal interest rate to fall in response to the drop in inflation and results in a rise in high-skilled consumption. However, low-skilled labor income decreases because of the huge drop in employment and small increase in real wages. Consequently, low-skilled consumption decreases. This result is consistent with Heathcote et al. (2010) in that low-skilled earnings dynamics are dominated by employment fluctuation. Therefore, a high-skilled productivity shock causes a rise in both consumption and labor income inequality.

6.2.3 High-skilled wage markup shock

I also consider the high-skilled wage markup shock as another idiosyncratic shock. When the high-skilled markup rises, the high-skilled nominal wage jumps up immediately, and the wage premium rises. As a consequence, firms reduce high-skilled employment by substituting low-skilled workers, which raises the low-skilled nominal wage in contrast to the case of high-skilled productivity shock. The increase in both high-skilled and low-skilled wages induces higher inflation. Consequently, the central bank raises nominal interest rate to stabilize such a rise in inflation resulting in a higher real interest rate, followed, in turn, by a drop in high skilled consumption. Again, since high-skilled wages are relatively flexible, there is only a small amount of decrease (increase) in employment (unemployment) of high-skilled workers. However, the relatively stickier low-skilled nominal wage induces two opposite consequences. On one hand, the strong increase in inflation overturns the muted increase in nominal wages and brings about a slight decline in real wages for low-skilled workers. On the other hand, relatively more staggered wages motivate firms to demand more low-skilled workers. Accordingly, low-skilled employment increases, which leads to a drop in unemployment. Given high substitutability of labor across sectors, ($\eta > 1$), a small decrease in relative wages leads to a larger increase low-skilled labor demand, and, as a result, low-skilled labor income actually increases and thus consumption increase as well. Moreover, sufficiently large increase in low-skilled consumption dominates the decrease in high-skilled consumption, and therefore, aggregate output increases. In sum, all of aggregate output, inflation, and real wages increase, whereas both consumption and income inequality falls, as illustrated in Figure 7.

6.3 The Role of Labor Market Assumption

As noted in Introduction, the labor market assumptions are important in discussing the disproportionate effect of monetary policy and the dynamics of inequality. Figure 9 plots dynamic responses of aggregate variables and inequality measures under two alternative labor market assumptions in comparison to the baseline model. Under the single labor market assumption, the differences in consumption occur because of LAMP. However, employments are identical so that all the workers face the same labor demand and wages even though their willingness to work is different. Thus, we cannot say anything about labor income inequality and the wage premium under this assumption. Consequently, the aggregate wage falls more than the baseline model because there is no strategic complementarities in wage setting without income inequality. Accordingly, marginal cost and inflation decrease more as well. If workers are homogeneous with segmented labor markets, the wage difference occurs only due to the financial friction (LAMP) which affects the consumption level and marginal rate of substitution.[31] However, the slope of sectoral wage Phillips curves are the same so that wage premium does not have an impact on aggregate wage inflation. Furthermore, since financially excluded workers have a relatively strong incentive to work (due to lower aggregate demand) under downward pressure on wages, their employment decreases less (because labors are the same from the perspective of the firms) than workers in other sectors. Therefore, income inequality decreases in response to a contractionary monetary policy shock which is in contrast with

[31] No difference in skills and market power implies no difference in elasticities of substitution between workers within a sector

the empirical evidence the literature have shown.

7 Optimal Monetary Policy

In the previous chapter, I showed that monetary policy has a disproportionate effect on labor market variables and eventually causes changes in consumption and labor income inequality. In this section, I approach the same phenomenon from the opposite direction by asking if inequality and heterogeneous response of households with different characteristics affect optimal monetary policy design. To this end, I derive the welfare loss function of the economy, and contemplate the optimal monetary policy. Following Bilbiie (2008), I assume that the social planner maximizes the convex combination of the utilities of the two types of households, weighted by the mass of agents of each type:

$$\mathcal{W}_t = \left\{ s\left(U(C_t^H) - V(N_t^H)\right) + (1-s)\left(U(C_t^L) - V(N_t^L)\right) \right\} \tag{7.1}$$

A Central Bank's loss function is obtained by the second order approximation of the welfare around the efficient steady state as in Woodford (2003)[32]:

$$\mathcal{L} = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t \left\{ (\sigma+\varphi)\, x_t^2 + \frac{\varepsilon_p}{\kappa_p}\left(\pi_t^p\right)^2 + \frac{s\varepsilon_w^H}{\kappa_w^H}\left(\pi_t^H\right)^2 + \frac{(1-s)\varepsilon_w^L}{\kappa_w^L}\left(\pi_t^L\right)^2 + \psi_c\left(\mathcal{G}_t^c\right)_t^2 + \psi_I\left(\mathcal{G}_t^I\right)^2 \right\}$$

where $\psi_c \equiv \frac{(\sigma-1)}{s(1-s)}$ and $\psi_I \equiv \frac{(1+\varphi)}{s(1-s)}\left(\frac{\eta}{\eta-1}\right)^2 > 0$. Because prices and wages are sticky, any change in those variables causes inefficient dispersion in prices and wages and hence inefficient output. This inefficiency is captured by inflation and output gap terms in loss function. Obviously, as prices and wages get stickier (κ_p and $\kappa_w^j \rightarrow 0$), the Central Bank puts more weight on the corresponding inflation. In comparison to the standard model, the loss function has two additional terms; consumption and income Gini coefficient which are associated with LAMP and segmented labor market respectively. Therefore, so long as the financial and labor market are segmented, changes in wages affect inequality and hence a welfare loss.[33] If high skilled labor is equally substitutable for the low skilled workers, a Central Bank does not need to be concerned about sectoral wage inflations, but aggregate wage inflation matters for welfare loss. As elasticity of

[32] see Appendix D for details.

[33] Recall that consumption inequality and income inequality are derived from the weighted sum of two sectoral wages and wage premium respectively.

substitution across labor sectors becomes larger within a plausible range of parameter suggested by Mollick (2011), loss from income inequality gets smaller. This occurs because when workers are perfectly substitutable, firms can fully accommodate a shock in relative wages (and hence income inequality) by substituting workers with different skills. Therefore, the effect of the shock on output distortion will be negligible. As population is distributed equally into two sectors (as $s \to \frac{1}{2}$), inequality measures become less important.

7.1 A new policy trade-off with inequality

It is well known that equilibrium with flexible prices and wages is not attainable if both prices and wages are sticky unless the natural wage is constant. There exists, that is, a policy trade-off between three standard target variables; output gap, price inflation and wage inflation. As stated above, without nominal rigidities in wage, wage inflations do not affect welfare, and thus the Central Bank only needs to be concerned with the variations of inequality in addition to output gap and price inflation. If there is no variation in inequality, a strict inflation targeting rule leads to an efficient equilibrium by achieving zero inflation and output gap simultaneously. However, inequality that arises from the differential in wage and consumption across labor sectors introduces a new trade-off so that a Central Bank cannot achieve the first best allocation even if wages are flexible. To distinguish the role of inequality in optimal monetary policy from the standard one that arises from nominal rigidities in wages, I consider an economy with flexible wages in this subsection. Equilibrium wages are determined at a marginal rate of substitution between consumption and labor supply for each type of household in any given period under flexible wages. Subtracting the low-skilled equilibrium wage from the high-skilled equilibrium wage, I obtain a relationship between the output gap and income inequality.[34] If I assume log-utility, $\sigma = 1$, then the equation is simplified further:

$$\mathcal{G}_t^I = (1-s)(\eta-1)x_t + \frac{s(1-s)(\eta-1)}{\eta}(a_t^H - a_t^L) \tag{7.2}$$

where I use $\widetilde{\omega}_t = (\sigma + \varphi)x_t$ which is obtained from the convex combination of two sectoral wages weighted by their population share. The equation (7.2) shows that a Central Bank is not able to

[34] $\widetilde{\omega}_t^R = \frac{\sigma}{1+\varphi\eta} c_t^R - \frac{1+\varphi}{1+\varphi\eta}\left(a_t^H - a_t^L\right)$ and $c_t^R = -\frac{1}{s}\left(\widetilde{\omega}_t + s(\eta-1)\widetilde{\omega}_t^R\right)$.

completely stabilize both income inequality and output gap at the same time after idiosyncratic productivity shocks even under flexible wages.

7.2 Dynamic response under optimal monetary policy

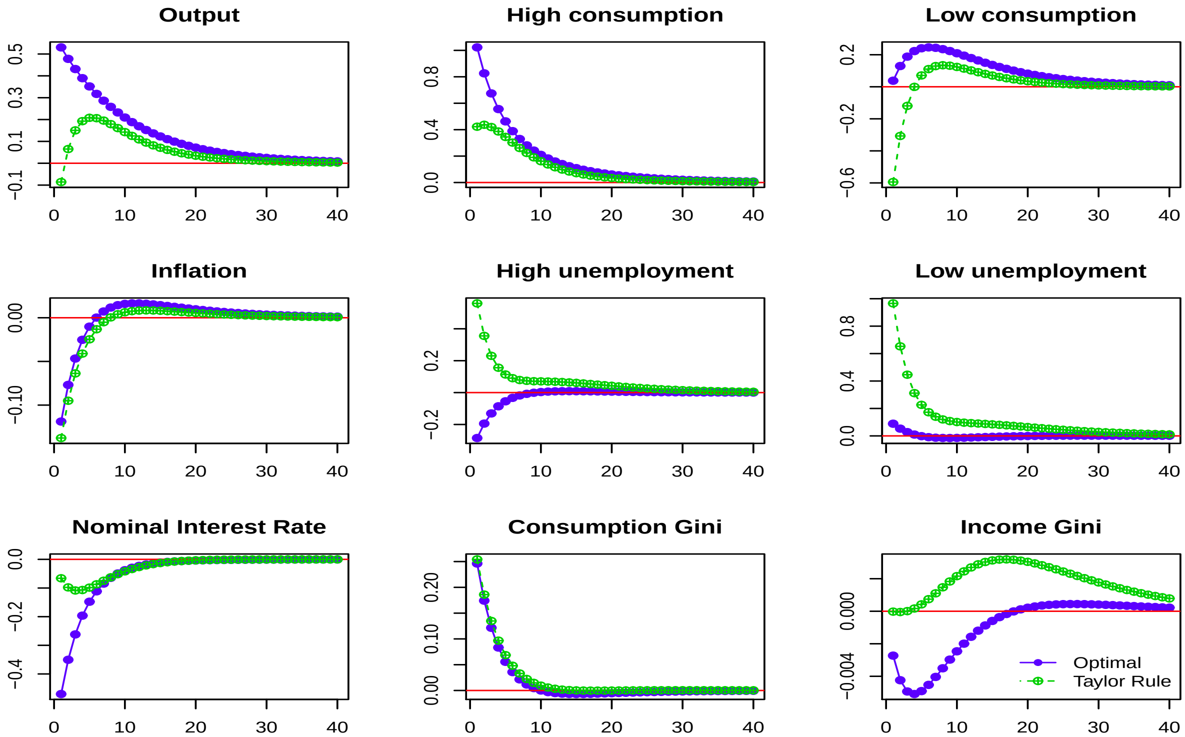
This section explores optimal monetary policy under full commitment in which a Central Bank minimizes the loss function above, (D.10), subject to the given constraints.[35] I compare the dynamic responses of endogenous variables under optimal monetary policy with those under the Taylor rule to see how they differ in response to aggregate as well as idiosyncratic shocks.

7.2.1 A positive technology shock

Figure 2 plots impulse responses to a positive aggregate technology shock. In response to the technology shock, optimal monetary policy lowers the nominal interest rate substantially, allowing output and high-skilled consumption to increase more than those variables under Taylor rule. Accordingly, the optimal monetary policy encourages firms to demand more workers as opposed to the Taylor rule. However, the shock does not cause a variation in the relative marginal productivity across sectors, and thus, the changes in employment rates are almost the same for both sectors initially. The increase in labor demand induces higher wages for both high-skilled and low-skilled workers. Once the nominal wage increases, however, the wage premium increases due to effectively stickier low-skilled nominal wages, and in turn low-skilled employment rises more than high skilled employment. As a result, income inequality falls opposite to that under Taylor rule. In addition, low-skilled workers are more willing to supply their labor relative to high-skilled workers and thus the low-skilled unemployment rate slightly increases while the high-skilled unemployment actually decreases, which is in stark contrast with huge increases in both high-skilled and low-skilled unemployment rates observed under the Taylor rule. On the other hand, the optimal monetary policy, which is much more accommodative, leads to a greater inflation and a less increase in real wages. However, labor income for both high and low skilled workers increases due to sufficient increase in employment, and accordingly, low-skilled consumption rises as well.

[35] Details for the constraints and the first order conditions are provided in Appendix E.

Figure 2: Dynamic responses to the positive technology shock



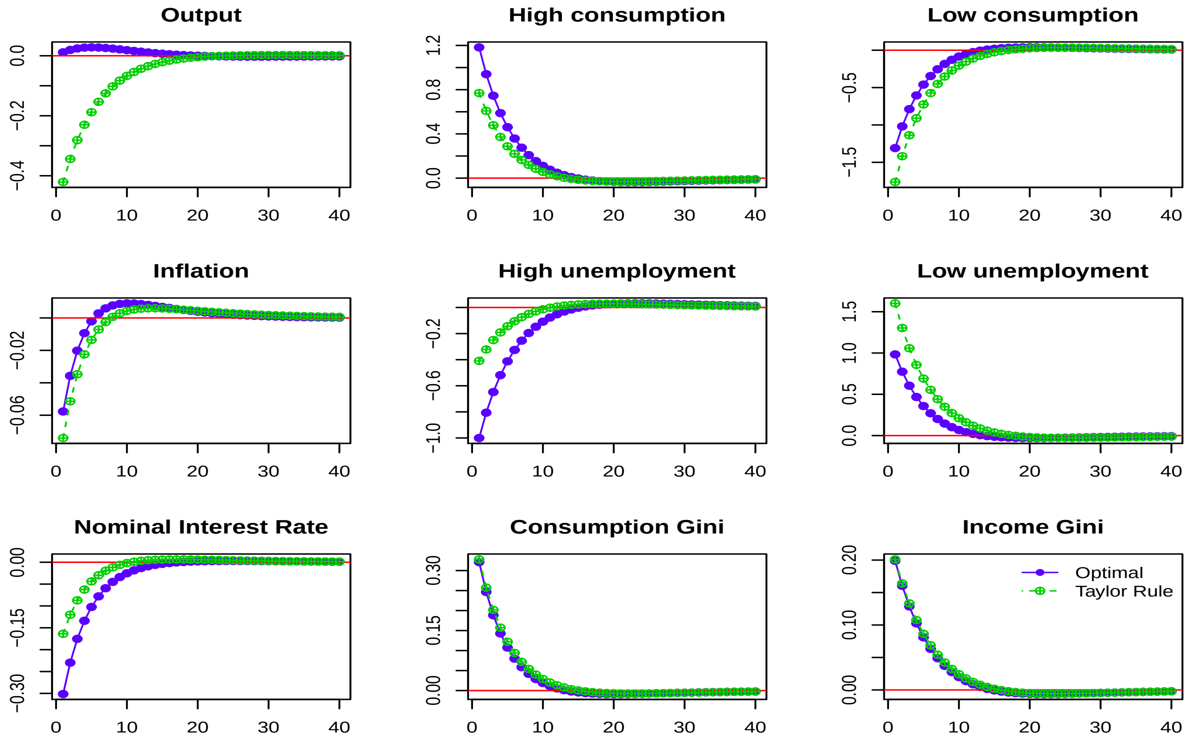
7.2.2 A high-skilled wage markup shock

Figure 12 shows the response of the same variables to a positive high-skilled wage markup shock. An exogenous increase in high-skilled wages induces a sharp decrease in high-skilled employment, and consequently high-skilled unemployment increases. A rise in real wages also leads to a higher marginal cost and inflation. However, since optimal monetary policy again aggressively responds to this change, the increase inflation is muted resulting in a higher real interest rate. Consequently, the high-skilled consumption decreases substantially. Such a huge drop in high-skilled consumption causes a drop in aggregate output followed by lower demand for low-skilled workers. Accordingly, low-skilled unemployment rather increases somewhat in contrast to that under Taylor rule. This, in turn, pushes low-skilled wages down, and as a result, the labor income for both high and low skilled workers decreases. However, a substantial drop in high-skilled employment induces lower income inequality. The optimal monetary policy in response to this idiosyncratic shock becomes remarkable in comparison to aggregate shock in that the optimal monetary policy generates effectively different path of inequality; the dynamic responses of inequality are muted under an optimal monetary policy relative to under the Taylor rule.

7.2.3 A positive high-skilled productivity shock

Figure 3 displays the dynamic responses of the same variables to an exogenous increase in high-skilled productivity. Firms demand more high-skilled workers immediately in response to the idiosyncratic shock by substituting low-skilled workers. Consequently, high-skilled nominal wages increase and low-skilled wages drop. However, the magnitude of the rise in high-skilled employment is diminished by the substantive increase in wages and low-skilled employment falls substantially due to stickier norminal wages. Therefore, aggregate employment decreases. Since both wages and employment drop for low-skilled workers, their consumption decrease as well. On the other hand, due to the greater productivity of high-skilled workers, marginal costs and thus inflation decrease.The optimal monetary policy responds to these changes in objective variables by lowering nominal interest rate aggressively. As a result, high-skilled consumption increases substantially dominating the decrease in low-skilled consumption. As a result, aggregate output actually rises which, in turn, offests the drop in low-skilled employment and pushs up high-skilled employment. Therefore, the low-skilled consumption decreases less in comparison to that under Taylor rule and their unemployment rate rises less severely. In addition, aggregate unemployment rate remains virtually unchanged in contrast to the significant increase of it under the Taylor rule.

Figure 3: Dynamic responses to the high-skilled productivity shock



7.3 Counter-factual experiments

In this section, I examine aggregate dynamics under three hypothetical scenarios in comparison to those of the present model (benchmark model).[36] In the benchmark model, a Central Bank sets an optimal interest rate, considering segmented labor markets under sticky wages. In Scenario 1, a Central Bank considers a single labor market in which the representative union sets a desired wage for each type of workers regardless of their skill level, but the wages are sticky. In Scenario 2, a Central Bank is aware of the segmented labor market, but considers flexible wages. In Scenario 3, a Central Bank considers a single labor market with flexible wages. All the Central Banks admit that some fraction of the total population is financially excluded.

7.3.1 Scenario 1: Single labor market with staggered wages

If workers are homogeneous with respect to the demand elasticity in a single labor market then loss function is simplified to the one in Ascari et al. (2011). In this scenario, workers are different

[36]See Appendix F for a detailed description of each scenario.

only in financial accessibility and face the same wages and labor demand. Since the representative unions of i-type of workers who maximize weighted average of both skilled life-time utility, all i-type workers face the same labor demand and wages regardless of their skill level. A Central Bank now cares only aggregate real wage gap arising from LAMP rather than sectoral wage gaps or inequality measures. As Ascari et al. (2011) noted, LAMP does not affects optimal monetary policy design, because consumption inequality only shows up in demand equation, (5.3), and supply block of the economy is not influenced by the income inequality, in fact, there is no income inequality in this scenario. In a special case when $\sigma = 1$, a Central Bank's loss function and the constraints are exactly the same as the one in Ascari et al. (2011). Evidently, if all agents are able to smooth consumption by holding bonds, that is $s = 1$, the loss function collapses to one in Erceg et al. (2000) as well.

7.3.2 Scenario 2: Segmented labor market with flexible wage case

When the economy approaches to a flexible wages, $\theta_w \to 0$ and $\frac{\varepsilon_w^j}{\kappa_w^j} \to 0$, welfare losses from wage dispersion become negligible and hence terms associated with wage inflations disappear. However, presence of Rule-of-Thumb agents (low-skilled workers) still matters for welfare loss because LAMP imposes different marginal rate of substitution across different skilled workers, wages, and consumption, in turn, which causes consumption and income inequality variation. In addition, since marginal cost (real wage gap in baseline model) become a proportional to output gap, inflation is directly affected by output fluctuation via (3.9). Therefore, the trade-off explained above takes place, and a Central Bank need to allow output gap variations in the face of changes in inequality. As a special case, when production function is Cobb-Douglas ($\eta = 1$), income inequality becomes constant, and thus, a Central Bank consider wage premium rather than income inequality. Therefore, an idiosyncratic productivity shock causes a trade-off between output gap and wage premium. In this case, the larger steady state high-skilled income share implies the more weight on wage premium variation.

7.3.3 Scenario 3: Single labor market with flexible wages

If there is a single labor market and the wages are flexible, the loss function is exactly the same as the standard New Keynesian model with LAMP such as Bilbiie (2008). In this case, welfare loss comes from price and output variation only. The existence of low-skilled workers who are financially constrained suggest a Central Bank to put more weight on output relative to standard NK model, since these workers are affected directly by output fluctuation but not inflation. If there is no cost-push shock, then a Central Bank can impose zero inflation and output gap by strict inflation targeting and the economy achieves the first best allocation.

7.3.4 Welfare analysis

As I expected, the welfare losses under the optimal monetary policy when a Central Bank ignores (or is not aware of) inequality are much greater than that of the baseline model. Table 2 reports the relative welfare loss of the first scenario (single labor market) in comparison to the baseline model after aggregate and idiosyncratic productivity shocks as well as sectoral wage markup shocks. If a Central Bank ignores inequality by focusing only on the aggregate variables, welfare loss is 0.87% higher than the baseline model after aggregate technology shock. Since aggregate shock does not cause trade-off with inequality, the extra losses come only from the stickier aggregate nominal wage adjustment caused by income inequality. As Figure 13 shows, aggregate dynamics of endogenous variables under different policies are not distinguishable after aggregation technology shock; they are different only if the Central Bank thinks differently on the wage stickiness. However, the optimal monetary policy induces significantly larger welfare loss in response to idiosyncratic shocks when a central bank do not care of inequality variation. The welfare losses are 5.19% and 5.16% greater than the benchmark model after positive high-skilled and low-skilled productivity shock respectively. Similarly, the loss are 1.8% and 1% larger after high-skilled and low-skilled markup shock respectively.

Table 2: Relative Welfare Losses

Scenario	Productivity shock			Markup shock	
	Aggregate	High-skilled	Low-skilled	High-skilled	Low-skilled
Baseline	1	1	1	1	1
Single labor market	1.000873	1.051882	1.051621	1.017812	1.010268

8 Conclusion

As data indicate, households with different characteristics behave very differently over the business cycle and income inequality moves countercyclically in response to monetary policy shock. In this paper, I have shown that segmented labor markets with limited asset market participation can account for the differentials in labor market variables and the dynamics of inequality. Any economic shock that affects the relative wage results in a variation in income inequality and the change in inequality amplifies aggregate dynamics through strategic complementarities in wage setting. In particular, a contractionary monetary policy, which has a disproportionate effect on labor market variables, lowers the wage premium and thus raises income inequality. A variation in income inequality enhances the stickiness of aggregate wage adjustments and leads to greater fluctuations in macroeconomic variables such as output, employment, and unemployment. Welfare analysis based on the central bank's loss function, which is obtained from the weighted average of households' life time utility, suggests that a Central Bank needs to react more aggressively to an output gap relative to the standard Taylor rule. In addition, when a Central Bank ignores heterogeneity in the labor market and thus inequality, its optimal monetary policy causes substantive welfare losses relative to those under the benchmark model in which a Central Bank takes into account inequality variation.

Finally, according to Reis (2013), "if financial stability is to be included as a separate goal for the Central Bank, it must pass certain tests: 1) there must be a measurable definition of financial stability, 2) there has to be a convincing case that monetary policy can achieve the target of bringing about a more stable financial system, and 3) financial stability must pose a trade-off with the other two goals, creating situations where prices and activity are stable but financial instability justifies a change in policy that potentially leads to a recession or causes inflation to exceed its target." Even though, financial stability might not be an appropriate target variable for a Central Bank as Reis (2013) mentioned, income inequality fulfills those three criteria. Income inequality is measurable, causes substantive welfare loss, and poses a trade-off with output gap. Thus, a Central Bank should pay attention to income inequality in addition to inflation and the output gap.

There are some useful extensions of the model. First, a fraction of the population who is excluded from the financial markets can vary over the business cycle. In a recession, more people will have limited access to the financial market, and, therefore, low skilled unemployment becomes more vulnerable to an economic shock. Accordingly, variations in income inequality will be larger, and more attention to inequality by a Central Bank will therefore be required. Second, even though labor income is the largest contributor to total income for households, other income sources may also affect households' behavior. For example, an expansionary monetary policy that lowers nominal interest rate raises asset prices. However, since high-skilled workers whose average labor income is larger than low-skilled workers tend to hold more financial assets, the income gap between two different types of households will be widened. In addition, when nominal interest rates hit the zero lower bound, an unconventional monetary policy intended to boost the economy may cause wider income inequality as shown in Saiki and Frost (2014). Therefore, other income sources such as capital income might have a significant impact that mitigates the positive effect of expansionary monetary policy on inequality and the result would enhance the portfolio channel of the monetary policy.

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Appendix A Efficient Equilibrium

When wage markups are gone, both labor markets become identical, and all the workers get the same wages, and hence, enjoy the same level of consumption as if representative agents. Thus, the marginal product of labor and marginal rate of substitution for workers of different skill levels are the same. I then define efficient equilibrium condition as:

$$mpn_t = \omega_t = mrs_t$$

$$\begin{aligned} a_t &= \sigma y_t + \varphi h_t + \xi_t = (\sigma + \varphi)\, y_t - \varphi a_t + \xi_t \\ y_t^E &= \frac{1+\varphi}{\sigma+\varphi} a_t - \frac{1}{\sigma+\varphi}\xi_t \quad \text{and} \quad \omega_t^E = a_t \end{aligned} \tag{A.1}$$

Appendix B Household's total labor income

Each household consists of a continuum of workers, and the total income of the household is just the sum of each worker's labor income. Therefore, j-skilled household's total income is written by

$$\begin{aligned} \mathcal{X}_t^j &= \int_0^1 \omega_t^j(i) \int_0^1 N^j(i,z) dz di = \int_0^1 \omega_t^j(i) \int_0^1 N_t^j(z) \frac{N_t^j(i,z)}{N_t^j(z)} dz di \\ &= \int_0^1 \omega_t^j(i) \int_0^1 N_t^j(z) \left(\frac{\omega_t^j(i)}{\omega_t^j} \right)^{-\varepsilon_w^j} dz di = \omega_t^j \underbrace{\int_0^1 \left(\frac{\omega_t^j(i)}{\omega_t^j} \right)^{1-\varepsilon_w^j} di}_{=\,1} \int_0^1 N_t^j(z) dz \\ &= \omega_t^j \int_0^1 H_t(z) \left(\frac{N_t^j(z)}{H_t(z)} \right) dz = \omega_t^j \int_0^1 H_t(z) \gamma \left(\frac{\omega_t^j}{\omega_t} \right)^{\eta} dz \\ &= \gamma \omega_t^j \left(\frac{\omega_t^j}{\omega_t} \right)^{-\eta} \frac{Y_t}{A_t} \underbrace{\int_0^1 \left(\frac{P_t(z)}{P_t} \right)^{-\varepsilon_p} dz}_{\Delta_t^p} \\ &\approx \omega_t^j - \eta \left(\omega_t^j - \omega_t \right) + y_t - a_t \end{aligned} \tag{B.1}$$

for $j \in \{H, L\}$ and where Δ_t^p is measure for the price dispersion and is second order term. Since the Gini coefficient for income $\mathcal{G}_t^I$ is given by $(1-s)\left(1-\frac{\mathcal{X}_t^L}{\mathcal{X}_t}\right)$, it is approximated as $-(1-s)\left(\widehat{\mathcal{X}}_t^j - \widehat{\mathcal{X}}_t\right)$:

$$
\begin{aligned}
& -(1-s)\left(\widehat{\mathcal{X}}_t^j - \widehat{\mathcal{X}}_t\right) \\
= & -(1-s)\left[\omega_t^L + \eta s \omega_t^R + y_t - a_t - \left\{s\left(\omega_t^H - \eta(1-s)\omega_t^R + y_t - a_t\right) + (1-s)\left(\omega_t^L + \eta s \omega_t^R + y_t - a_t\right)\right\}\right] \\
= & \quad -s(1-s)(\eta-1)\,\omega_t^R
\end{aligned} \tag{B.2}
$$

Appendix C Wage Phillips curve

C.1 High-skilled wage Phillips Curve ($\kappa_w^H \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\varepsilon_w^H\varphi)}$)

$$
\begin{aligned}
& \pi_t^H = \beta E_t \pi_{t+1}^H - \kappa_w^H \mu_t^H + \mu_t^{Hn} \\
\Rightarrow\ & \mu_t^H = \omega_t^H - \sigma c_t^H - \varphi n_t^H - \xi_t \\
& = \omega_t^H - \sigma\left(\frac{c_t - (1-s)c_t^L}{s}\right) - \varphi n_t^H - \xi_t = \omega_t^H - \frac{\sigma}{s}c_t + \frac{\sigma(1-s)}{s}c_t^L - \varphi n_t^H - \xi_t \\
& = \omega_t^H - \frac{\sigma}{s}c_t + \frac{\sigma(1-s)}{s}\left(\omega_t^L + n_t^L\right) - \varphi\left\{-\eta(1-s)(\omega_t^H - \omega_t^L) + y_t - a_t\right\} - \xi_t \\
& = \underbrace{(1+\eta(1-s)(\sigma+\varphi))}_{\equiv\lambda_H^H}\omega_t^H - \underbrace{\left(\eta(1-s)(\sigma+\varphi) - \frac{\sigma(1-s)}{s}\right)}_{\equiv\lambda_H^L}\omega_t^L \\
& + \left(-\frac{\sigma}{s} + \frac{\sigma(1-s)}{s} - \varphi\right)y_t + \left(-\frac{\sigma(1-s)}{s} + \varphi\right)a_t - \xi_t \\
& = -(\sigma+\varphi)y_t + \left(-\frac{\sigma(1-s)}{s} + \varphi\right)a_t + \lambda_H^H\omega_t^H - \lambda_H^L\omega_t^L - \xi_t \\
& = -(\sigma+\varphi)x_t + \lambda_H^H\widetilde{\omega}_t^H - \lambda_H^L\widetilde{\omega}_t^L \\
\Rightarrow\ & \pi_t^H = \beta E_t\pi_{t+1}^H + \kappa_w^H(\sigma+\varphi)x_t - \kappa_w^H\lambda_H^H\widetilde{\omega}_t^H + \kappa_w^H\lambda_H^L\widetilde{\omega}_t^L + \mu_t^{Hn}
\end{aligned} \tag{C.1}
$$

This also can be written in terms of relative consumption and wage

$$
= \beta E_t\pi_{t+1}^H + \kappa_w^H(\sigma+\varphi)x_t + \kappa_w^H\left((\sigma+\varphi)\eta s - \varphi\eta\right)\widetilde{\omega}_t^R + \kappa_w^H\sigma c_t^R - \kappa_w^H\left(\widetilde{\omega}_t^H - \sigma\widetilde{\omega}_t^L\right) + \mu_t^{Hn} \tag{C.2}
$$

C.2 Low-skilled wage Phillips Curve

$$
\begin{aligned}
\pi_t^L &= \beta E_t \pi_{t+1}^L - \kappa_w^L \mu_t^L + \mu_t^{Ln} \\
\Rightarrow \mu_t^L &= \omega_t^L - mrs_t^L = \omega_t^L - \sigma c_t^L - \varphi n_t^L - \xi_t \\
&= \omega_t^L - \sigma(\omega_t^L + n_t^L) - \varphi n_t^L - \xi_t = (1-\sigma)\omega_t^L - (\sigma+\varphi) n_t^L - \xi_t \\
&= (1-\sigma)\omega_t^L - (\sigma+\varphi)\left(\eta s\left(\omega_t^H - \omega_t^L\right) + y_t - a_t\right) - \xi_t \\
&= -(\sigma+\varphi)\eta s \omega_t^H + \left((1-\sigma) + (\sigma+\varphi)\eta s\right)\omega_t^L - (\sigma+\varphi)(y_t - a_t) - \xi_t \\
&= -\left\{\left((\sigma-1) - (\sigma+\varphi)\eta s\right)\widetilde{\omega}_t^L + (\sigma+\varphi)\eta s \widetilde{\omega}_t^H\right\} - (\sigma+\varphi) x_t \\
\Rightarrow \pi_t^L &= \beta E_t \pi_{t+1}^L + \kappa_w^L (\sigma+\varphi) x_t + \kappa_w^L (\sigma+\varphi)\eta s \widetilde{\omega}_t^H - \kappa_w^L\left((\sigma+\varphi)\eta s - (\sigma-1)\right)\widetilde{\omega}_t^L + \mu_t^{Ln} && \text{(C.3)} \\
\text{or,} \quad \pi_t^L &= \beta E_t \pi_{t+1}^L + \kappa_w^L (\sigma+\varphi) x_t + \kappa_w^L (\sigma+\varphi)\eta s \widetilde{\omega}_t^R + \kappa_w^L (\sigma-1)\widetilde{\omega}_t^L + \mu_t^{Ln} && \text{(C.4)}
\end{aligned}
$$

C.3 Aggregate wage Phillips curve

By definition, I aggregate wage Phillips is a convex combination of two sectoral wage Phillips curves weighted by corresponding population share.

$$
\begin{aligned}
\pi_t^w &= s\pi_t^H + (1-s)\pi_t^L \\
&= \beta \pi_{t+1}^w + \underbrace{\left(s\kappa_w^H + (1-s)\kappa_w^L\right)}_{\equiv\ \kappa_w}(\sigma+\varphi)\,x_t - \left(\kappa_w^H - \kappa_w^L\right)\eta s(1-s)\,(\sigma+\varphi)\,\widetilde{\omega}_t^R \\
&\quad - \left(\kappa_w^H - \kappa_w^L\right)(1-s)\sigma\widetilde{\omega}_t^L - \left(s\kappa_w^H \widetilde{\omega}_t^H + (1-s)\kappa_w^L \widetilde{\omega}_t^L\right) + \underbrace{\left(s\mu_t^{Hn} + (1-s)\mu_t^{Ln}\right)}_{\equiv\ \epsilon_t} \\
\text{(and if } \sigma = 1) \quad &= \beta \pi_{t+1}^w + \kappa_w\,(1+\varphi)\,x_t + \underbrace{\left(\kappa_w^H - \kappa_w^L\right)\frac{\eta\,(1+\varphi)}{\eta-1}}_{\equiv\ \Upsilon}\mathcal{G}_t^I - \kappa_w^H \widetilde{\omega}_t + \epsilon_t && \text{(C.5)}
\end{aligned}
$$

C.4 Special Case with log utility and homogeneous labor ($\sigma = 1$, $\varepsilon_w^H = \varepsilon_w^L$)

If $\varepsilon_w^H = \varepsilon_w^L$, (C.5) is then simplified further,

$$
\pi_t^w = \beta \pi_{t+1}^w + \kappa_w\,(1+\varphi)\,x_t - \kappa_w \widetilde{\omega}_t + \epsilon_t \tag{C.6}
$$

and subtracting (C.4) from (C.2) I obtain,

$$\widetilde{\omega}_t^R = \psi_R \widetilde{\omega}_{t-1}^R + \beta\psi_R E_t \widetilde{\omega}_{t+1}^R + \psi_R \kappa_w c_t^R + \epsilon_t^D \tag{C.7}$$

where $\psi_R \equiv \frac{1}{1+\beta+\kappa_w(1+\varphi\eta)}$ and $\epsilon_t^D \equiv \psi_R\left(\mu_t^{Hn} - \mu_t^{Ln}\right)$

Appendix D Utility-based Loss Function (Woodford (2003))

$$W_t \equiv \left\{ s\left(U(C_t^H) - V(N_t^H)\right) + (1-s)\left(U(C_t^L) - V(N_t^L)\right)\right\}$$

- Efficient Steady State:

$$\begin{aligned} C(z) = C = Y \ \text{ and } \ \frac{Y}{H} &= A = 1 \Rightarrow C = H \\ C = C^H = C^L, \quad H &= N^H = N^L \\ \frac{W}{P} = MRS = MPN \Leftrightarrow \frac{W}{P} &= C^\sigma H^\varphi = A = 1 \Rightarrow U_c = V_N \end{aligned}$$

- Utility from Consumption $\left(\text{for a variable X, } \frac{X_t - X}{X} \approx x_t + \frac{1}{2}x_t^2\right)$

$$\begin{aligned} U(C_t^j) &\approx U(C) + U_c C\left(\frac{C_t^j - C}{C}\right) + \frac{U_{cc}C^2}{2}\left(\frac{C_t^j - C}{C}\right)^2 + \mathcal{O}\left(||\zeta||\right)^3 \\ U(C_t^j) - U(C) &\approx U_c C\left\{\left(\frac{C_t^j - C}{C}\right) + \frac{U_{cc}C}{2U_c}\left(\frac{C_t^j - C}{C}\right)^2\right\} + \mathcal{O}\left(||\zeta||\right)^3 \\ &\approx U_c C\left\{c_t^j + \frac{1}{2}\left(c_t^j\right)^2 - \frac{\sigma}{2}\left(c_t^j\right)^2\right\} + \mathcal{O}\left(||\zeta||\right)^3 \end{aligned} \tag{D.1}$$

- Disutility from Labor Supply

$$
\begin{aligned}
V(N_t^j) &\approx V(N) + V_j N \left(\frac{N_t^j - N}{N} \right) + \frac{V_{jj} N^2}{2} \left(\frac{N_t^j - N}{N} \right)^2 \\
&\quad + V_\chi \left(\chi_t - 1 \right) + V_{\chi N} N \left(\frac{\chi_t - 1}{1} \right) \left(\frac{N_t^j - N}{N} \right) + V_{\chi\chi} \left(\chi_t - 1 \right)^2 + \mathcal{O}\left(||\zeta||^3 \right) \\
V(N_t^j) - V(N) &\approx V_j N \left\{ \left(\frac{N_t^j - N}{N} \right) + \frac{V_{jj} N}{2 V_j} \left(\frac{N_t^j - N}{N} \right)^2 + \left(\frac{N_t^j - N}{N} \right) \xi_t \right\} + t.i.p + \mathcal{O}\left(||\zeta||^3 \right) \\
&\approx V_j N \left\{ n_t^j + \frac{1}{2} \left(n_t^j \right)^2 + \frac{\varphi}{2} \left(n_t^j \right)^2 + n_t^j \xi_t \right\} + t.i.p + \mathcal{O}\left(||\zeta||^3 \right) \qquad \text{(D.2)}
\end{aligned}
$$

- Combine (D.1) and (D.2)

$$
\frac{W_t - W}{U_c C} \approx \left[\left\{ c_t + \frac{1-\sigma}{2} \left(s (c_t^H)^2 + (1-s)(c_t^L)^2 \right) \right\} - \left\{ h_t + \frac{1+\varphi}{2} \left(s (n_t^H)^2 + (1-s)(n_t^L)^2 \right) + h_t \xi_t \right\} \right] \tag{D.3}
$$

where $U_c C = V_j N$, $c_t = s c_t^H + (1-s) c_t^L$, and $h_t = s n_t^H + (1-s) n_t^L$

- $s \left(c_t^H \right)^2 + (1-s) \left(c_t^L \right)^2$

$$
c_t^L = \omega_t^L + n_t^L = \omega_t^L + \eta s \omega_t^R + \widehat{\Delta}_t^L + \widehat{\Delta}_t^p + y_t - a_t = \widehat{\Delta}_t^L + \widehat{\Delta}_t^p + \underbrace{(1-\eta s)\omega_t^L + \eta s \omega_t^H}_{\equiv \Phi_t} + y_t - a_t
$$

$$
\left(c_t^L \right)^2 = \Phi_t^2 + y_t^2 + 2\Phi_t y_t - 2\Phi_t a_t - 2 y_t a_t + t.i.p + \mathcal{O}\left(||\zeta||^3 \right) \tag{D.4}
$$

$$
\begin{aligned}
c_t^H &= \frac{c_t-(1-s)c_t^L}{s} = \frac{1}{s}c_t - \frac{1-s}{s}c_t^L \\
\left(c_t^H\right)^2 &= \left(\frac{1}{s}\right)^2 c_t^2 + \left(\frac{1-s}{s}\right)^2 \left(c_t^L\right)^2 - 2\left(\frac{1}{s}\frac{1-s}{s}\right) c_t c_t^L \\
&= \left(\frac{1}{s}\right)^2 c_t^2 + \left(\frac{1-s}{s}\right)^2 \left(\Phi_t^2 + y_t^2 + 2\Phi_t y_t - 2\Phi_t a_t - 2y_t a_t\right) - 2\left(\frac{1-s}{s^2}\right) c_t\left(\Phi_t + y_t - a_t\right) \\
&= \underbrace{\left(\frac{1}{s^2} + \left(\frac{1-s}{s}\right)^2 - 2\frac{1-s}{s^2}\right)}_{=1} y_t^2 + \left(\frac{1-s}{s}\right)^2 \left(\Phi_t^2 - 2\Phi_t a_t\right) \\
&+ 2\underbrace{\left(\left(\frac{1-s}{s}\right)^2 - \frac{1-s}{s^2}\right)}_{=-(1-s)/s} \left(\Phi_t y_t - y_t a_t\right)
\end{aligned}
$$

$$
\begin{aligned}
&\Rightarrow s\left(c_t^H\right)^2 + (1-s)\left(c_t^L\right)^2 \\
&\quad = y_t^2 + \left((1-s) + \frac{(1-s)^2}{s}\right)\left(\Phi_t^2 - 2\Phi_t a_t\right) + 2\left((1-s)-(1-s)\right)\left(\Phi_t y_t - y_t a_t\right) \\
&\quad = y_t^2 + \frac{1-s}{s}\left(\Phi_t^2 - 2\Phi_t a_t\right) \qquad \text{(D.5)}
\end{aligned}
$$

- $sn_t^H + (1-s)n_t^L$

$$
\begin{aligned}
&\Rightarrow s\left(\widehat{\Delta}_t^H + \widehat{\Delta}_t^p - s(1-s)\omega_t^R + y_t - a_t\right) + (1-s)\left(\widehat{\Delta}_t^L + \widehat{\Delta}_t^p + \eta s\omega_t^R + y_t - a_t\right) \\
&\quad = \underbrace{s\widehat{\Delta}_t^H + (1-s)\widehat{\Delta}_t^L}_{\equiv\widehat{\Delta}_t^w} + \widehat{\Delta}_t^p + y_t - a_t \qquad \text{(D.6)}
\end{aligned}
$$

- $s\left(n_t^H\right)^2 + (1-s)\left(n_t^L\right)^2$

$$
\begin{aligned}
&\Rightarrow s\left\{(\eta(1-s))^2\left(\omega_t^R\right)^2 + y_t^2 - 2\eta(1-s)\omega_t^R y_t + 2\eta(1-s)\omega_t^R a_t - 2y_t a_t\right\} \\
&\quad + (1-s)\left\{(\eta s)^2\left(\omega_t^R\right)^2 + y_t^2 + 2\eta s\omega_t^R y_t - 2\eta s\omega_t^R a_t - 2y_t a_t\right\} \\
&\quad = \eta^2 s(1-s)\left(\omega_t^R\right)^2 + y_t^2 - 2y_t a_t \qquad \text{(D.7)}
\end{aligned}
$$

- substituting (D.5), (D.6) and (D.7) for (D.3) we get,

$$\frac{W_t - W}{U_c C} \approx -\frac{1}{2}\left(\begin{array}{c} 2\left(\widehat{\Delta}_t^w + \widehat{\Delta}_t^p\right) + (\sigma+\varphi)\, y_t^2 - (1+\varphi)\, 2y_t a_t + 2y_t\xi_t \\ \\ +\frac{(\sigma-1)(1-s)}{s}\left(\Phi_t - a_t\right)^2 + (1+\varphi)\eta^2 s(1-s)\left(\omega_t^R\right)^2 \end{array}\right) + t.i.p + \mathcal{O}\left(||\zeta||^3\right) \tag{D.8}$$

where $(\sigma+\varphi)y_t y_t^E = ((1+\varphi)a_t - \xi_t)\, y_t$ and $\Phi_t^E = a_t$ and $\omega_t^{R,E} = 0$. Now, replacing inequality measure,(5.1) and (5.2), for the last two terms, I get

$$\frac{W_t - W}{U_c C} \approx -\frac{1}{2}\left(\begin{array}{c} 2\left(\widehat{\Delta}_t^w + \widehat{\Delta}_t^p\right) + (\sigma+\varphi)\, y_t^2 - (1+\varphi)\, 2y_t a_t + 2y_t\xi_t \\ \\ +\underbrace{\frac{(\sigma-1)}{s(1-s)}}_{\psi_c}\left(\mathcal{G}_t^c\right)^2 + \underbrace{\frac{(1+\varphi)}{s(1-s)}\left(\frac{\eta}{1-\eta}\right)^2}_{\psi_I}\left(\mathcal{G}_t^I\right)^2 \end{array}\right) + t.i.p + \mathcal{O}\left(||\zeta||^3\right) \tag{D.9}$$

In addition, $\widehat{\Delta}_t^P \approx \frac{\varepsilon_p}{2} Var_z\left\{p_t(z)\right\}$ and $\widehat{\Delta}_t^j \approx \frac{\varepsilon_w^j}{2} Var_i\left\{w_t^j(i)\right\}$ for $j \in \{H, L\}$ and can be expressed in terms of corresponding inflation.

$$\sum_{t=0}^{\infty}\beta^t Var_p\left\{p_t(z)\right\} = \frac{\theta_p}{(1-\theta_p)(1-\beta\theta_p)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^p\right)^2$$

$$\text{Similarly,}\quad \sum_{t=0}^{\infty}\beta^t Var_i\left\{w_t^j(i)\right\} = \frac{\theta_w}{(1-\theta_w)(1-\beta\theta_w)}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^j\right)^2 \quad \text{for } \; j \in \{H, L\}$$

Finally, we obtain the Central Bank's loss function as

$$L = \frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varepsilon_p}{\kappa_p}\left(\pi_t^p\right)^2 + (\sigma+\varphi)\, x_t^2 + \frac{s\varepsilon_w^H}{\kappa_w^H}\left(\pi_t^H\right)^2 + \frac{(1-s)\varepsilon_w^L}{\kappa_w^L}\left(\pi_t^L\right)^2 + \psi_c\left(\mathcal{G}_t^c\right)^2 + \psi_I\left(\mathcal{G}_t^I\right)^2\right\} \tag{D.10}$$

Appendix E Optimal monetary policy under commitment

Minimize (D.10) subject to (3.9) (C.1),(C.3) and (6.1), in addition to inequality measures, (5.1) and (5.2), with corresponding Lagrange multiplier $\phi_{i,t}$ for $i = 1, 2, 3, \cdots, 8$ and for $t = 0, 1, 2, \cdots$

w.r.t.	First Order Conditions
x_t	$(\sigma+\varphi)\,x_t-\kappa_w^H\,(\sigma+\varphi)\,\phi_{2,t}-\kappa_w^L\,(\sigma+\varphi)\,\phi_{3,t}=0$
π_t^p	$\frac{\varepsilon_p}{\kappa_p}\pi_t^p+\Delta\phi_{1,t}+\phi_{4,t}+\phi_{5,t}=0$
π_t^H	$\frac{s\varepsilon_w^H}{\kappa_w^H}\pi_t^H+\Delta\phi_{2,t}-\phi_{4,t}=0$
π_t^L	$\frac{(1-s)\varepsilon_w^L}{\kappa_w^L}\pi_t^L+\Delta\phi_{3,t}-\phi_{5,t}=0$
$\widetilde{\omega}_t^H$	$\kappa_w^H\lambda_H^H\phi_{2,t}-\kappa_w^L\,(\sigma+\varphi)\,\eta s\phi_{3,t}+\phi_{5,t}-\beta\phi_{5,t+1}-s\phi_{6,t}+\eta s(1-s)\phi_{7,t}-s(1-s)(1-\eta)\phi_{8,t}=0$
$\widetilde{\omega}_t^L$	$-\kappa_w^H\lambda_H^L\phi_{2,t}+\kappa_w^L\left((\sigma+\varphi)\eta s-(\sigma-1)\right)\phi_{3,t}+\phi_{5,t}-\beta\phi_{5,t+1}-(1-s)\phi_{6,t}$
	$+(1-s)(1-\eta s)\phi_{7,t}+s(1-s)(1-\eta)\phi_{8,t}=0$
$\widetilde{\omega}_t$	$-\kappa_p\phi_{1,t}+\phi_{6,t}=0$
$\mathcal{G}_t^c$	$\psi_c\mathcal{G}_t^c+\phi_{7,t}=0$
$\mathcal{G}_t^I$	$\psi_I\mathcal{G}_t^I+\phi_{8,t}=0$

Appendix F Counter-factual Experiment

F.1 Scenario 1: sticky wage + single labor market model

$$\mathcal{L}=\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left\{(\sigma+\varphi)\,x_t^2+\frac{\varepsilon_p}{\kappa_p}\left(\pi_t^p\right)^2+\frac{\varepsilon_w}{\kappa_w}\left(\pi_t^w\right)^2+\frac{(\sigma-1)(1-s)}{s}\left(\widetilde{\omega}_t\right)^2\right\}$$

subject to

$$\begin{aligned}
\pi_t^p&=\beta E_t\pi_{t+1}^p+\kappa_p\widetilde{\omega}_t\\
\pi_t^w&=\beta E_t\pi_{t+1}^w+\kappa_w\,(\sigma+\varphi)\,x_t-\kappa_w\widetilde{\omega}_t\\
\widetilde{\omega}_t&=\widetilde{\omega}_{t-1}+\pi_t^w-\pi_t^p-\Delta\omega_t^E
\end{aligned}$$

w.r.t.	First Order Conditions
x_t	$(\sigma+\varphi)\,x_t-\kappa_w\,(\sigma+\varphi)\,\phi_{2,t}=0$
π_t^p	$\frac{\varepsilon_p}{\kappa_p}\pi_t^p+\Delta\phi_{1,t}+\phi_{3,t}=0$
π_t^w	$\frac{\varepsilon_w}{\kappa_w}\pi_t^w+\Delta\phi_{2,t}-\phi_{3,t}=0$
$\widetilde{\omega}_t$	$\frac{(\sigma-1)(1-s)}{s}\widetilde{\omega}_t-\kappa_p\phi_{1,t}+\kappa_w\phi_{2,t}-\beta\phi_{3,t+1}+\phi_{3,t}=0$

F.2 Scenario 2: flexible wage + segmented labor market model

$$L=\frac{1}{2}\sum_{t=0}^{\infty}\beta^t\left\{\frac{\varepsilon_p}{\kappa_p}\left(\pi_t^p\right)^2+(\sigma+\varphi)\,x_t^2+\psi_c\left(\mathcal{G}_t^c\right)^2+\psi_I\left(\mathcal{G}_t^I\right)^2\right\}$$

subject to

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \kappa_p (\sigma + \varphi) x_t$$

with (6.1), in addition to inequality measures, (5.1) and (5.2)

w.r.t.	First Order Conditions
x_t	$(\sigma + \varphi) x_t - \kappa_p (\sigma + \varphi) \phi_{1,t} = 0$
π_t^p	$\frac{\varepsilon_p}{\kappa_p} \pi_t^p + \Delta \phi_{1,t} + \phi_{2,t} + \phi_{3,t} = 0$
$\widetilde{\omega}_t^H$	$\phi_{2,t} - \beta \phi_{2,t+1} + \eta s(1-s) \phi_{4,t} - s(1-s)(1-\eta) \phi_{5,t} = 0$
$\widetilde{\omega}_t^L$	$\phi_{3,t} - \beta \phi_{3,t+1} + (1-s)(1-\eta s) \phi_{4,t} + s(1-s)(1-\eta) \phi_{5,t} = 0$
$\mathcal{G}_t^c$	$\psi_c \mathcal{G}_t^c + \phi_{4,t} = 0$
$\mathcal{G}_t^I$	$\psi_I \mathcal{G}_t^I + \phi_{5,t} = 0$

F.3 Scenario 3: flexible wage + single labor market Model

$$\mathcal{L} = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left\{ (\sigma + \varphi) x_t^2 + \frac{\varepsilon_p}{\kappa_p} \left(\pi_t^p \right)^2 \right\}$$

subject to

$$\pi_t^p = \beta E_t \pi_{t+1}^p + \kappa_p (\sigma + \varphi) x_t$$

w.r.t.	First Order Conditions
x_t	$(\sigma + \varphi) x_t - \kappa_p (\sigma + \varphi) \phi_{1,t} = 0$
π_t^p	$\frac{\varepsilon_p}{\kappa_p} \pi_t^p + \Delta \phi_{1,t} = 0$

Appendix G An extension with idiosyncratic productivity shock

G.1 Labor Demand

$$\min_{N_t^H, N_t^L} W_t^H N_t^H + W_t^L N_t^L \quad \text{s.t. } H_t \geq \left[\gamma_H^{\frac{1}{\eta}} \left(A_t^H N_t^H \right)^{\frac{\eta-1}{\eta}} + \gamma_L^{\frac{1}{\eta}} \left(A_t^L N_t^L \right)^{\frac{\eta-1}{\eta}} \right]$$

$$\text{F.O.C:} \quad \varpi_t^j \equiv \frac{W_t^j}{A_t^j} = \lambda_t \left\{ \gamma^{\frac{1}{\eta}} \left(\frac{H_t}{A_t^j N_t^j} \right)^{\frac{1}{\eta}} \right\} \quad \text{for } j \in \{H, L\}$$

Convex combination of the wages after taking $1-\eta$ power and using the definition of H_t, I get

$$\left[\gamma \left(\varpi_t^H \right)^{1-\eta} + (1-\gamma) \left(\varpi_t^L \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} = \left[\gamma \left(\frac{W_t^H}{A_t^H} \right)^{1-\eta} + (1-\gamma) \left(\frac{W_t^L}{A_t^L} \right)^{1-\eta} \right]^{\frac{1}{1-\eta}} = \lambda_t \equiv \varpi_t$$

replace λ_t in F.O.C

$$\begin{aligned} N_t^j = \gamma_j \left(\frac{\varpi_t^j}{\varpi_t} \right)^{-\eta} \frac{Y_t}{A_t A_t^j} &\Rightarrow n_t^H \approx -\eta \left(\widehat{\varpi}_t^j - \widehat{\varpi}_t \right) + y_t - a_t - a_t^j \\ &\Rightarrow n_t^H - n_t^L \equiv n_t^R = -\eta \left(\omega_t^R - a_t^R \right) - a_t^R = -\eta \omega_t^R - (1+\eta) a_t^R \end{aligned}$$

G.2 Marginal Cost

$$\min_{N_t^H, N_t^L} W_t^H N_t^H + W_t^L N_t^L \quad \text{s.t. } Y_t \geq A_t \left[\gamma_H^{\frac{1}{\eta}} \left(A_t^H N_t^H \right)^{\frac{\eta-1}{\eta}} + \gamma_L^{\frac{1}{\eta}} \left(A_t^L N_t^L \right)^{\frac{\eta-1}{\eta}} \right]$$

$$\text{F.O.C:} \quad \varpi_t^j = \lambda_t A_t \left\{ \gamma^{\frac{1}{\eta}} \left(\frac{H_t}{A_t^j N_t^j} \right)^{\frac{1}{\eta}} \right\} \quad \text{for } j \in \{H, L\}$$

Similarly,

$$\gamma_j \left(\varpi_t^j\right)^{1-\eta} = \lambda_t^{1-\eta} A_t^{1-\eta} \left\{ \gamma_j^{\frac{1}{\eta}} \left(\frac{j_t}{A_t^j N_t^j} \right)^{\frac{1-\eta}{\eta}} \right\} \quad \text{for } \; j \in \{H, L\}$$

$$\Rightarrow \left(\frac{\varpi_t}{A_t}\right)^{1-\eta} = \lambda_t^{1-\eta} \Rightarrow mc_t = \widehat{\varpi}_t - a_t \Rightarrow \underbrace{sw_t^H + (1-s)w_t^L}_{\equiv\, w_t} - \underbrace{sa_t^H + (1-s)a_t^L}_{\equiv\, a_t^c} - a_t$$

which is obtained by a convex combination with weight of population share.

G.3 Efficient level of output

$$MRS_t^j = \chi_t \left(C_t^j\right)^\sigma \left(N_t^j\right)^\varphi = \frac{W_t^j}{P_t} = \gamma_j^{\frac{1}{\eta}} A_t A_t^j \left(\frac{H_t}{A_t^j N_t^j}\right)^{\frac{1}{\eta}} = MPN_t^j$$

$$\Rightarrow \sigma c_t^j + \varphi n_t^j + \xi_t = \omega_t^j = a_t + a_t^j + \frac{1}{\eta}\left(h_t - a_t^j - n_t^j\right)$$

Convex combination of the two sectoral efficient conditions gives

$$\sigma y_t + \varphi\left(sn_t^H + (1-s)n_t^L\right) + \xi_t = a_t + a_t^c + \frac{1}{\eta} \underbrace{\left(h_t - s\left(a_t^H + n_t^h\right) - (1-s)\left(a_t^L + n_t^L\right)\right)}_{=0}$$

$$\sigma y_t + \varphi\left(y_t - a_t - a_t^c\right) + \xi_t = a_t + a_t^c$$

$$y_t^E = \frac{1+\varphi}{\sigma+\varphi}\left(a_t + a_t^c\right) - \frac{1}{\sigma+\varphi}\xi_t$$

$$w_t^E = a_t + a_t^c \quad \text{and} \quad w_t^{H,E} = a_t + a_t^H \quad \& \quad w_t^{L,E} = a_t + a_t^L$$

$$w_t^{R,E} = a_t^H - a_t^L$$

G.4 Log-linearized Equations

1. $x_t = E_t x_{t+1} - \frac{1}{\sigma}\left\{i_t - E_t \pi_{t+1}^P - r_t^E\right\} + \frac{1}{s}\Delta\mathcal{G}_{t+1}^c$ (IS curve)
2. $\pi_t^p = \beta E_t \pi_{t+1}^P + \kappa_p \widetilde{\omega}_t$ (NKPC)
3. $\pi_t^H = \beta E_t \pi_{t+1}^H + \kappa_w^H (\sigma+\varphi) x_t - \kappa_w^H \lambda_H^H \widetilde{\omega}_t^H + \kappa_w^H \lambda_H^L \widetilde{\omega}_t^L + \mu_t^H$ (high-skilled NKWPC)
4. $\pi_t^L = \beta E_t \pi_{t+1}^L + \kappa_w^L (\sigma+\varphi) x_t + \kappa_w^L \lambda_L^H \widetilde{\omega}_t^H - \kappa_w^L \lambda_L^L \widetilde{\omega}_t^L + \mu_t^L$ (low-skilled NKWPC)
5. $\mathcal{G}_t^c = -(1-s)\left(\eta s \widetilde{\omega}_t^H + (1-\eta s)\widetilde{\omega}_t^L\right)$ (Consumption Gini coefficient)

6. $\widetilde{\omega}_t^H = \widetilde{\omega}_{t-1}^H + \pi_t^H - \pi_t^p - \Delta\omega_t^{H,E}$

7. $\widetilde{\omega}_t^L = \widetilde{\omega}_{t-1}^L + \pi_t^L - \pi_t^p - \Delta\omega_t^{L,E}$

8. $\widetilde{\omega}_t = s\widetilde{\omega}_t^H + (1-s)\widetilde{\omega}_t^L$

9. $i_t = \rho_i i_{t-1} + (1-\rho_i)\left(\phi_\pi \pi_t^p + \phi_x x_t\right) + \nu_t$ (Monetary policy rule)

10. $x_t = y_t - y_t^E$

11. $y_t^E = \frac{1+\varphi}{\sigma+\varphi}\left(a_t + a_t^c\right) - \frac{1}{\sigma+\varphi}\xi_t$

12. $r_t^E = \sigma \Delta y_{t+1}^E$

13. $\omega_t^{H,E} = a_t + a_t^H$

14. $\omega_t^{L,E} = a_t + a_t^L$

15. $a_t = \rho_a a_{t-1} + \epsilon_t^a$ (Labor aggregation technology shock)

16. $a_t^H = \rho_{a,H} a_{t-1}^H + \epsilon_t^{a,H}$ (Labor aggregation technology shock)

17. $a_t^L = \rho_{a,L} a_{t-1}^L + \epsilon_t^{a,L}$ (high-skilled productivity shock)

18. $\mu_t^H = \rho_H \mu_{t-1}^H + \epsilon_t^H$ (low-skilled productivity shock)

19. $\mu_t^L = \rho_L \mu_{t-1}^L + \epsilon_t^L$ (low-skilled wage markup shock)

20. $\nu_t = \rho_\nu \nu_{t-1} + \epsilon_t^\nu$ (Monetary policy shock)

21. $\xi_t = \rho_\xi \xi_{t-1} + \epsilon_t^\xi$ (Labor Supply shock)

Appendix H Additional Figures

Figure 4: Dynamic responses to the positive technology shock

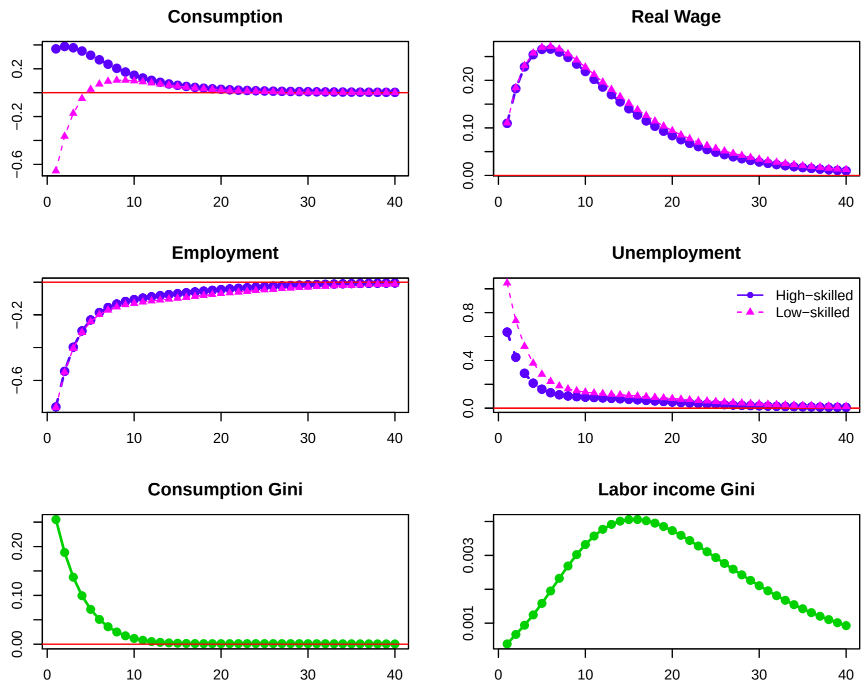


Figure 5: Dynamic responses to the positive high-skilled productivity shock

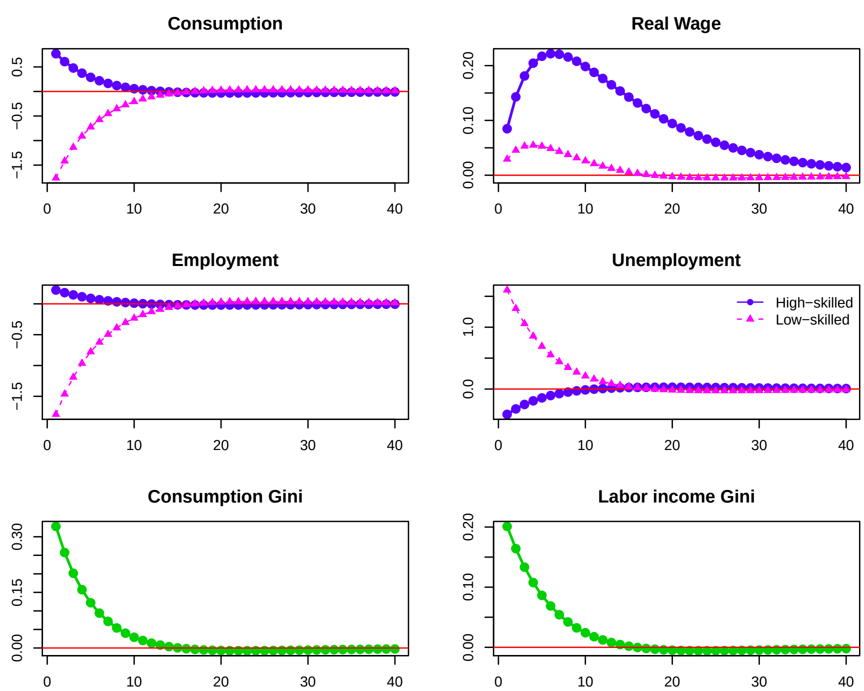


Figure 6: Dynamic responses to the positive low-skilled technology shock

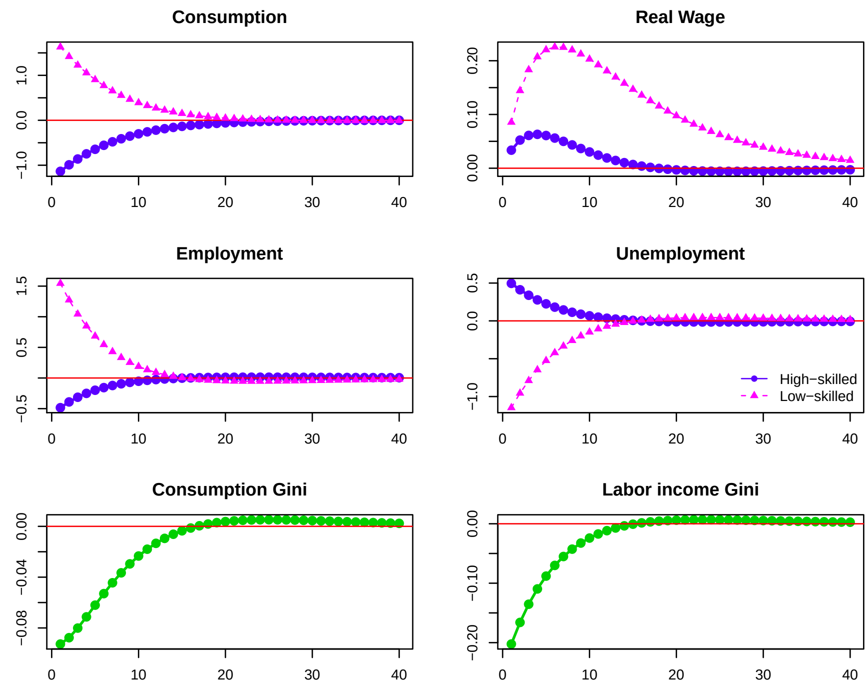


Figure 7: High skilled wage markup shock

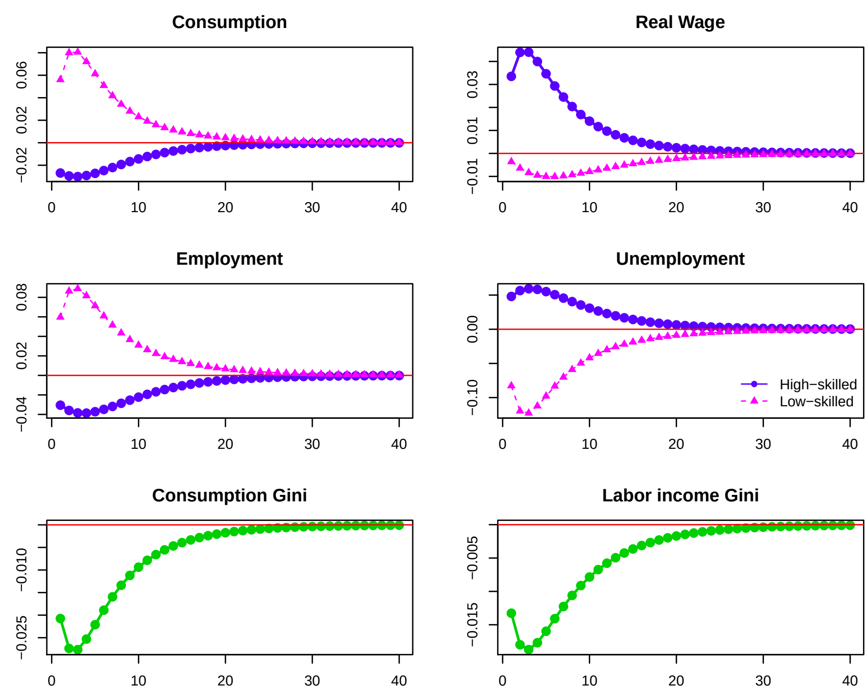


Figure 8: Low skilled wage markup shock

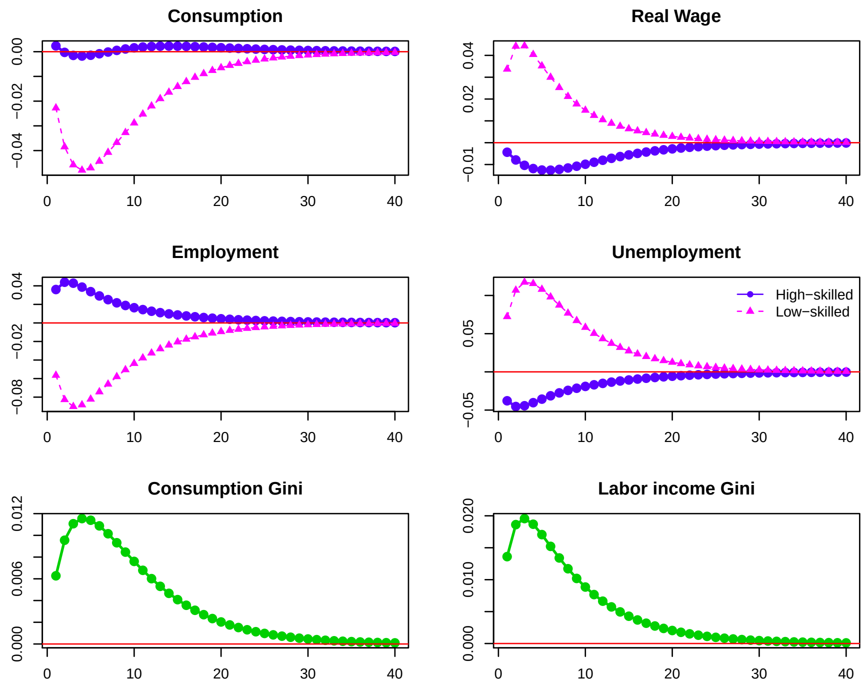
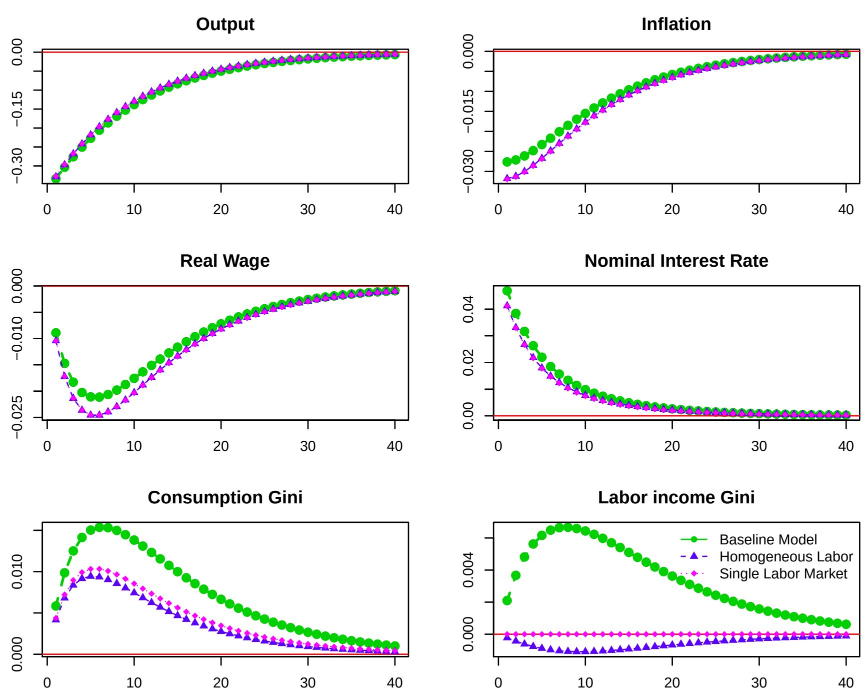
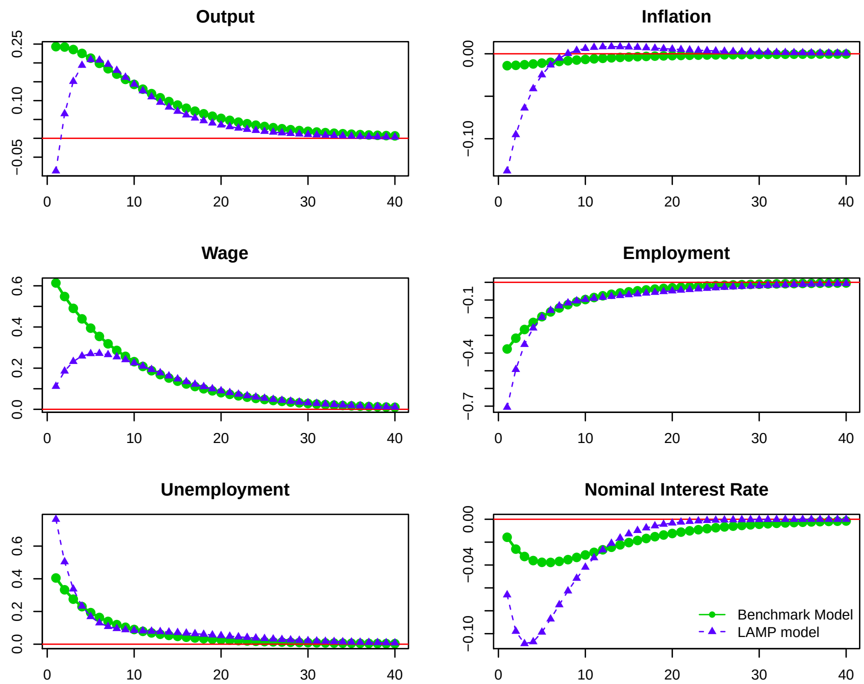


Figure 9: Role of labor market assumption: IRFs to a Monetary policy shock



NOTE: Homogeneous labor market implies the same labor demand elasticity ($\varepsilon_w^H = \varepsilon_w^L$). The single labor market features the same labor demand and wages in addition to homogeneous labor

Figure 10: Dynamic responses to positive technology shock



Note: I consider Galí (2011) as a benchmark model which is a standard NK model with staggered wages

Figure 11: Dynamic responses to the low-skilled productivity shock

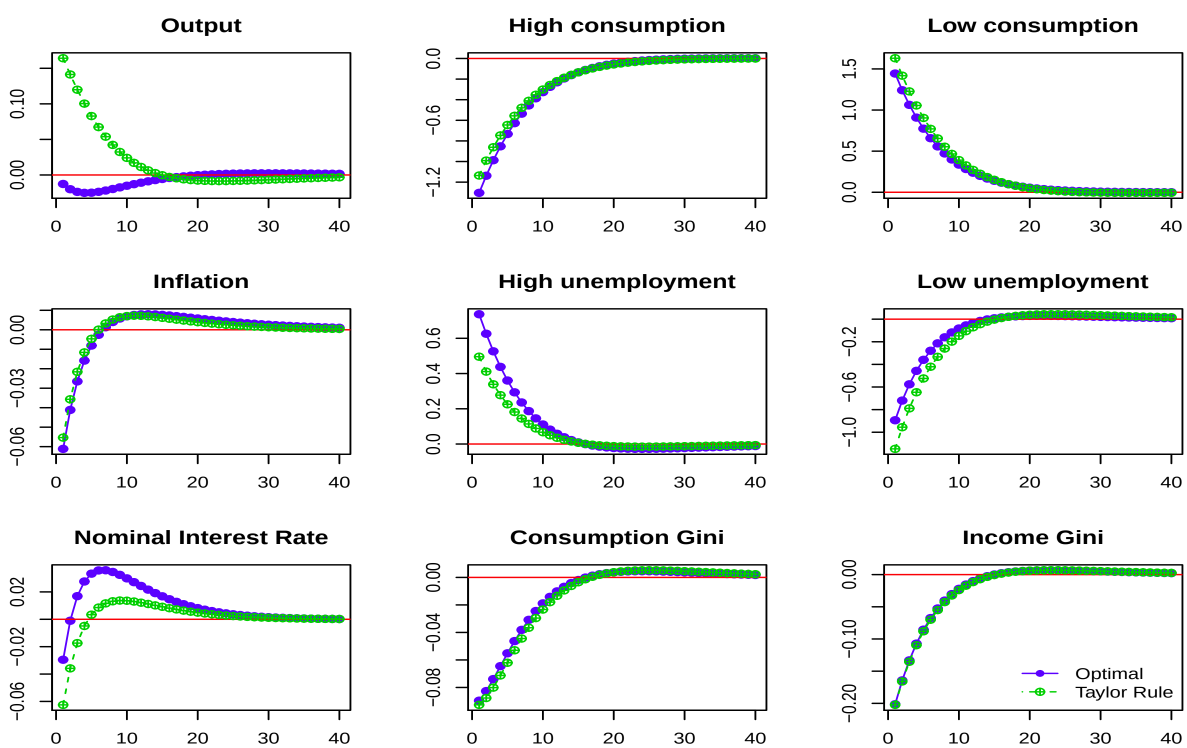


Figure 12: Dynamic responses to the high-skilled wage markup shock

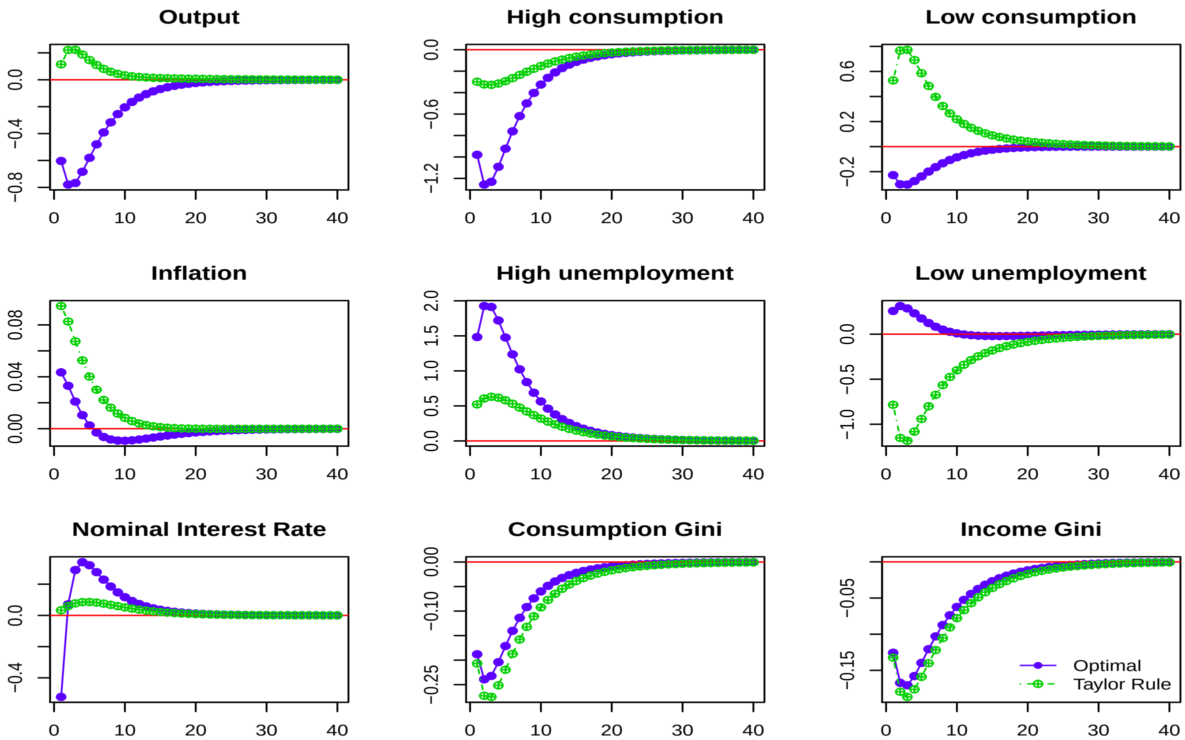
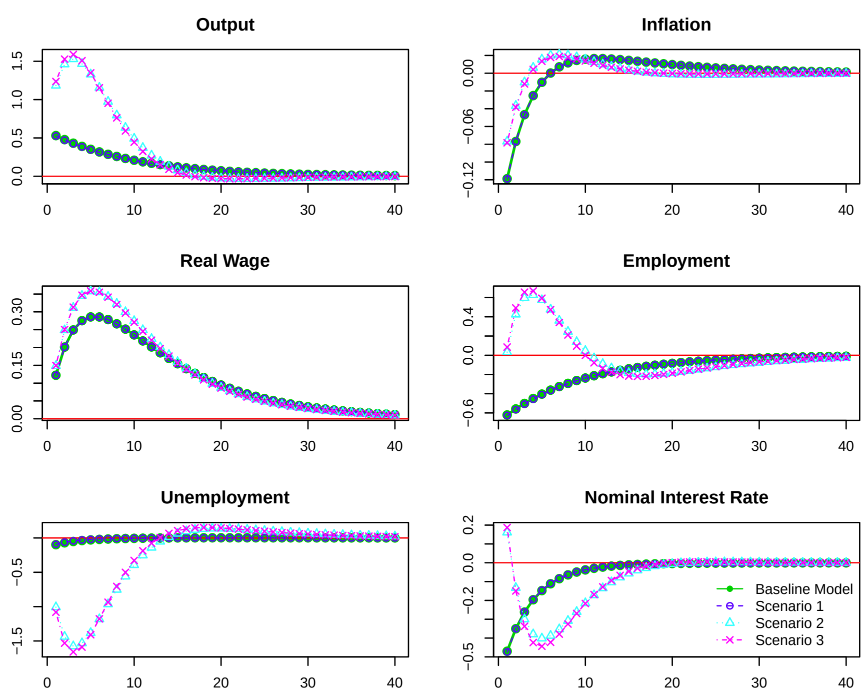
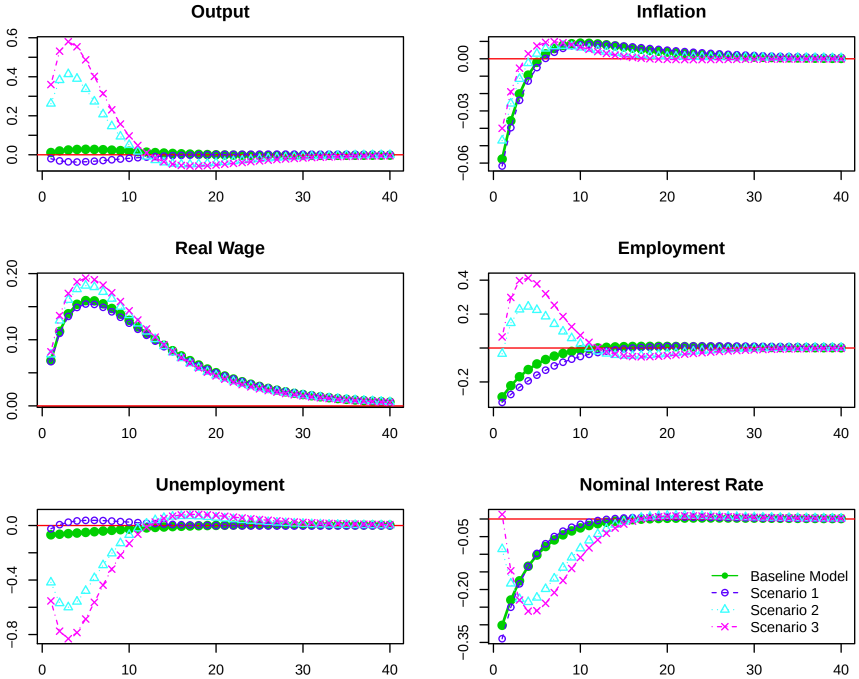


Figure 13: Optimal responses to the positive technology shock



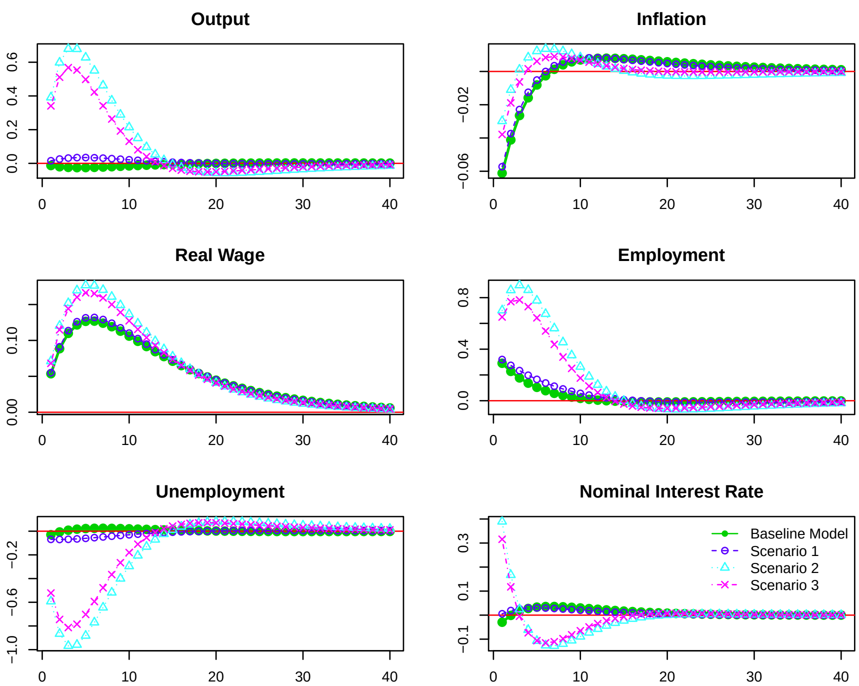
Note: Baseline: Segmented labor market with staggered wages; Scenario 1: Single labor market with staggered wages; Scenario 2: Segmented labor market with flexible wages; Scenario 3: Single labor market with flexible wages

Figure 14: Optimal responses to the positive high-skilled productivity shock



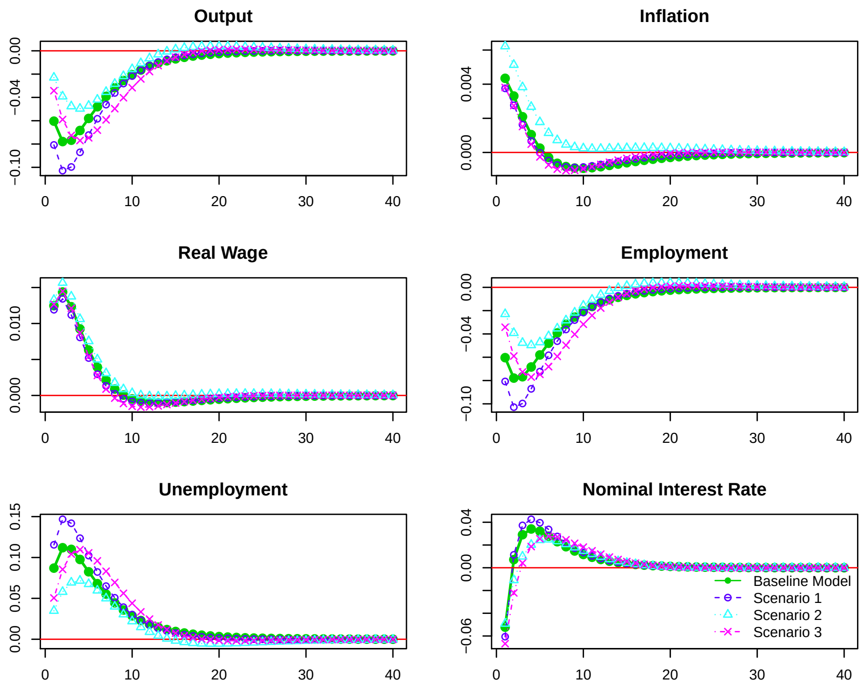
Note: Baseline: Segmented labor market with staggered wages; Scenario 1: Single labor market with staggered wages; Scenario 2: Segmented labor market with flexible wages; Scenario 3: Single labor market with flexible wages

Figure 15: Optimal responses to the positive low-skilled productivity shock



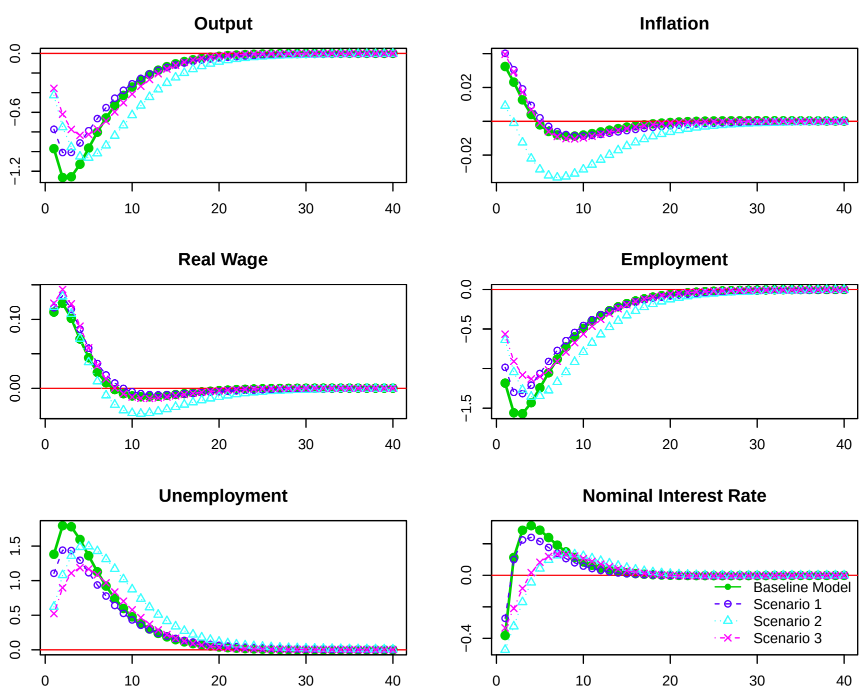
Note: Baseline: Segmented labor market with staggered wages; Scenario 1: Single labor market with staggered wages; Scenario 2: Segmented labor market with flexible wages; Scenario 3: Single labor market with flexible wages

Figure 16: Optimal responses to the positive high-skilled wage markup shock



Note: Baseline: Segmented labor market with staggered wages; Scenario 1: Single labor market with staggered wages; Scenario 2: Segmented labor market with flexible wages; Scenario 3: Single labor market with flexible wages

Figure 17: Optimal responses to the positive low-skilled wage markup shock



Note: Baseline: Segmented labor market with staggered wages; Scenario 1: Single labor market with staggered wages; Scenario 2: Segmented labor market with flexible wages; Scenario 3: Single labor market with flexible wages

Table 3: Relative Welfare Losses

Scenario	Productivity shock			Markup shock	
	Aggregate	High-skilled	Low-skilled	High-skilled	Low-skilled
Baseline	1	1	1	1	1
1	1.000873	1.051882	1.051621	1.017812	1.010268
2	30.951561	1.542624	4.387239	1.093651	1.079710
3	30.677522	2.246138	3.020883	1.022344	1.025460