Dynamics of Real Exchange Rates and the Taylor Rule: Importance of Taylor-rule Fundamentals, Monetary Policy Shocks, and Risk-premium Shocks

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Abstract: We derive two alternative representations of the real exchange rate under the Taylor rule with interest rate smoothing: one based on a first-order difference equation and the other based on a second-order difference equation. By comparing these two alternative representations, we evaluate the relative importance of Taylor-rule fundamentals, monetary policy shock and risk-premium shock in the dynamics of the real exchange rate. Under the assumption that the persistence of the risk-premium shock is as high as the degree of interest rate smoothing in monetary policy, we report that the Taylor-rule fundamentals can account for 3%-17% of variations of real exchange rate. The relative contribution of monetary policy shocks is also limited ranging between only 3% and 15%. However, the relative contribution of risk-premium shocks in real exchange rate variations range between 49% and 89% across countries.

Keywords: Exchange rate, Monetary Policy Shocks, Present-value relation, Risk-premium shocks, Taylor rule

JEL classification: E52, F31, F41

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1. Introduction

The difficulty of explaining the dynamics of the exchange rate based on macroeconomic fundamentals has been a well-known puzzle since Meese and Rogoff (1983). Obstfeld and Rogoff (2001) refer to this as the “exchange-rate disconnect puzzle.”[1] Recent studies, however, have made some progress in connecting the exchange rate with macroeconomic fundamentals. A growing body of literature shows the relevance and importance of Taylor-rule fundamentals in explaining the dynamics of the exchange rate. For example, Benigno (2004) theoretically demonstrates that the persistence of the real exchange rate is related to the degree of interest rate smoothing in the Taylor rule. Engel and West (2005, 2006), Clarida and Waldman (2007), Mark (2009), Molodtsova and Papell (2009), and Kim et al. (2014) report some empirical success in explaining the dynamics of the exchange rate under Taylor rules.

The purpose of this paper is to investigate the relative importance of Taylor-rule fundamentals, monetary policy shocks and risk-premium shocks within the Taylor-rule-based real exchange rate model.[2] For this purpose, we first present two alternative representations of the real exchange rate under the Taylor rule with interesting smoothing: one based on a solution for a first-order difference equation and the other based on a solution for a second-order difference equation. We then utilize the differences in these two alternative representations to quantify the relative contributions of Taylor-rule fundamentals, monetary policy shocks and

[1] “Exchange-rate disconnect” puzzle in Obstfeld and Rogoff (2001) has two meanings. One is that macro variables have limited ability to explain exchange rate, and the other is that the exchange rate has a small impact on the real economy.

[2] In this study, monetary policy shocks are a part of interest rate movements that is not explained by the Taylor rule with interest smoothing, and risk-premium shocks mean deviations from the uncovered interest rate parity (UIP) condition.

risk-premium shocks in real exchange variations.

Previous studies report that Taylor-rule fundamentals explain stylized facts of exchange rates and have significant forecast ability for movements of exchange rates. In this study, we formally quantify the relative contribution of Taylor-rule fundamentals to variations of the real exchange rates via a variance decomposition exercise. As a result, we are able to check if Taylor-rule fundamentals can explain a substantial portion of exchange rate movements unlike other macroeconomic models.

The importance of monetary shocks in real exchange dynamics has also been investigated by researchers focusing on the exchange rate's delayed response to monetary policy within the structural vector autoregression (VAR) frameworks (see Eichenbaum and Evans (1995), Clarida and Gali (1994), Faust and Rogers (2003), Kim and Roubini (2000), Bjornland (2009), etc.). However, the results seem to be sensitive to the identifying assumptions employed. For example, Clarida and Gali (1994) report that the contribution of monetary policy shocks in exchange rate variations is very limited. Juvenal (2011) and Faust and Rogers (2003) also cast doubt on the idea that monetary policy shocks play the main role in exchange rate dynamics. Conversely, Eichenbaum and Evans (1995) demonstrate that monetary policy shocks can explain up to 43% of real exchange rate movements through variance decompositions in VAR, and argue that a tight relation between monetary policy and real exchange rate exists. Rogers (1999) also shows that the contribution of monetary policy shocks to the variations of exchange rate lies in the range between 19% and 60%. In addition to the structural VAR approach, Bergin (2006), who performs a structural estimation of a new Keynesian model with the risk-premium shock, claims that the monetary policy shock, not the risk-premium shock, is the primary driver of the exchange rate. He argues that more than 50%

of variations in the real exchange rate can be accounted for by monetary policy shocks.

While quantifying the relative contribution of Taylor-rule fundamentals, monetary policy shock, and risk-premium shock in real exchange rate variations, our approach is distinctive in the sense that it does not require any assumptions that are often employed in the structural VAR approach in identifying those shocks. Instead, we utilize a simple model with the uncovered interest rate parity (UIP) and the Taylor rule to gauge the relative importance of Taylor-rule fundamentals, monetary policy shocks and risk-premium shocks. Methodologically, we first estimate the present value of Taylor-rule fundamentals by employing VAR, as in Engel and West (2006).[3] Using the estimated present value of the fundamentals, we calculate model-based exchange rates and error terms of the two alternative representations of the real exchange rate dynamics under interest rate smoothing. By analyzing the components of the error terms in these two alternative representations and comparing the estimated variance and covariance of these estimated error terms, we present approximate measures of the relative importance of Taylor-rule fundamentals, monetary and risk-premium shocks.

Under the assumption that the persistence of risk-premium shocks is as high as the degree of interest rate smoothing in monetary policy,[4] we find that Taylor-rule fundamentals can explain about up to 17% of fluctuations of real exchange rates, while the largest contribution to explain real exchange rate movements comes from the risk-premium shocks. On the contrary, the relative contribution of monetary policy shock is also limited ranging

[3] Allowing deviations from the UIP in the short-run, we can decompose the monetary policy shock in Engel and West (2006) into the monetary policy shock and risk-premium shock in this study.

[4] Engel (2014) shows that a persistent but stationary process is needed for the risk-premium shock to explain the hump-shaped pattern of exchange rate predictability over different prediction horizons.

between 3% and 15%. This limited contribution of monetary policy shocks is comparable to the results of previous studies by Clarida and Gali (1994), Juvenal (2011), and Faust and Rogers (2003), but the results are in sharp contrast to those of Eichenbaum and Evans (1995), Rogers (1999), and Bergin (2006).

This paper is organized as follows. In Section 2, we provide two alternative solutions for a Taylor-rule-based real exchange rate model under interest rate smoothing, and discuss how to construct error terms from the two alternative representations. In Section 3, we explain a procedure for evaluating the relative importance of Taylor-rule fundamentals, monetary policy shock and risk-premium shock based on an empirical evaluation of the models implied by the two alternative solutions. Section 4 provides the empirical results, and Section 5 concludes.

2. Two Alternative Solutions for a Taylor-Rule-Based Real Exchange Rate Model with Interest Rate Smoothing

In this section, we consider two alternative solutions for a Taylor-rule-based model of real exchange rate in the presence of interest rate smoothing. For this purpose, we consider the following monetary policy rules for the home and the foreign countries. A superscript 'h' denotes the home country (non-U.S. countries in the empirical analysis), and a superscript 'f' denotes the foreign country (the U.S. in the empirical analysis):

Target Interest Rate

$$i_t^{h*} = \gamma_q(q_t - c_0) + \gamma_\pi E_t\left(\pi_{t+1}^h\right) + \gamma_y y_t^h, \tag{1}$$

$$i_t^{f*} = \gamma_\pi E_t(\pi_{t+1}^f) + \gamma_y y_t^f, \quad (2)$$

$$\gamma_\pi > 1,\ \gamma_y > 0,\ \gamma_q > 0$$

where i_t^{h*} (i_t^{f*}) is the target interest rate for the home country (the foreign country); q_t is the real exchange rate defined as the log of nominal exchange rate (s_t) minus the log of the relative price ($p_t^h - p_t^f$); c_0 is the steady-state level of the real exchange rate; π_{t+1}^h (π_{t+1}^f) is the inflation rate for the home country (the foreign country); and y_t^h (y_t^f) is the output gap for the home country (the foreign country). Equations (1) and (2) suggest that the home country sets the target interest rate in response to the expected inflation rate, output gap, and deviation of real exchange rate from its steady-state level, whereas the foreign country sets the target interest rate in response to the expected inflation rate and output gap only.

Interest Rate Smoothing

$$i_t^h = (1-\rho)i_t^{h*} + \rho i_{t-1}^h + u_{mt}^h, \quad (3)$$

$$i_t^f = (1-\rho)i_t^{f*} + \rho i_{t-1}^f + u_{mt}^f, \quad (4)$$

$$0 < \rho < 1,$$

where i_t^h (i_t^f) denotes the interest rate for the home country (the foreign country), $u_{m,t}^h$ ($u_{m,t}^f$) denotes the monetary policy shock for the home country (the foreign country), and the ρ parameter refers to the degree of interest rate smoothing. Equations (3) and (4) suggest that both the home and the foreign countries gradually change their interest rates to reach the target interest rates.

In addition to the above monetary policy rules, we assume the following interest parity for the real exchange rate q_t:

$$i_t^h - i_t^f = E_t(q_{t+1}) - q_t + E_t\left(\pi_{t+1}^h - \pi_{t+1}^f\right) + u_t^r \tag{5}$$

where u_t^r denotes the risk-premium shock causing deviations from the UIP relation under rational expectations. The UIP holds in the long run (i.e., $E[u_t^r] = 0$), but not in the short run.

In what follows, we consider two alternative solutions for the model given by equation (1) through equation (5): one based on a first-order difference equation for q_t and the other based on a second-order difference equation for q_t.

2.1. A Solution Based on a First-Order Difference Equation for the Real Exchange Rate

Combining equations (1)–(5) results in the following expression:

$$q_t = \beta\tilde{\gamma}_q c_0 + \beta(1-\tilde{\gamma}_\pi)E_t\left(f_{1,t+1}\right) - \beta\tilde{\gamma}_y f_{2,t} - \beta\rho f_{3,t-1} + \beta E_t(q_{t+1}) + u_t' \tag{6}$$

where $\tilde{\gamma}_q = (1-\rho)\gamma_q$, $\tilde{\gamma}_\pi = (1-\rho)\gamma_\pi$, $\tilde{\gamma}_y = (1-\rho)\gamma_y$, $\beta = \frac{1}{1+\tilde{\gamma}_q}$, $f_{1,t} = \pi_t^h - \pi_t^f$, $f_{2,t} = y_t^h - y_t^f$, $f_{3,t} = i_t^h - i_t^f$, and $u_t' = \beta(u_t^r + u_{mt}^f - u_{mt}^h)$. Then, by iterating equation (6) in the forward direction, we obtain:

$$q_t = a_1 + \beta(1-\tilde{\gamma}_\pi)\sum_{i=0}^{\infty}\beta^i E_t\left(f_{1,t+1+i}\right) - \beta\tilde{\gamma}_y\sum_{i=0}^{\infty}\beta^i E_t\left(f_{2,t+i}\right)$$

$$-\beta\rho\sum_{i=0}^{\infty}\beta^i E_t\left(f_{3,t-1+i}\right) + \epsilon_{1t} \tag{7}$$

where $a_1 = \frac{\beta\tilde{\gamma}_q c_0}{1-\beta}$ and $\epsilon_{1t} = \sum_{i=0}^{\infty}\beta^{i+1} E_t\left(u_{m,t+i}^f - u_{m,t+i}^h + u_{t+i}^r\right)$. This solution suggests that the real exchange rate is a function of the present value for the inflation differential, output gap differential, and interest rate differential, which is similar to Engel and West (2006). Also, Equation (7) leads us to the following relation between actual and model-based real exchange rate:

$$q_t = q_t^* + \epsilon_{1t} \tag{8}$$

where the model-based real exchange rate, q_t^*, is

$$q_t^* = a_1 + \beta(1-\tilde{\gamma}_\pi)\sum_{i=0}^{\infty}\beta^i E_t\big(f_{1,t+1+i}\big) - \beta\tilde{\gamma}_y \sum_{i=0}^{\infty}\beta^i E_t\big(f_{2,t+i}\big)$$
$$-\beta\rho\sum_{i=0}^{\infty}\beta^i E_t\big(f_{3,t-1+i}\big) \tag{9}$$

2.2. A Solution Based on a Second-Order Difference Equation for the Real Exchange Rate

For an alternative solution to the model, we first note that the equation for the interest parity in (5) can be rewritten as:

$$i_t^h - i_t^f = q_{t+1} - q_t + \pi_{t+1}^h - \pi_{t+1}^f + u_t^r + \xi_{t+1} \tag{10}$$

where ξ_t is the forecast error that results from replacing $E_t(q_{t+1})$ and $E_t\big(\pi_{t+1}^h - \pi_{t+1}^f\big)$ with q_{t+1} and $\pi_{t+1}^h - \pi_{t+1}^f$, respectively.[5] Then, by combining equations (1)–(4) and equation (10), we obtain the following second-order difference equation for the real exchange rate, q_t:

$$E_t(q_{t+1}) - \big(1+\tilde{\gamma}_q+\rho\big)q_t + \rho q_{t-1}$$
$$= -\tilde{\gamma}_q c_0 + (\tilde{\gamma}_\pi - 1)E_t\big(f_{1,t+1}\big) + \tilde{\gamma}_y f_{2,t} + \rho f_{1,t} - u_t \tag{11}$$

[5] We assume that forecast errors (ξ_t) have no serial correlations.

where $u_t = (u_t^r - \rho u_{t-1}^r) + (u_{mt}^f - u_{mt}^h) - \rho \xi_t$.

Then, we can solve the above second-order difference equation by applying the procedure proposed by Sargent (1987), in order to obtain the following present-value form:

$$q_t = a_2 + \frac{1}{\delta_2} q_{t-1} + \frac{1-\widetilde{\gamma}_\pi}{\rho\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t\big(f_{1,t+1+i}\big) - \frac{\widetilde{\gamma}_y}{\rho\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t\big(f_{2,t+i}\big)$$

$$-\frac{1}{\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t\big(f_{1,t+i}\big) + \epsilon_{2t}, \qquad (12)$$

where δ_1 and δ_2 are the roots of the characteristic equation, $L^2 - \frac{1+\widetilde{\gamma}_q+\rho}{\rho} L + \frac{1}{\rho} = 0$, and $\epsilon_{2t} = \frac{1}{\rho\delta_2}\big[\sum_{i=0}^{\infty} \delta_1^i E_t\big(u_{m,t+i}^f - u_{m,t+i}^h + u_{t+i}^r - \rho u_{t+i-1}^r\big) - \rho\xi_t\big]$. Note that under plausible values for γ_q, γ_π, γ_y, and ρ, we can show that $0 < \delta_1 < 1$ and $\delta_2 > 1$.

Similarly to the fact that q_t can be expressed by either Equation (7) or Equation (12), we have an alternative expression for q_t^*. By rewriting q_t^* in the form of the second-order difference equation, q_t^* can be written as follows:

$$q_t^* = a_2 + \frac{1}{\delta_2} q_{t-1}^* + \frac{1-\widetilde{\gamma}_\pi}{\rho\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t\big(f_{1,t+1+i}\big) - \frac{\widetilde{\gamma}_y}{\rho\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t\big(f_{2,t+i}\big) -$$

$$\frac{1}{\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t\big(f_{1,t+i}\big) \qquad (13)$$

The proof of the equivalence between Equation (9) and Equation (13) is provided in Appendix. By combining Equation (12) and Equation (13), we obtain

$$q_t = a_2 + \frac{1}{\delta_2} q_{t-1}^* + \frac{1-\widetilde{\gamma}_\pi}{\rho\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t\big(f_{1,t+1+i}\big) - \frac{\widetilde{\gamma}_y}{\rho\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t\big(f_{2,t+i}\big)$$

$$-\frac{1}{\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t\big(f_{1,t+i}\big) + \epsilon_{2t} + \frac{1}{\delta_2}(q_{t-1} - q_{t-1}^*)$$

$$= q_t^* + \epsilon_{2t} + \frac{1}{\delta_2}(q_{t-1} - q_{t-1}^*) = q_t^* + \epsilon_{2t} + \frac{1}{\delta_2}\epsilon_{1t-1} \tag{14}$$

The comparison of equations (8) and (14) implies that $\epsilon_{2t} = \epsilon_{1t} - \frac{1}{\delta_2}\epsilon_{1t-1}$. We utilize this relation and Equation (8) to construct ϵ_{1t} and ϵ_{2t}, and evaluate the importance of Taylor-rule fundamentals, the monetary policy and risk-premium shocks in the dynamics of the real exchange rate.

3. Evaluating the Relative Importance of the Shocks Based on Two Alternative Solutions of the Model

Although Equations (7) and (12) are derived under identical sets of assumptions, ϵ_{1t} and ϵ_{2t} rely on monetary policy shocks and risk-premium shocks in a different way. In this section, we focus on the differences in the disturbance terms in equations (7) and (12), given below:

$$\epsilon_{1t} = \sum_{i=0}^{\infty} \beta^{i+1} E_t\left(u_{m,t+i}^f - u_{m,t+i}^h + u_{t+i}^r\right) \tag{15}$$

$$\epsilon_{2t} = \frac{1}{\rho\delta_2}\left[\sum_{i=0}^{\infty} \delta_1^i E_t\left(u_{m,t+i}^f - u_{m,t+i}^h + u_{t+i}^r - \rho u_{t+i-1}^r\right) - \rho\xi_t\right]. \tag{16}$$

Following Engel (2014), we assume that the risk-premium shock u_t^r follows a stationary AR(1) process, given below:

$$u_t^r = \alpha u_{t-1}^r + \eta_t, \tag{18}$$

where η_t, the innovation to the risk premium shock, is serially uncorrelated. If we further assume that the persistence parameter (α) for the risk-premium shock is approximately the same as that of the degree of interest rate smoothing (ρ),[6] equations (15) and (16) can be written as:

$$\epsilon_{1t} = \beta\left(u_{mt}^{f} - u_{mt}^{h}\right) + \frac{\beta}{1-\alpha\beta} u_t^r, \tag{18}$$

$$\epsilon_{2t} = \frac{1}{\rho\delta_2}\left[\sum_{i=0}^{\infty} \delta_1^i \, E_t\left(\left(u_{m,t+i}^{f} - u_{m,t+i}^{h}\right) + \left(u_{t+i}^{r} - \alpha u_{t+i-1}^{r}\right) + (\alpha - \rho) u_{t+i-1}^{r}\right) - \rho\xi_t\right]$$

$$= \frac{1}{\rho\delta_2}\left(u_{mt}^{f} - u_{mt}^{h}\right) + \frac{1}{\rho\delta_2}\eta_t + \frac{1}{\rho\delta_2}\frac{1}{1-\delta_1}(\alpha - \rho) u_{t-1}^{r} - \frac{1}{\delta_2}\xi_t. \tag{19}$$

Thus, if $\alpha \approx \rho$, the variances of ϵ_{1t} and ϵ_{2t} are given by:

$$\mathrm{Var}(\epsilon_{1t}) = \beta^2 \sigma_m^2 + \frac{\beta^2}{(1-\alpha\beta)^2}\frac{\sigma_\eta^2}{1-\alpha^2} + 2\frac{\beta^2}{1-\alpha\beta}\sigma_{m,\eta} \tag{20}$$

$$\mathrm{Var}(\epsilon_{2t}) \approx \frac{1}{\rho^2\delta_2^2}\sigma_m^2 + \frac{1}{\delta_2^2}\mathrm{Var}(\xi_t) + \frac{1}{\rho^2\delta_2^2}\sigma_\eta^2 + \frac{2}{\rho^2\delta_2^2}\sigma_{m,\eta} \tag{21}$$

$$\mathrm{Cov}(\epsilon_{1t}, \epsilon_{2t}) = \frac{\beta}{\rho\delta_2}\sigma_m^2 + \frac{\beta}{(1-\alpha\beta)\rho\delta_2}\sigma_\eta^2 + \frac{\beta}{\rho\delta_2}\left(1 + \frac{1}{1-\alpha\beta}\right)\sigma_{m,\eta} \tag{22}$$

where $\sigma_m^2 = \mathrm{Var}(u_{mt}^{f} - u_{mt}^{h})$, $\sigma_\eta^2 = \mathrm{Var}(\eta_t)$ and $\sigma_{m,\eta} = \mathrm{Cov}(u_{mt}^{f} - u_{mt}^{h}, \eta_t)$. By defining $k \equiv \frac{1}{\delta_2^2}\mathrm{Var}(\xi_t)/\mathrm{Var}(\epsilon_{2t})$, equations (20) – (22) can be expressed in a matrix form as follows:

$$\begin{bmatrix} \mathrm{Var}(\epsilon_{1t}) \\ (1-k)\mathrm{Var}(\epsilon_{2t}) \\ \mathrm{Cov}(\epsilon_{1t}, \epsilon_{2t}) \end{bmatrix} = \begin{bmatrix} \beta^2 & \frac{\beta^2}{(1-\alpha\beta)^2(1-\alpha^2)} & 2\frac{\beta^2}{1-\alpha\beta} \\ \frac{1}{\rho^2\delta_2^2} & \frac{1}{\rho^2\delta_2^2} & \frac{2}{\rho^2\delta_2^2} \\ \frac{\beta}{\rho\delta_2} & \frac{\beta}{(1-\alpha\beta)\rho\delta_2} & \frac{\beta}{\rho\delta_2}\left(1 + \frac{1}{1-\alpha\beta}\right) \end{bmatrix} \begin{bmatrix} \sigma_m^2 \\ \sigma_\eta^2 \\ \sigma_{m,\eta} \end{bmatrix} \tag{23}$$

[6] Even if we allow serial correlation for monetary policy shock, that assumption does not affect the following discussion qualitatively. Hence, the no serial correlation assumption for monetary policy shock is innocuous.

Since we can estimate ϵ_{1t} and ϵ_{2t} from equations (8) and (14), we can have estimates for $\mathrm{Var}(\epsilon_{1t})$, $\mathrm{Var}(\epsilon_{2t})$, and $\mathrm{Cov}(\epsilon_{1t}, \epsilon_{2t})$. Then, for a given k, we can quantify the magnitudes of σ_m^2, σ_η^2, and $\sigma_{m,\eta}$ using the relation in equation (23). Since we do not know how important the forecast error (ξ_t) is in ϵ_{2t}, we will compute σ_m^2, σ_η^2, and $\sigma_{m,\eta}$ by varying the value for k within its plausible range. Once possible values for σ_m^2, σ_η^2, and $\sigma_{m,\eta}$ are obtained for a given k, we can evaluate the relative contributions from monetary policy shocks, risk-premium shocks, and covariance between those two. Note that equation (8) implies the following relation:

$$\mathrm{Var}(q_t) = \mathrm{Var}(q_t^*) + 2Cov(q_t^*, \epsilon_{1t}) + \mathrm{Var}(\epsilon_{1t})$$

$$= \mathrm{Var}(q_t^*) + 2Cov(q_t^*, \epsilon_{1t}) + \beta^2 \sigma_m^2 + \frac{\beta^2}{(1-\alpha\beta)^2} \frac{\sigma_\eta^2}{1-\alpha^2} + 2\frac{\beta^2}{1-\alpha\beta} \sigma_{m,\eta} \qquad (24)$$

Dividing both sides of equation (24) by $\mathrm{Var}(q_t)$, we obtain the following expression:

$$1 = \frac{\mathrm{Var}(q_t^*)}{\mathrm{Var}(q_t)} + 2\frac{Cov(q_t^*, \epsilon_{1t})}{\mathrm{Var}(q_t)} + \beta^2 \frac{\sigma_m^2}{\mathrm{Var}(q_t)} + \frac{\beta^2}{(1-\alpha\beta)^2(1-\alpha^2)} \frac{\sigma_\eta^2}{\mathrm{Var}(q_t)} + 2\frac{\beta^2}{1-\alpha\beta} \frac{\sigma_{m,\eta}}{\mathrm{Var}(q_t)} \qquad (25)$$

We interpret the first term in the right-hand-side of equation (25) as the contribution of Taylor-rule fundamentals in the variation of the real exchange rate, the third term as the contribution of monetary policy shocks, the fourth term as the risk-premium shocks. The second and the last term are contributions of the covariance terms in the variance of the real exchange rate. The second term is the covariance between fundamentals and the composite of shocks, while the last term is the covariance between monetary policy shocks and the risk-premium shocks. By varying the value of k within its plausible range, we can gauge the range of contributions from fundamentals, monetary policy shocks, risk-premium shocks and covariances.

4. Empirical Results

4.1. Data

We estimate and analyze the two alternative solutions to the model for the bilateral quarterly U.S. exchange rates versus those of Germany, Canada, Japan, Switzerland, and the UK (the U.S. is the foreign country). For Germany, the sample period covers 1979: I–1998: IV. For the other countries, the sample period covers 1979: I–2006: IV. The start of the sample is chosen to match the beginning of the Volcker monetary regime shift. The sample ends right before the introduction of the euro for Germany, and it ends right before the global financial crisis for the other countries.

Real GDP series, Consumer Price indices (CPI), policy interest rates, and the exchange rate are all obtained from International Financial Statistics (IFS). The output gap for each country is obtained by detrending real GDP with a quadratic time trend. Owing to the German unification in 1990, the CPI for West Germany (1979: I–1991: IV) and the CPI for unified Germany (1992: I–1998: IV) are combined. To smooth the break in the CPI, the CPI for unified Germany from 1992 is scaled up by the average ratio between the West German CPI and unified German CPI during 1991. Interest rates are the federal fund rate for the U.S., and the money market rates for other countries.

4.2. A VAR Model for the Present Value of Fundamentals

In order to construct the model-based real exchange rate (q_t^*) from Equation (9), we first need the parameter values for the Taylor rule. For the γ_q, γ_π, and γ_y parameters, we use the same values set by Engel and West (2006): $\gamma_q = 0.1$, $\gamma_\pi = 1.75$, and $\gamma_y = 0.25$. We set $\rho = 0.9$, which is roughly the estimate for the interest rate smoothing parameter of the Taylor rule in Clarida, Gali, and Gertler (1998). These values imply discount rates near one in equations (7) and (12), which means that $\beta \approx 0.99$, $\delta_1 \approx 0.94$, and $\delta_2 \approx 1.18$ in those equations. The values for β and δ_1 imply that all the discount rates in equations (7) and (12) are near one, as emphasized in Engel and West (2005). Furthermore, the value of δ_2 also suggests that the coefficient of the lagged real exchange rate ($\frac{1}{\delta_2}$) in equation (12) is also near one, which can explain why the real exchange rate is extremely persistent under interest rate smoothing.

Next, to estimate the present values of the fundamentals, we employ the following VAR model, as in Engel and West (2006) and Mark (2009):

$$z_t = \mu + \Phi_1 z_{t-1} + \Phi_2 z_{t-2} + \cdots + \Phi_p z_{t-p} + e_t, \tag{25}$$

where $z_t = [f_{1,t} \quad f_{2,t} \quad f_{3,t}]'$ is a vector of fundamentals: $f_{1,t}$, $f_{2,t}$, and $f_{3,t}$ denote the inflation rate differential $\left(\pi_t^h - \pi_t^f\right)$, output gap differential $(y_t^h - y_t^f)$, and interest rate differential $(i_t^h - i_t^f)$, respectively. We set the lag length of the VAR to 4 (i.e. $p = 4$).

By suppressing the intercept term for an expositional purpose, the VAR representation in (14) can be written in the following companion form:

$$Z_t = F Z_{t-1} + \tilde{e}_t \tag{26}$$

where $Z_t = \begin{bmatrix} z_t \\ z_{t-1} \\ Z_{t-2} \\ \vdots \\ z_{t-p+1} \end{bmatrix}$, $F = \begin{bmatrix} \Phi_1 & \Phi_2 & \cdots & \Phi_p \\ I_3 & 0 & \cdots & 0 \\ 0 & I_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$, and $\tilde{e}_t = \begin{bmatrix} e_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$. Then, the present-value terms in Equation (7) can be written as follows:

$$\sum_{i=0}^{\infty} \beta^i E_t(f_{1,t+1+i}) = \frac{1-\tilde{\gamma}_\pi}{1+\tilde{\gamma}_q} \omega_1{}' F(I_{12} - \beta F)^{-1} Z_t, \tag{27}$$

$$\sum_{i=0}^{\infty} \beta^i E_t(f_{2,t+i}) = \frac{-\tilde{\gamma}_y}{1+\tilde{\gamma}_q} \omega_2{}'(I_{12} - \beta F)^{-1} Z_t, \tag{28}$$

$$\sum_{i=0}^{\infty} \beta^i E_t(f_{3,t-1+i}) = \frac{-\rho}{1+\tilde{\gamma}_q} [f_{3,t-1} + \beta \omega_3{}'(I_{12} - \beta F)^{-1} Z_t], \tag{29}$$

where ω_n is the selection vector. For example, $\omega_1 = [1\ 0\ 0\ \cdots 0]$, $\omega_2 = [0\ 1\ 0\ \cdots 0]$, and so on. Once the present-value terms are constructed, then we can obtain $\epsilon_{1t} = q_t - q_t^*$ and further compute $\epsilon_{2t} = \epsilon_{1t} - \frac{1}{\delta_2}\epsilon_{1t-1}$ from the estimate of ϵ_{1t}.

4.3. Results

In this section, we estimate model-based real exchange rates and compare ϵ_{1t} and ϵ_{2t} in order to quantify the relative contributions of Taylor-rule fundamentals, monetary shocks and risk-premium shocks in explaining real exchange rate variations.

Table 1 compares autocorrelations of the actual exchange rate with those of q_t^* in Equation (9). The stylized pattern of exchange rate is that the real exchange rate is highly autocorrelated, whereas the growth rates of real and nominal exchange rates have low autocorrelations (see Bekaert (1996) and Chari, Kehoe, and McGrattan (2002)). This pattern is shown in the second column of each panel in Table 1. The third column of Table 1 shows that the model-based exchange rates from Equations (9) display properties similar to the stylized pattern shown in the second column. Similar patterns can be observed for Corr(Δs_t, Δq_t) from both actual and model-based real exchange rates.

Table 2 presents cross-correlations between actual data and the counterparts from the model-based exchange rate series. Correlations between the growth rates of actual exchange rates and the growth rates of model-based exchange rates are generally low. Figure 1 compares model-based real exchange rates from Equation (9) with actual real exchange rates across countries. As implied by Tables 1, and 2, model-based real exchange rates seem a smooth trend of actual real exchange rates in all countries. The results in these tables and figures are quite similar to those in Engel and West (2006). We also compare movements of ϵ_{1t} and ϵ_{2t} in Figure 2. As shown in Figure 2, ϵ_{1t} is much more persistent and has greater variability than ϵ_{2t}.

Utilizing estimates of ϵ_{1t} and ϵ_{2t} and the relation in Equation (23), we can compute the magnitudes of σ_m^2, σ_η^2, and $\sigma_{m,\eta}$, and further quantify relative contributions of each term in Equation (24) to the variation in the real exchange rate. The results from this variance decomposition exercise are shown in Table 3. Notable points emerge from Table 3.

First, even though previous studies report that the Taylor-rule fundamentals have significant forecast ability for future movements of the exchange rate and model-based exchange rate from the Taylor-rule fundamentals can replicate some stylized facts, the contribution of the Taylor-rule fundamentals to the variation of the real exchange rate seems quite limited. The first column of Table 3 shows that the relative contribution of the Taylor-rule fundamentals ranges between 3% and 17%. Even if we consider the covariance between model-based exchange rate and residuals, which is the composite of shocks, the Taylor-rule fundamentals can never explain the majority of variations in the real exchange rate.

Second, as shown in the third column of Table 3, the importance of monetary policy shock is small in understanding fluctuations of real exchange rates. For the countries under consideration, the relative contributions of monetary policy shock in the real exchange rate dynamics range between 3% and 15%. This limited contribution of monetary policy shock is in line with previous studies such as Clarida and Gali (1994), Juvenal (2011), and Faust and Rogers (2003).

Third, the largest contribution to explain movements of the real exchange rate comes from the risk-premium shock in all countries examined, which is shown in the fourth column of Table 3. The risk-premium shock can explain 49% - 89% of variations in the real exchange rate. Interestingly, as we assume greater variability for forecast errors (ξ_t) in the variance of ϵ_{2t}, the contribution of the risk-premium declines monotonically except Canada. The large contribution from the risk-premium shock with the small contribution from the monetary policy shock is in sharp contrast to Bergin (2008) arguing that the monetary policy shock, not the risk-premium shock, is the primary driver of the exchange rate.

Fourth, the covariance between the monetary policy shock and risk-premium shock is not zero but quite low when $k = \frac{1}{\delta_2^2}\mathrm{Var}(\xi_t)/\mathrm{Var}(\epsilon_{2t})$ is low. The low covariance between the monetary policy shock and risk-premium shock seems consistent with the estimated correlation in Bergin (2008). As k rises, however, the covariance between the monetary policy shock and risk-premium shock increases and the contribution from the risk-premium shock declines in most cases. The negative relation between the covariance and the contribution of the risk-premium shock and the positive correlation between the monetary policy shock and risk-premium shock can be interpreted as supportive evidence to the models in McCallum (1994), Obstfeld and Rogoff (2002), and Backus et al. (2010) where the monetary policy accounts for deviations of the UIP.

Finally, we also conduct the robustness tests to see if the results in Table 3 hold under different parameter values. The results of these robustness tests are provided in Table 4. Consistent with Table 3, the contribution of the Taylor rule fundamentals continues to be below 20%, and the importance of the monetary policy shock is quite limited ranging from 4.3% to 15%. Furthermore, the 41% - 94% of the variations in the real exchange rate can be explained by the risk-premium shock, but the importance of the risk-premium shock declines as the interest rate smoothing parameter (ρ) or the persistence of the risk-premium shock (α) becomes lower. The correlation between the monetary policy shock and the risk-premium shock is mostly positive in Table 4.

5. Summary and Conclusion

When interest rate smoothing is incorporated in a Taylor-rule-based real exchange rate model, two alternative solutions exist: one is based on a first-order difference equation and the other is based on a second-order difference equation. These two alternative solutions have different implications for the error terms. Under the assumption that the persistence of risk-premium shocks is as high as the degree of interest rate smoothing, the difference in those two error terms enables us to find the following: Although many studies report that exchange rate movements are tightly related with Taylor-rule fundamentals, the contribution of Taylor-rule fundamentals ranges between 3% and 17%. Monetary policy shock, which can explain 3% and 15% of variations of the real exchange rate, is not the main driving factor for the dynamics of real exchange rates. Instead, persistent risk-premium shocks are a major part of exchange rate movements. The results from our approach comparing two alternative representations are robust under different parameter values and across countries.

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Appendix 1. Equivalence of equations (9) and (13) in the absence of shocks

In this appendix, we show that equation (13) is equivalent to equation (9). From equation (13),

$$q_t^* = a_2 + \frac{1}{\delta_2} q_{t-1}^* + \frac{1-\tilde{\gamma}_\pi}{\rho\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t(f_{1,t+1+i}) - \frac{\tilde{\gamma}_y}{\rho\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t(f_{2,t+i}) - \frac{1}{\delta_2} \sum_{i=0}^{\infty} \delta_1^i E_t(f_{1,t+i})$$

Applying the lag operator to both sides, we have the following expression:

$$q_t^* - \frac{1}{\delta_2} q_{t-1}^* = a_2 + \frac{1}{\rho\delta_2} \frac{1-\tilde{\gamma}_\pi}{1-\delta_1 L^{-1}} E_t(f_{1,t+1}) - \frac{1}{\rho\delta_2} \frac{\tilde{\gamma}_y}{1-\delta_1 L^{-1}} f_{2,t} - \frac{1}{\delta_2} \frac{1}{1-\delta_1 L^{-1}} f_{1,t}.$$

$$\frac{\rho}{\delta_2}(\delta_2 - L) q_t^* = \frac{\tilde{a}_2}{1-\delta_1 L^{-1}} + \frac{1}{\delta_2} \frac{1-\tilde{\gamma}_\pi}{1-\delta_1 L^{-1}} E_t(f_{1,t+1}) - \frac{1}{\delta_2} \frac{\tilde{\gamma}_y}{1-\delta_1 L^{-1}} f_{2,t} - \frac{\rho}{\delta_2} \frac{1}{1-\delta_1 L^{-1}} f_{1,t}.$$

$$\rho(L - \delta_2)(1 - \delta_1 L^{-1}) q_t^* = -\delta_2 \tilde{a}_2 + (\tilde{\gamma}_\pi - 1) E_t(f_{1,t+1}) + \tilde{\gamma}_y f_{2,t} + \rho f_{1,t}.$$

$$\rho L^{-1}(L - \delta_2)(L - \delta_1) q_t^* = -\delta_2 \tilde{a}_2 + (\tilde{\gamma}_\pi - 1) E_t(f_{1,t+1}) + \tilde{\gamma}_y f_{2,t} + \rho f_{1,t}.$$

$$\rho L^{-1}\left(L^2 - \frac{1+\tilde{\gamma}_q+\rho}{\rho} L + \frac{1}{\rho}\right) q_t^* = -\delta_2 \tilde{a}_2 + (\tilde{\gamma}_\pi - 1) E_t(f_{1,t+1}) + \tilde{\gamma}_y f_{2,t} + \rho f_{1,t}.$$

$$\left(\rho L - (1 + \tilde{\gamma}_q + \rho) + L^{-1}\right) q_t^* = -\delta_2 \tilde{a}_2 + (\tilde{\gamma}_\pi - 1) E_t(f_{1,t+1}) + \tilde{\gamma}_y f_{2,t} + \rho f_{1,t}.$$

$$\rho q_{t-1}^* - \left(1 + \tilde{\gamma}_q + \rho\right) q_t^* + E_t(q_{t+1}^*) = -\delta_2 \tilde{a}_2 + (\tilde{\gamma}_\pi - 1) E_t(f_{1,t+1}) + \tilde{\gamma}_y f_{2,t} + \rho f_{1,t}.$$

$$E_t(q_{t+1}^*) - q_t^* = -\delta_2 \tilde{a}_2 + \tilde{\gamma}_q q_t^* + (\tilde{\gamma}_\pi - 1) E_t(f_{1,t+1}) + \tilde{\gamma}_y f_{2,t} + \rho(f_{1,t} + q_t^* - q_{t-1}^*).$$

Under the UIP with neither shocks nor errors for the model-based real exchange rate, $i_{t-1}^h - i_{t-1}^f = f_{3,t-1} = q_t^* - q_{t-1}^* + \pi_t^h - \pi_t^f$.

Hence, $\left(1 + \tilde{\gamma}_q\right) q_t^* = \delta_2 \tilde{a}_2 + (1 - \tilde{\gamma}_\pi) E_t(f_{1,t+1}) - \tilde{\gamma}_y f_{2,t} - \rho f_{3,t-1} + E_t(q_{t+1}^*)$.

$$q_t^* = \beta \tilde{\gamma}_q c_0 + \beta(1 - \tilde{\gamma}_\pi) E_t(f_{1,t+1}) - \beta \tilde{\gamma}_y f_{2,t} - \beta\rho f_{3,t-1} + \beta E_t(q_{t+1}^*)$$

where $a_2 = \frac{\tilde{\gamma}_q c_0}{(1-\delta_1)\delta_2}$ and $\tilde{a}_2 = (1-\delta_1)a_2$. It is straightforward to show that the last expression can be written as follows:

$$q_t^* = a_1 + \beta(1-\tilde{\gamma}_\pi)\sum_{i=0}^{\infty}\beta^i E_t(f_{1,t+1+i})$$

$$-\beta\tilde{\gamma}_y\sum_{i=0}^{\infty}\beta^i E_t(f_{2,t+i}) - \beta\rho\sum_{i=0}^{\infty}\beta^i E_t(f_{3,t-1+i})$$

where $a_1 = \frac{\beta\tilde{\gamma}_q c_0}{1-\beta}$. Since the last equation is equivalent to equation (9), equations (9) and (13) are equivalent in the absence of all shocks like the model-based real exchange rate.

Table 1. Comparison of Autocorrelations and Correlations

Variables	Actual data (q_t)	Model-based real exchange rate (q_t^*)
Canada		
q_t	0.9678	0.9094
Δq_t	0.0457	0.0866
$\Delta s_t = \Delta q_t + \pi_t$	0.0372	0.0700
Corr(Δs_t, Δq_t)	0.9813	0.9903
Germany		
q_t	0.9231	0.9466
Δq_t	0.1429	0.0481
$\Delta s_t = \Delta q_t + \pi_t$	0.1493	0.0486
Corr(Δs_t, Δq_t)	0.9948	0.9517
Japan		
q_t	0.9451	0.8845
Δq_t	0.0532	0.0518
$\Delta s_t = \Delta q_t + \pi_t$	0.0655	0.0267
Corr(Δs_t, Δq_t)	0.9948	0.9943
Switzerland		
q_t	0.9221	0.9213
Δq_t	0.0328	0.0761
$\Delta s_t = \Delta q_t + \pi_t$	0.0377	0.0716
Corr(Δs_t, Δq_t)	0.9961	0.9981
UK		
q_t	0.8740	0.8818
Δq_t	0.1111	0.0647
$\Delta s_t = \Delta q_t + \pi_t$	0.1235	0.0605
Corr(Δs_t, Δq_t)	0.9883	0.9937

Note: This table compares the first-order autocorrelations. Corr(Δs_t, Δq_t) denotes correlation between Δs_t and Δq_t.

Table 2. Comparison of Cross-Correlations

	Corr(Δq_t^* , Δq_t)	Corr(Δs_t^* , Δs_t)
Canada	0.0155	0.0488
Germany	0.1344	0.1375
Japan	0.0553	0.0214
Switzerland	-0.0666	-0.1137
UK	-0.0420	-0.0937

Note: This table shows the cross-correlations between Δq_t^* and Δq_t or between Δs_t^* and Δs_t.

Table 3. Variance Decomposition of the Real Exchange Rate

	Fundamentals $\frac{\text{Var}(q_t^*)}{\text{Var}(q_t)}$	Monetary policy shock $\frac{\beta^2\sigma_m^2}{\text{Var}(q_t)}$	Risk-premium shock $\frac{\frac{\beta^2}{(1-\alpha\beta)^2}\frac{\sigma_\eta^2}{1-\alpha^2}}{\text{Var}(q_t)}$	Covariance between monetary shock and risk-premium shock $\frac{2\frac{\beta^2}{1-\alpha\beta}\sigma_{m,\eta}}{\text{Var}(q_t)}$	Covariance $\frac{2Cov(q_t^*,\epsilon_{1t})}{\text{Var}(q_t)}$
$k \equiv \frac{\text{Var}(\xi_t)}{\delta_2^2\text{Var}(\epsilon_{2t})} = 0.1$					
Canada	0.0474	0.0554	0.5910	0.1756	0.1307
Germany	0.1470	0.1206	0.5508	0.0588	0.1228
Japan	0.0307	0.0874	0.6916	0.1763	0.0141
Switzerland	0.1684	0.1501	0.6577	0.0775	-0.0538
UK	0.0933	0.1456	0.8852	0.0462	-0.1702
$k \equiv \frac{\text{Var}(\xi_t)}{\delta_2^2\text{Var}(\epsilon_{2t})} = 0.3$					
Canada	0.0474	0.0344	0.5559	0.2527	0.1307
Germany	0.1470	0.0851	0.5211	0.1240	0.1228
Japan	0.0307	0.0574	0.6666	0.2312	0.0141
Switzerland	0.1684	0.1058	0.6207	0.1589	-0.0538
UK	0.0933	0.1033	0.8499	0.1238	-0.1702
$k \equiv \frac{\text{Var}(\xi_t)}{\delta_2^2\text{Var}(\epsilon_{2t})} = 0.5$					
Canada	0.0474	0.0134	0.5734	0.2142	0.1307
Germany	0.1470	0.0495	0.4915	0.1892	0.1228
Japan	0.0307	0.0275	0.6416	0.2862	0.0141
Switzerland	0.1684	0.0614	0.5837	0.2402	-0.0538
UK	0.0933	0.0610	0.8146	0.2014	-0.1702

Notes: This variance decomposition exercise is conducted under the assumption that $\gamma_\pi = 1.75$, $\gamma_y = 0.25$, $\gamma_q = 0.1$, $\rho = \alpha = 0.9$. Since $q_t = q_t^* + \epsilon_{1t} = q_t^* + \beta\left(u_{mt}^f - u_{mt}^h\right) + \frac{\beta}{1-\alpha\beta}u_t^r$, $\text{Var}(q_t) = \text{Var}(q_t^*) + 2Cov(q_t^*,\epsilon_{1t}) + \beta^2\sigma_m^2 + \frac{\beta^2}{(1-\alpha\beta)^2}\frac{\sigma_\eta^2}{1-\alpha^2} + 2\frac{\beta^2}{1-\alpha\beta}\sigma_{m,\eta}$.

Table 4. Variance Decomposition of the Real Exchange Rate: Robustness Check

	Fundamentals $\frac{\text{Var}(q_t^*)}{\text{Var}(q_t)}$	Monetary policy shock $\frac{\beta^2\sigma_m^2}{\text{Var}(q_t)}$	Risk-premium shock $\frac{\frac{\beta^2}{(1-\alpha\beta)^2}\frac{\sigma_\eta^2}{1-\alpha^2}}{\text{Var}(q_t)}$	Covariance between monetary shock and risk-premium shock $\frac{2\frac{\beta^2}{1-\alpha\beta}\sigma_{m,\eta}}{\text{Var}(q_t)}$	Covariance $\frac{2Cov(q_t^*,\epsilon_{1t})}{\text{Var}(q_t)}$
$\gamma_\pi = 2,\ \gamma_y = 0.25,\ \gamma_q = 0.1,\ \rho = \alpha = 0.9$					
Canada	0.0471	0.0554	0.5920	0.1760	0.1296
Germany	0.1503	0.1211	0.5503	0.0581	0.1203
Japan	0.0307	0.0875	0.6916	0.1762	0.0140
Switzerland	0.1690	0.1504	0.6575	0.0772	-0.0541
UK	0.0922	0.1454	0.8855	0.0465	-0.1695
$\gamma_\pi = 1.75,\ \gamma_y = 0.5,\ \gamma_q = 0.1,\ \rho = \alpha = 0.9$					
Canada	0.0519	0.0562	0.5892	0.1742	0.1284
Germany	0.1995	0.1262	0.5317	0.0492	0.0935
Japan	0.0397	0.0892	0.6887	0.1728	0.0096
Switzerland	0.1841	0.1534	0.6500	0.0730	-0.0605
UK	0.0966	0.1458	0.8904	0.0481	-0.1808
$\gamma_\pi = 1.75,\ \gamma_y = 0.25,\ \gamma_q = 0.05,\ \rho = \alpha = 0.9$					
Canada	0.0508	0.0536	0.6245	0.1405	0.1306
Germany	0.1615	0.1187	0.5814	0.0274	0.1109
Japan	0.0321	0.0845	0.7337	0.1372	0.0125
Switzerland	0.1839	0.1481	0.6997	0.0410	-0.0727
UK	0.0985	0.1449	0.9405	-0.0015	-0.1824
$\gamma_\pi = 1.75,\ \gamma_y = 0.25,\ \gamma_q = 0.1,\ \rho = \alpha = 0.75$					
Canada	0.0319	0.0119	0.4757	0.3681	0.1124
Germany	0.1555	0.0901	0.4133	0.2154	0.1257
Japan	0.0321	0.0426	0.5313	0.3781	0.0159
Switzerland	0.1111	0.1050	0.4875	0.2693	0.0271
UK	0.0561	0.0807	0.6722	0.2994	-0.1084

Notes: This variance decomposition exercise is conducted under the assumption that $k \equiv \frac{\text{Var}(\xi_t)}{\delta_2^2\text{Var}(\epsilon_{2t})} = 0.1$. Since $q_t = q_t^* + \epsilon_{1t} = q_t^* + \beta(u_{mt}^f - u_{mt}^h) + \frac{\beta}{1-\alpha\beta}u_t^r$, $\text{Var}(q_t) = \text{Var}(q_t^*) + 2Cov(q_t^*,\epsilon_{1t}) + \beta^2\sigma_m^2 + \frac{\beta^2}{(1-\alpha\beta)^2}\frac{\sigma_\eta^2}{1-\alpha^2} + 2\frac{\beta^2}{1-\alpha\beta}\sigma_{m,\eta}$.

Figure 1. Actual and Theoretical Real Exchange Rate Series

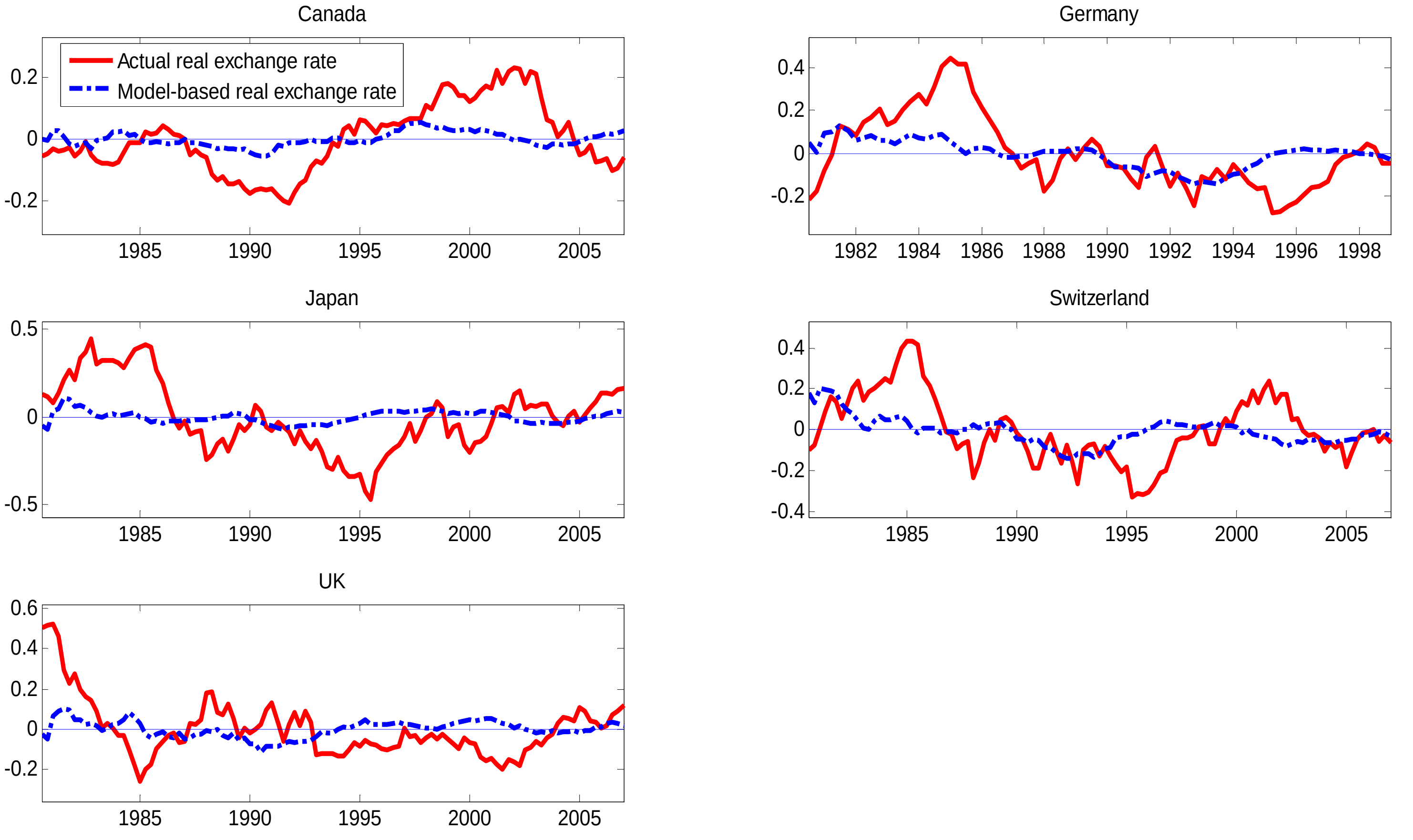

Figure 2. Comparison of ϵ_{1t} and ϵ_{2t}