Wage Negotiations in Multi-worker Firms and Stochastic Bargaining Powers of Existing Workers*

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Abstract

This paper develops a business cycle search and matching model that can generate realistic labor market dynamics including labor share in the US. To this end, I introduce an alternative mechanism of wage negotiations and bargaining shocks in an environment where a firm hires more than one worker and the firm faces diminishing marginal product of labor (MPL). When Nash bargaining with a marginal worker breaks down, a firm negotiates wages with existing workers collectively and produces with them. Due to diminishing MPL, the breakdown of the negotiation with the marginal worker negatively affects the bargaining position of the firm with existing workers (one fewer workers) since MPL is higher with one fewer workers. How much the firm internalizes this negative effect depends on stochastic bargaining powers of existing workers which can be identified through labor share data. The stochastic bargaining power of existing workers provides an additional margin to increase the volatility of labor market variables. In contrast to the prediction of Ríos-Rull and Santaeulalia-Llopis (2010), in which the effect of productivity shocks is dampened when labor share overshoots due to huge wealth effects from the overshooting property, this paper presents a model in which the labor share overshoots and the volatility of employment closely matches that of US data.

Keywords: wage negotiation, multi-worker firms, existing workers, bargaining shocks, business cycle

JEL Classification Numbers: E24, E32, E60, H55, I38, J65

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1 Introduction

This paper develops a business cycle search and matching model that can generate realistic labor market dynamics including labor share in the US. In literature, two different bargaining protocols are used in the search and matching model where a firm hires more than one worker and the firm faces diminishing marginal product of labor (MPL). One is the Stole and Zwiebel (1996) type bargaining protocol as in Elsby and Michaels (2013), Acemoglu and Hawkins (2014), and Hawkins (2015). In these papers, a breakdown of a negotiation with a marginal worker negatively affects the bargaining position of the firm with other workers (one fewer workers) since MPL is higher with one fewer workers. The other is a standard bargaining protocol as in Merz (1995), Andolfatto (1996), and Cheron and Langot (2004). In these papers, a breakdown of a negotiation does not affect the bargaining with other workers because they implicitly assume that MPL does not change when the firm bargains with other workers. I interpret these two bargaining protocols as two extreme cases: in terms of relative bargaining powers between a firm and other workers. I will call other workers existing workers. If other workers have all the bargaining powers[1], then the firm has to fully internalize the negative effects from the breakdown of the negotiation with a marginal worker. However, if the firm has all the bargaining powers[2], the firm does not internalize any negative effects from the breakdown by ignoring that MPL is higher with one fewer workers. Given the two extreme cases, I am looking at cases between the two extremes by introducing stochastic bargaining powers of other workers.

In this paper, when Nash bargaining with a marginal worker breaks down, a firm negotiates wages with existing workers. The bargaining powers of existing workers are stochastic. Due to diminishing MPL, the breakdown of the negotiation with the marginal worker negatively affects the bargaining position of the firm with existing workers (one fewer workers) since MPL is higher with one fewer workers. How much the firm internalizes this negative effect depends on the stochastic bargaining powers of existing workers which can be identified through labor share data. During expansions, it is relatively difficult for the firm to hire workers, so existing

[1] This is my interpretation of the Stole and Zwiebel type bargaining protocol

[2] This is my interpretation of the standard bargaining protocol

workers might have higher bargaining powers. If the firm fails to hire a marginal worker due to a breakdown of negotiations, the firm has to pay higher wages to existing workers in order to produce goods with them. Since the failure to hire marginal workers is more costly during expansions, the firm has more incentives to hire marginal workers by offering higher wages to forgo the higher cost associated with the breakdown. During recessions, the opposite happens. Through this mechanism, the stochastic bargaining powers of existing workers provide an additional margin to increase the volatility of labor market variables. The calibrated model generates more volatile total hours, employment, hours per worker while labor share overshoots in response to productivity shocks as documented in Ríos-Rull and Santaeulalia-Llopis (2010). In particular, the volatility of employment in the model is similar to the actual US data. In contrast to the prediction of Ríos-Rull and Santaeulalia-Llopis (2010), in which the effect of productivity shocks is dampened when labor share overshoots due to huge wealth effects from the overshooting property of labor share, this paper presents a model in which the labor share overshoots in response to productivity shocks and the volatility of employment closely matches that of US data.

This paper is related to several studies which can be classified into three groups. First, the baseline model is based on Andolfatto (1996). His model embeds search and matching framework into an otherwise standard RBC model, and has both extensive margins and intensive margins. By incorporating search and matching framework in labor markets, the model improves the standard RBC model along several dimensions. However, the volatility of labor market variables is still far lower than that of actual data. The Andolfatto model also has highly pro-cyclical real wages and labor productivity, which have weakly pro-cyclical counterparts in actual data. Several papers have addressed these problems. Nakajima (2012) analyzes several volatility problems by explicitly distinguishing between leisure and unemployment benefits for the outside options of households. This distinction is consistent with the calibration proposed by Hagedorn and Manovskii (2008). However, the main focus of Nakajima (2012) is unemployment and vacancies than employment and hours per worker, which are my main interest. Cheron and Langot (2004) address the second failure of Andolfatto (1996) by using non-separable preference between consumptions and leisure such that the outside options of households can move counter-cyclically. This proposal results in less pro-cyclical real wages and labor productivity. However, this paper is not interested in the volatility of labor market variables in general.

The second branch of papers related to my paper is literature on the Stole and Zwiebel type bargaining and its applications to business cycle dynamics[3]. Krause and Lubik (2013) (KL, henceforth) incorporate the Stole and Zwiebel type bargaining protocol into a simple RBC search and matching model to evaluate the quantitative effects of the bargaining protocol on business cycle dynamics. They show that the aggregate effects of the bargaining protocol are negligible. Table 1 summarizes business cycle moments for the modified KL model[4]. The performance of KL model is almost the same as the Andolfatto model, and both models perform poorly in replicating moments along several dimensions. In contrast to KL, this paper introduces the stochastic bargaining with existing workers when the match with a marginal worker fails, and the bargaining powers of existing workers vary stochastically. The time-varying incentives to hire workers for firms, resulting from the stochastic bargaining, provide a new margin to increase the volatility of labor market variables. Later, it turns out that Andolfatto and KL are two extreme cases where bargaining powers of existing workers are fixed at 0, and 1, respectively, in the baseline model.

Lastly, this paper is also related to papers studying labor share. Recently, Rios-Rull and Santaeulalia-Llopis (2010) document several properties of labor share dynamics based on US data. In particular, they propose redistributive shocks that can be identified by using labor share data in the US, and point out the importance of the dynamic property of labor share (overshooting). They showed that labor share overshoots in response to productivity shocks, and the dynamic overshooting response of labor share drastically dampens the role of

[3]Elsby and Michaels (2013), Acemoglu and Hawkins (2014), and Hawkins (2015) also study the labor marker fluctuations with the Stole and Zwiebel type bargaining, but the main focus of these papers is unemployment and vacancies than employment and hours per worker.

[4]The original model in KL does not have capital and intensive margins. Therefore, I add the Stole and Zwiebel type bargaining to the Andolfatto model rather than to the original model in KL in order to assure fair comparison of the two models.

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho\,(x, Output)$			$\rho\,(x_t, x_{t-1})$		
Variable (x)	Data	Andolfatto	KL	Data	Andolfatto	KL	Data	Andolfatto	KL
Output	1.54 (1.00)	1.31 (1.00)	1.31 (1.00)	-	-	-	0.86	0.82	0.82
Total Hours	1.38 (0.90)	0.70 (0.53)	0.70 (0.53)	0.85	0.92	0.92	0.88	0.91	0.91
Employment	1.00 (0.65)	0.68 (0.52)	0.69 (0.53)	0.81	0.78	0.78	0.91	0.89	0.88
Hours per Worker	0.49 (0.32)	0.19 (0.15)	0.19 (0.15)	0.74	0.55	0.54	0.56	0.58	0.58
Wages	0.91 (0.59)	0.62 (0.47)	0.64 (0.49)	0.34	0.94	0.94	0.69	0.65	0.64
Labor Productivity	0.82 (0.53)	0.72 (0.55)	0.72 (0.55)	0.45	0.92	0.92	0.57	0.62	0.62
Labor Share	0.74 (0.48)	0.12 (0.09)	0.09 (0.07)	-0.08	-0.72	-0.70	0.78	0.51	0.50
Vacancies	13.23 (8.59)	3.65 (2.79)	3.69 (2.82)	0.90	0.80	0.80	0.91	0.54	0.54

1) All data are in logs and filtered using the HP filter with a smoothing parameter of 1600.
2) In the Andolfatto model, I use the standard bargaining protocol
3) In the KL model, I use the Stole and Zwiebel type bargaining protocol

Table 1: Business cycle moments in data and models over 1960:Q1-2012:Q1

productivity shocks on labor markets due to huge wealth effects. My model also generates the overshooting property of labor share, but total hours, employment, and hours per worker are still more volatile than the benchmark Andolfatto model. Different from Rios-Rull and Santaeulalia-Llopis (2010), the search and matching framework weakens wealth effects from the overshooting of labor share and more incentives for firms to hire workers offset the huge reduction of total hours.

The main contribution of this paper is as follows. First, I incorporate the stochastic bargaining with existing workers into the Andolfatto model, which has both extensive and intensive margins in the labor market[5]. To the best of my knowledge, I first study the effect of the stochastic bargaining powers of existing workers on wage negotiations in multi-worker firms. The bargaining powers of existing workers can be time-varying because when labor markets are tighter, mostly in booms, existing workers are more valuable as the firm will have difficulty finding new workers. However, when the labor market is less tight, mostly in recessions, existing workers become less attractive to firms, which could easily find new workers. This reason makes the bargaining powers of existing workers possibly pro-cyclical with some lags. Another possible explanation could be related to the entry and exit of firms. In booms, firms compete

[5] I include intensive margins for two reasons. First, labor share is important for identifying bargaining shocks, and for the labor share in the model to be consistent with actual data, I need to include intensive margins. Second, bargaining shocks directly affect intensive margins because the bargaining powers of existing workers affect the relative usefulness of intensive and extensive margins for the firm.

with each other because of higher entry rates of new firms and lower exit rates of existing firms. These situations reduce the monopolistic or bargaining powers of firms over existing workers. However, during recessions, the opposite happens. Given this explanation, the bargaining powers of existing workers move pro-cyclically with some lags based on the pro-cyclical entry and the counter-cyclical exit rates. The inclusion of the stochastic bargaining with existing workers improves the capacity of the standard RBC search and matching model, especially in the volatility of total hours, employment, hours per worker, and labor share.

Second, I identify bargaining shocks by using labor share data. I provide the link between the bargaining powers of workers and the movement of labor share in the US. In addition, my model generates an overshooting property of labor share, but the effect of productivity shocks on labor market variables are still significant in contrast to the prediction of Rios-Rull and Santaeulalia-Llopis (2010). In their model, the effect of productivity shocks is dampened when labor share overshoots because of huge wealth effects from the overshooting property. In contrast to their model, the baseline model has a search and matching framework, and the nature of this framework weakens wealth effects resulting from the overshooting of labor share. On top of these differences, more incentives for firms to hire workers offset the huge reduction of total hours in booms.

The remainder of the paper is structured as follows. Section 2 introduces the baseline model with the stochastic bargaining powers of existing workers. Section 3 discusses the calibration of the baseline model. Section 4 shows quantitative analysis of the model. Section 5 discusses the robustness of the baseline model. Finally, Section 6 concludes and proposes the further research.

2 Model

I develop a model based on a standard RBC search and matching model, the Andolfatto (1996) model. The main difference between this paper and the Andolfatto model is the outside option of a firm in the bargaining with a marginal worker. I explicitly consider the outside option of a firm when the firm bargains with a marginal worker. The outside option of the firm is bargaining with existing workers and producing goods with them. The issue with bargaining with existing workers is the wages the firm pays. In this paper, these wages depend on the bargaining powers of existing workers. If the bargaining powers of existing workers are high, then existing workers will receive higher wages, but if the bargaining powers of existing workers are low, then they will receive lower wages. Note that these wages are not realized if the match with the marginal worker is successful while they still affect the equilibrium wages. In this paper, matches are always successful because the surplus of a new match is always positive. Therefore, wages bargained with existing workers would not be realized in equilibrium. Furthermore, I assume the bargaining powers of existing workers stochastically evolves. Except for the stochastic bargaining with existing workers, the baseline model is similar to Andolfatto (1996), and Cheron and Langot (2004).

2.1 Matching

I assume that the period in the model is a quarter. The timing of my model is as follows: (1) shocks are realized, (2) wages and hours per worker are bargained over with marginal workers, (3) if matches are not successful, the firm bargains wages with existing workers (4) workers are matched with the firm (5) production takes place and the firm posts vacancies, and (6) separations occur.

Since labor markets are frictional, the unemployed search for jobs and firms post vacancies to hire workers. The number of matches is determined by constant returns to scale matching function $M = M(V, 1-N)$, which depends on the total number of vacancies, V, and the total number of the unemployed, $U \equiv 1 - N$. For later use, I define $\theta = V/(1-N)$ as market tightness in labor markets. Also, I define the job-finding rate $p(\theta) \equiv M/(1-N) = M(\theta, 1)$ and the job-filling rate $q(\theta) \equiv M/V = M(1, 1/\theta)$. Finally, I assume that workers are separated at the exogenous and constant rate $\chi \in (0, 1)$. Therefore, we have the following law of motion of total employment.

$$N^{'} = (1-\chi) N + M (V, 1-N)$$

2.2 Household

There is a continuum of identical and infinitely lived households of measure one. The measure of members in each household is also normalized to 1. The aggregate states in this economy are given by $S = \{z, \gamma; K, N\}$, where z is the aggregate productivity and γ is the bargaining weight of existing workers, which varies stochastically. K is the aggregate capital stock, and N is total employment. The individual state variables of the household are $s_H = \{a, n\}$, where a is the amount of assets they hold and n is the measure of the employed in household. I can write the household problem as follows:

$$\Omega (S, s_H) = \max_{c, a^{'}} u(c) + n\widetilde{u} (1 - h(S, s_H)) + (1-n) \widetilde{u} (1) + \beta E \left[\Omega \left(S^{'}, s_H^{'} \right) \right] \tag{1}$$

$$s.t.$$

$$c + a^{'} + T(S) = w(S, s_H) h(S, s_H) n + (1-n) b + (1 + r(S)) a + \Pi (S)$$

$$n^{'} = (1-\chi) n + p(S) (1-n)$$

$$S' = G(S)$$

where $u(c)$ is utility from consumption, a is the assets household holds, $\widetilde{u}(\cdot)$ is utility from leisure, $T(S)$ is the lump-sum tax, $\Pi(S) = F(z, k, nh) - w(S, s_F) h(S, s_F) n - (r(S) + \delta) k - \kappa v$ is the dividend which will be defined in the firm's problem. $p(S) = M/(1-N)$ is the job-finding rate and $G(S)$ is the law of motion of aggregate state variables. Household takes wages $w(S, s_H)$ and hours per worker $h(S, s_H)$ as given. They are jointly determined via Nash bargaining.

The household consumes (c), accumulate assets (a) which they rent to a firm, and supplies labor. The n fraction of members in each household is matched with the firm and employed. And the $1-n$ fraction of members is unemployed, searches for jobs, and they collect unemployment benefits (b) from the government. I assume that there is no search cost, and so every member who is not employed searches for the job.[6] I also assume that there is a perfect insur-

[6]In this sense, u in my model is the non-employed. I do not distinguish between the unemployed and the

ance for unemployment within the household as noted in Andolfatto (1996).[7] As a result, every member receives the same consumption level. Note that this implies unemployed members are better off than those who are employed since they receive the same consumption level but the unemployed enjoy a full amount of leisure.[8]

The first order conditions of household's problem give[9]

$$\beta E\left[\frac{u_c^{'}}{u_c}\left(1+r^{'}\right)\right] = 1 \tag{2}$$

This is a standard Euler equation for the household.

2.3 Firm

There exists a representative firm. The firm produces goods using a constant returns to scale production technology $F(z,k,nh)$, where z is the aggregate productivity . Given the aggregate state S, and the individual state variable of the firm $s_F = \{n\}$, I can write firm's recursive problem as follows:

$$\begin{aligned} J(S,s_F) &= \max_{v,k} \Pi(S) + E\left[\widetilde{\beta}(S,S')\,J(S',s_F')\right] \quad (3)\\ &= \max_{v,k} F(z,k,nh) - w(S,s_F)\,h(S,s_F)\,n - (r(S)+\delta)\,k - \kappa v + E\left[\widetilde{\beta}(S,S')\,J(S',s_F')\right] \\ s.t. & \\ n' &= (1-\chi)\,n + q(S)\,v \\ S' &= G(S) \end{aligned}$$

where $\widetilde{\beta}\left(S,S^{'}\right) = \beta u_c\left(c\left(S^{'}\right)\right)/u_c\left(c\left(S\right)\right)$ is the stochastic discount factor, κ is the cost of posting vacancies, and $q(S) = M/V$ is the job-filling rate. Again, $G(S)$ is the law of motion of aggregate state variables. The firm hires workers and rent capital from the households, and posts vacancies to hire more workers in the next period. Firms also take wages $w(S,s_F)$ and

non-employed like Andolfatto (1996). Since the measure of the unemployment rate in model and data are inconsistent, I do not report any statistics regarding unemployment in this paper.

[7]Separable utility functions over consumption and leisure satisfy this assumption.

[8]I can relax this assumption. As noted in Cheron and Langot (2004), Nakajima (2012), if I use non-separable utility functions over consumption and leisure, the employed receive higher levels of consumption than the unemployed. Consequently, the employed are better off in equilibrium. If I use non-separable utility functions, the performance of the model would be better, especially for labor productivity and real wages. However, I do not use these utility functions because I prefer to setup the baseline model in a more parsimonious way.

[9]I will drop state variables for simple notations.

hours per worker $h(S, s_F)$ as given. They are jointly determined via Nash bargaining. From the first order conditions, we have two equilibrium conditions.

$$r = F_k - \delta \tag{4}$$
$$\kappa = qE\left[\widetilde{\beta} J^{m'}\right] \tag{5}$$

where J^m is a marginal value of an additional employee to the firm. The first condition is an equation for the equilibrium rental rate. The second equation is a job creation condition, which implies the firm posts vacancies up to the point where the marginal cost of posting vacancies equals to the value of an additional worker discounted by the probability that the firm meets a marginal worker.

2.4 The bargaining with a marginal worker

As stated before, wages, w, and hours per workers, h, are jointly determined via Nash bargaining between a worker and a firm each period. Formally, Nash bargaining problem can be written as follows:

$$\begin{aligned}(w,h) &= arg \max_{w,h} (\Omega^m)^{\mu} (J^m)^{1-\mu} \\ &= arg \max_{w,h} \left(\frac{V^E - V^U}{u_c}\right)^{\mu} \left(\lim_{\Delta \to 0} \frac{J[n+\Delta] - J^B[n]}{\Delta}\right)^{1-\mu} \end{aligned} \tag{6}$$

The first component, Ω^m, denotes the marginal value of employment for the worker[10] and the second component, J^m, represents the marginal value of an additional employee to the firm. μ is the bargaining weight of a marginal worker. V^E is the value of employment for the worker and V^U is the value of unemployment for the worker, which is the outside option of the worker. $J[n+\Delta]$ is the value of the firm when the match with the $(n+\Delta)$-th worker is successful and $J^B[n]$ is the value of the firm when the negotiation breaks down, which is the outside option of the firm. The only difference between the bargaining problem in this paper and the standard Nash bargaining is the outside option of the firm, $J^B[n]$, which is defined within the marginal

[10]Note that this value is discounted by the marginal utility of consumption so that the unit of this term can be converted to consumption goods

value of an additional employee to the firm.

2.4.1 The marginal value of employment for the worker

I can define the marginal value of employment for the worker as follows.

$$\begin{aligned}
\Omega^m = \frac{V^E - V^U}{u_c} \quad &\equiv \quad \frac{1}{u_c}\left[whu_c + u^l\left(1-h\right) + \left(1-\chi\right)\beta E\left[V^{E'}\right] + \chi\beta E\left[V^{U'}\right]\right] \\
&\quad -\frac{1}{u_c}\left[bu_c + u^l\left(1\right) + p\beta E\left[V^{E'}\right] + \left(1-p\right)\beta E\left[V^{U'}\right]\right] \\
&= \quad wh - b - \frac{u^l\left(1\right) - u^l\left(1-h\right)}{u_c} + \left(1-\chi-p\right)\beta E\left[\frac{V^{E'} - V^{U'}}{u_c}\right] \\
&= \quad wh - b - \frac{u^l\left(1\right) - u^l\left(1-h\right)}{u_c} + \left(1-\chi-p\right)\beta E\left[\frac{u_c' \Omega^{m'}}{u_c}\right] \qquad (7)
\end{aligned}$$

Note that the bracket in the first line is the value of working which includes utility from consumption, utility from leisure, and the continuation value of employment for the worker. The bracket in the second line is the outside option of the worker which consists of utility from consumption, utility from leisure, and the continuation value of unemployment for the worker. From the value function of the household, we also have

$$\frac{\Omega_n}{u_c} \quad = \quad wh - b - \frac{u^l\left(1\right) - u^l\left(1-h\right)}{u_c} + \left(1-\chi-p\right)\beta E\left[\frac{\Omega_n'}{u_c}\right] \qquad (8)$$

From the equation (7) and (8), we have

$$\Omega^m = \frac{\Omega_n}{u_c} \qquad (9)$$

Therefore, the marginal value of employment for the worker that I defined before is the same as the partial derivative of the value function of the household with respect to the number of the employed in household, n.

2.4.2 The marginal value of an additional employee to the firm

The marginal value of an additional employee to the firm is not trivial because the outside option of the firm can be defined in different ways. The outside option of the firm in the bargaining with a marginal worker is bargaining wages with existing workers and producing goods with them. The key component of the outside option for the firm is the wages the firm pays to existing workers. Let w^e be the wages negotiated between the firm and existing workers when the match with a marginal worker breaks down[11]. I can define the value of an additional employee to the firm, J^m, as follows:

$$\begin{aligned} J^m &\equiv \lim_{\Delta\to 0}\frac{J[n+\Delta]-J^B[n]}{\Delta} \\ &= \lim_{\Delta\to 0}\frac{1}{\Delta}\left[\left(F(z,k,(n+\Delta)h)-w[n+\Delta](n+\Delta)h-(r+\delta)k-\kappa v+\beta E\left[\frac{u'_c}{u_c}J\left[(n+\Delta)'\right]\right]\right)\right. \\ &\quad \left.-\left(F(z,k,nh)-w^e[n]nh-(r+\delta)k-\kappa v+\beta E\left[\frac{u'_c}{u_c}J^B[n']\right]\right)\right] \\ &= F_n-\lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)h-w^e[n]nh}{\Delta}+(1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right] \end{aligned} \tag{10}$$

$J[n+\Delta]$ denotes the value of the firm when the match with a marginal worker is successful[12]. $J^B[n]$ denotes the value of the firm when the negotiation with the marginal worker breaks down, which is the outside option of the firm. In this case, the firm do not hire new workers and continues to produce goods with existing workers by continuing wage negotiations with them afterward. $w[n+\Delta]$ is Nash bargained wages with the $(n+\Delta)$-th worker and $w^e[n]$ is wages for existing workers when the match breaks down. The second line is the value of the firm when the firm hires $\triangle$ more workers, which includes the level of output less wage bills with workers including newly hired ones and costs of posting vacancies, and the continuation value of the firm. The third line is the outside option of the firm, which consists of the level of output less wage bills with existing workers and costs of posting vacancies, and the continuation value of the firm. The derivation of the last equation can be found in the Appendix. If the firm has all the bargaining powers, then the firm does not internalize any negative effects from the breakdown of the negotiation with the marginal worker by ignoring that MPL is higher with one fewer workers. In this case, the firm pays existing workers the same wages as the firm would have paid the marginal worker. Then, we have $w^e[n]=w[n+\Delta]$.

[11]As stated before, these wages, w^e, would not be realized in equilibrium. These wages show up in the outside option of the firm, but the match with a marginal worker is always successful in this paper because the match surplus is always positive. Consequently, the wages for existing workers are not realized in equilibrium while they still affect equilibrium wages and other variables.

[12]I drop aggregate state variables for simple notations here

Proposition 1

Suppose $w^e[n] = w[n+\Delta]$. Then, the marginal value of an additional employee to the firm reduces to

$$J^m = F_n - w[n]h + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right] \tag{11}$$

Proof. See Appendix. □ This is the standard marginal value of an additional employee to the firm in literature where wages are determined via the standard bargaining protocol as in Merz (1995), Andofatto (1996), and Cheron and Langot (2004). Also, note that this equation can be directly derived by differentiating the firm's value function J with respect to n, under the assumption that wages are not a function of n. On the other hand, if existing workers have all the bargaining powers, then the firm should fully internalize the negative effects from the breakdown. In this case, the firm continues Nash bargaining with one fewer workers, and we have $w^e = w[n]$, where $w[n]$ is Nash bargained wages with n-th worker.

Proposition 2

Suppose $w^e[n] = w[n]$. Then, the marginal value of an additional employee to the firm reduces to

$$J^m = F_n - w[n]h - \frac{\partial w[n]}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right] \tag{12}$$

Proof. See Appendix. □ This is the marginal value of an additional employee to the firm when wages are determined via the Stole and Zwiebel bargaining protocol as in Elsby and Michaels (2013), Acemoglu and Hawkins (2014), and Hawkins (2015). Note that this equation can be directly derived by differentiating the firm's value function J with respect to n, under the assumption that wages are an explicit function of n. The partial derivative term $\frac{\partial w}{\partial n}$ will be turned out to be negative later.

In this paper, I assume that wages, $w^e[n]$, are determined based on the bargaining weights of existing workers, γ. More specifically, I assume $w^e[n] \equiv \gamma w[n] + (1-\gamma)w[n+\Delta]$. For example, if existing workers have higher bargaining powers, they receive wages closer to $w[n]$, and if they have lower bargaining powers, they receive wages closer to $w[n+\Delta]$.

Proposition 3

Suppose $w^e[n] = \gamma w[n] + (1-\gamma) w[n+\Delta]$. Then, the marginal value of an additional employee to the firm reduces to

$$J^m = F_n - w[n] h - \gamma \frac{\partial w[n]}{\partial n} nh + (1-\chi) \beta E \left[\frac{u'_c}{u_c} J^{m'} \right] \tag{13}$$

Proof. See Appendix. □ By construction, if $\gamma = 0$, then *Proposition 3* reduces to *Proposition 1* (standard bargaining protocol), and if $\gamma = 1$, then *Proposition 3* reduces to *Proposition 2* (Stole and Zweibel bargaining protocol). Note that the marginal value of an additional employee to the firm depends on the stochastic bargaining weight of existing workers through the term, $\gamma \frac{\partial w[n]}{\partial n} nh$. This is the main contribution of this paper. The inclusion of the stochastic bargaining bargaining with existing workers provides an additional margin to increase the volatility of labor market variables basically through the term, $\gamma \frac{\partial w[n]}{\partial n} nh$ within the the marginal value of an additional employee to the firm.

2.4.3 Stochastic bargaining powers of existing workers, γ

The bargaining weight of existing workers, γ, can be time-varying because when labor markets are tighter, mostly in booms, existing workers are more valuable as the firm will have difficulty finding new workers. However, when the labor market is less tight, mostly in recessions, existing workers become less attractive to firms, which could easily find new workers. This reason makes the bargaining weight of existing workers possibly pro-cyclical with some lags. Another possible explanation could be related to the entry and exit of firms over business cycles, which are abstracted from in this paper. In booms, several firms compete with a specific firm because of higher entry rates of new firms and lower exit rates of existing firms. These situations reduce the monopolistic or bargaining powers of the firm over existing workers. However, during recessions, the opposite happens. Given this explanation, the bargaining weight of existing workers moves pro-cyclically with some lags based on the pro-cyclical entry and the counter-cyclical exit rates.

Since the baseline model does not have any endogenous mechanism to generate time-varying bargaining weight of existing workers, I will assume that$\gamma \in [0,1]$ varies stochastically and call innovations to γ bargaining shocks. I will show, in the calibration section, that bargaining shocks can be identified by using labor share data from US once we have the solution to the first order differential equation from the wage bill equation. I set a fixed bargaining weight for marginal workers, μ, while I allow the bargaining weights of existing workers, γ, to vary over time. In the robustness section, I show the time-varying bargaining weight of a marginal worker is quantitatively not an important factor given the constructed shock series of bargaining weight of a marginal worker, μ_t, by using labor share data. I will discuss it more in the robustness section.

2.4.4 Solutions to the bargaining with a marginal worker

Now, we turn to the bargaining problem which is the same as standard Nash bargaining given the marginal value of employment for the worker and the marginal value of an additional employee to the firm.

$$\Omega^m = wh - b - \frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right] \tag{14}$$

$$J^m = F_n - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right] \tag{15}$$

Given the bargaining weight of the marginal worker, $\mu \in [0,1]$, and the bargaining powers of existing workers , $\gamma \in [0,1]$, wages and hours per worker are determined via the following standard bargaining problem.

$$(w,h) = arg\max_{w,h}(\Omega^m)^{\mu}(J^m)^{1-\mu} \tag{16}$$

I will write w instead of $w[n]$ for simple notations hereafter. From the first order conditions with respect to w and h, we have the following two equations.

$$wh = \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \tag{17}$$

$$\frac{\widetilde{u}_l(1-h)}{u_c} = F_{nh} - \gamma\frac{\partial w}{\partial n}n \tag{18}$$

where $F_{nh} = \frac{\partial F(z,k,nh)}{\partial(nh)}$[13]. The equation (17) is the wage bill equation and the equation (18) is an intra-temporal condition for hours per worker. Note that we have additional terms, $\gamma\frac{\partial w}{\partial n}nh$ and $\gamma\frac{\partial w}{\partial n}n$ in the equation (17) and (18) compared to the standard bargaining case. The term $\frac{\partial w}{\partial n}$ can be calculated by solving the first order differential equation, which will be defined from the wage bill equation shortly. Equation (17) is similar to the wage bill equation as in Cheron and Langot (2004) except for the second term in the right hand side, $\gamma\frac{\partial w}{\partial n}nh$. I can rewrite the wage bill equation (17) as the first order differential equation with respect to wages w. Assuming a Cobb-Douglas production function, $F(z,k,nh) = e^z k^\alpha (nh)^{1-\alpha}$, the solution to the first order differential equation is given as

$$w = \mu\left(\frac{1-\alpha}{1-\mu\gamma\alpha}e^z k^\alpha h^{-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa h^{-1}\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right)h^{-1} \tag{19}$$

From equation (19), we have

$$\frac{\partial w}{\partial n} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\gamma\alpha}e^z k^\alpha h^{-\alpha} n^{-\alpha-1} < 0 \tag{20}$$

$$\gamma\frac{\partial w}{\partial n} = -\frac{\mu\gamma\alpha(1-\alpha)}{1-\mu\gamma\alpha}e^z k^\alpha h^{-\alpha} n^{-\alpha-1} < 0 \tag{21}$$

Using the equation (21), we can rewrite two important conditions (17) and (18) as follows:

$$wh = \mu\left(\frac{1-\alpha}{1-\mu\gamma\alpha}e^z k^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \tag{22}$$

$$\frac{\widetilde{u}_l(1-h)}{u_c} = \frac{(1-\alpha)}{1-\mu\gamma\alpha}e^z k^\alpha (nh)^{-\alpha} \tag{23}$$

[13] Following Andolfatto (1996) and Cheron and Langot (2004), it is assumed that a weight of each worker is small so that F_{nh} is taken as given by both the worker and the firm during the wage bargaining.

Stochastic bargaining weight of existing workers, γ, shows up in the equations for both intensive and extensive margins. This implies that bargaining shocks possibly increase the volatility of both margins. If $\gamma = 0$, we have similar conditions as in literature which uses standard bargaining protocol.

$$wh = \mu \left((1-\alpha) e^z k^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa \right) + (1-\mu) \left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b \right) \tag{24}$$

$$\frac{\widetilde{u}_l(1-h)}{u_c} = (1-\alpha) e^z k^\alpha (nh)^{-\alpha} \tag{25}$$

If $\gamma = 1$, the the conditions become similar to ones in KL[14].

$$wh = \mu \left(\frac{1-\alpha}{1-\mu\alpha} e^z k^\alpha h^{1-\alpha} n^{-\alpha} + \frac{V}{1-N}\kappa \right) + (1-\mu) \left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b \right) \tag{26}$$

$$\frac{\widetilde{u}_l(1-h)}{u_c} = \frac{1-\alpha}{1-\mu\alpha} e^z k^\alpha (nh)^{-\alpha} \tag{27}$$

2.5 Government

The government simply raises revenue in order to pay out unemployment benefits b to unemployed members within the household. Therefore, the government budget constraint is

$$T(S) = (1-n)\, b \tag{28}$$

2.6 Equilibrium

A *recursive equilibrium* is a set of functions; the household's value function $\Omega(S, s_H)$, the household's policy functions $c(S, s_H), a'(S, s_H)$, the firm's value function $J(S, s_F)$, the firm's policy functions $v(S, s_f), k(S, s_f)$, aggregate prices $r(S), \widetilde{\beta}(S, S')$, taxes $T(S)$, dividends $\Pi(S)$, the law of motion for aggregate state variables $G(S)$, asuch that

(1) Household's policy functions solve the household's problem

[14] Since KL does not have intensive margins, they do not have equation (26).

(2) Firm's policy functions solve the firm's problem

(3) $\widetilde{\beta}(S, S') = \beta u(c(S'))/u_c(c(S))$

(4) Wages and hours per worker $(w(S), h(S))$ are the solution to the bargaining problem

(5) Asset market and goods market clear

(6) The government budget constraint is balanced

(7) The law of motion $G(S)$ is consistent with individual decisions

3 Calibration

First, I define the matching function and the aggregate production function to be

$$F(z, k, nh) = e^z k (nh)^{1-\alpha} \tag{29}$$

$$M = \omega V^{\psi} (1-N)^{1-\psi} \tag{30}$$

where $\alpha \in (0,1), \psi \in (0,1)$. I specify the household's utility function as follows

$$u(c) = log(c) \tag{31}$$

$$\widetilde{u}(1-h) = \phi_e \frac{(1-h)^{1-\eta}}{1-\eta} \tag{32}$$

Including the parameters in the functions defined above, I have 19 parameters to be calibrated. Parameters can be categorized into three groups based on the way to calibrate them. The first set of parameters are predetermined parameters outside the model. The second set of parameters are parameters for shock processes, which will be estimated from constructed shock processes from US data. The last group of parameters is parameters to be determined in the model by using the steady state conditions and relevant targets.

3.1 Predetermined parameters (5)

I basically follow Andolfatto (1996) for the discount factor $\beta = 0.99$, the separation rate $\chi = 0.15$, the depreciation rate $\delta = 0.025$, the Cobb Douglas parameter for capital $\alpha = 0.36$, and the coefficient for vacancies in the matching function $\psi = 0.60$. Note that since the labor

Parameters	Description	Value	Source
β	Discount factor	0.99	Annual rate of return 4%
χ	Separation rate	0.15	Andolfatto (1996)
δ	Depreciation rate	0.025	Andolfatto (1996)
α	Cobb-Douglas parameter for capital	0.36	Andolfatto (1996)
ψ	Coefficient for vacancies in matching function	0.60	Free parameter, Andolfatto (1996)

Table 2: Predetermined parameters

market is not competitive in this paper, I cannot use labor share data to calibrate α. Table 2 summarizes predetermined parameters.

3.2 Parameters for shock processes (7)

Productivity shocks can be constructed as a series of the measure Solow residual. From the aggregate production function, we have

$$\hat{z}_t = \hat{y}_t - \alpha\hat{k}_t - (1-\alpha)\,\hat{n}_t - (1-\alpha)\,\hat{h}_t \tag{33}$$

where hats denote log-deviations from a linear trend for each variable over the period 1960:Q1-2012:Q1. I normalize $\bar{z} = 1$.

For bargaining shocks, we can use the solution to the first order differential equation we solved before

$$\frac{\partial w}{\partial n} = -\frac{\mu\alpha\,(1-\alpha)}{1-\mu\gamma\alpha} z k^{\alpha} h^{-\alpha} n^{-\alpha-1} \tag{34}$$

$$\frac{\partial w}{\partial n}\frac{n}{w} = -\frac{\mu\alpha\,(1-\alpha)}{1-\mu\gamma\alpha}\frac{y}{nhw} = -\frac{\mu\alpha\,(1-\alpha)}{1-\mu\gamma\alpha}\frac{1}{labor\,share} \tag{35}$$

$$labor\,share = \frac{\mu\alpha\,(1-\alpha)}{1-\mu\gamma\alpha}\frac{1}{(-\epsilon_{w,n})} \tag{36}$$

where $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n}\frac{n}{w}$.[15] I assume the elasticity $\epsilon_{w,n}$ does not move much around the steady-state value $\overline{\epsilon_{w,n}} \equiv \overline{\frac{\partial w}{\partial n}\frac{n}{w}} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\bar{\gamma}\alpha}\frac{1}{\overline{labor\,share}}$, and I will show this assumption is innocuous in the quantitative analysis section. From equation (36), I can construct series of γ_t given series of

[15]Note that $\epsilon_{w,n} < 0$.

Parameters	Value	Remarks
ρ_{zz}	0.9616	P-value = 0.000
$\rho_{\gamma z}$	-0.0053	P-value = 0.169
$\rho_{z\gamma}$	0.3937	P-value = 0.000
$\rho_{\gamma\gamma}$	0.9336	P-value = 0.000
σ_z	0.0070	-
σ_γ	0.0399	-
$\sigma_{z\gamma}$	-0.0001	$\rho(\varepsilon_z, \varepsilon_\gamma)$ =-0.3643

Table 3: Shock processes

the labor share data from US.

$$(labor\,share)_t = \frac{\mu\alpha(1-\alpha)}{1-\mu\gamma_t\alpha}\frac{1}{(-\overline{\epsilon_{w,n}})} \tag{37}$$

$$\gamma_t = \frac{1}{\mu\alpha} - \frac{(1-\alpha)}{(labor\,share)_t(-\overline{\epsilon_{w,n}})} \tag{38}$$

The series of labor share are constructed from US data. The detail can be found in the data appendix. Given signs of parameters and $\overline{\epsilon_{w,n}} < 0$, there exists the positive relationship between bargaining shocks γ_t and labor share. This implies that the higher labor share is related to the higher bargaining powers of existing workers.

$$\frac{\partial(labor\,share)}{\partial\gamma} = \frac{(\mu\alpha)^2(1-\alpha)}{(1-\mu\gamma\alpha)^2(-\overline{\epsilon_{w,n}})} > 0 \tag{39}$$

Based on several information criteria such as FPE, AIC, HQIC, and SBIC, I specify VAR(1) system for detrended shock series $\hat{z}, \hat{\gamma}$ to estimate shock processes.

$$\begin{pmatrix}\hat{z}'\\ \hat{\gamma}'\end{pmatrix} = \begin{pmatrix}\rho_{zz} & \rho_{\gamma z}\\ \rho_{z\gamma} & \rho_{\gamma\gamma}\end{pmatrix}\begin{pmatrix}\hat{z}\\ \hat{\gamma}\end{pmatrix} + \begin{pmatrix}\varepsilon_z'\\ \varepsilon_\gamma'\end{pmatrix} \tag{40}$$

$$\begin{pmatrix}\varepsilon_z\\ \varepsilon_\gamma\end{pmatrix} \sim N\left(0, \begin{pmatrix}\sigma_{zz}^2 & \sigma_{z\gamma}\\ \sigma_{z\gamma} & \sigma_{\gamma\gamma}^2\end{pmatrix}\right) \tag{41}$$

Table 3 summarizes parameters estimated using VAR(1) system above. All coefficient parameters except for $\rho_{\gamma z}$ are significant. Note that we have $\rho_{z\gamma} = 0.3937$, which means today's productivity shocks increase tomorrow's bargaining powers of existing workers. This is key

Target	Value	Source
Frisch elasticity of hours for those employed	0.50	Andolfatto (1996)
Steady-state employment to population ratio	0.60	Data (1960:Q1-2012:Q1)
Steady-state hours per worker	0.39	Data (1960:Q1-2012:Q1)
Steady-state job-filling rate	0.90	Andolfatto (1996)
Vacancy expenditure to output ratio	0.0218	Silva & Toledo (2009)
Replacement ratio	0.40	Shimer (2005)
$\mu = \bar{\gamma}$	-	Jointly determined in the model

Table 4: Targets

Parameters	Description	Baseline
η	Curvature parameter for leisure	3.0940
ϕ_e	Scale parameter for leisure	0.9136
κ	Cost of posting vacancies	0.1905
ω	Matching efficiency	0.5156
b	Unemployment Benefits	0.4080
μ	Bargaining weight of a marginal worker	0.5697
$\overline{\gamma}$	Bargaining weight of existing workers	0.5697

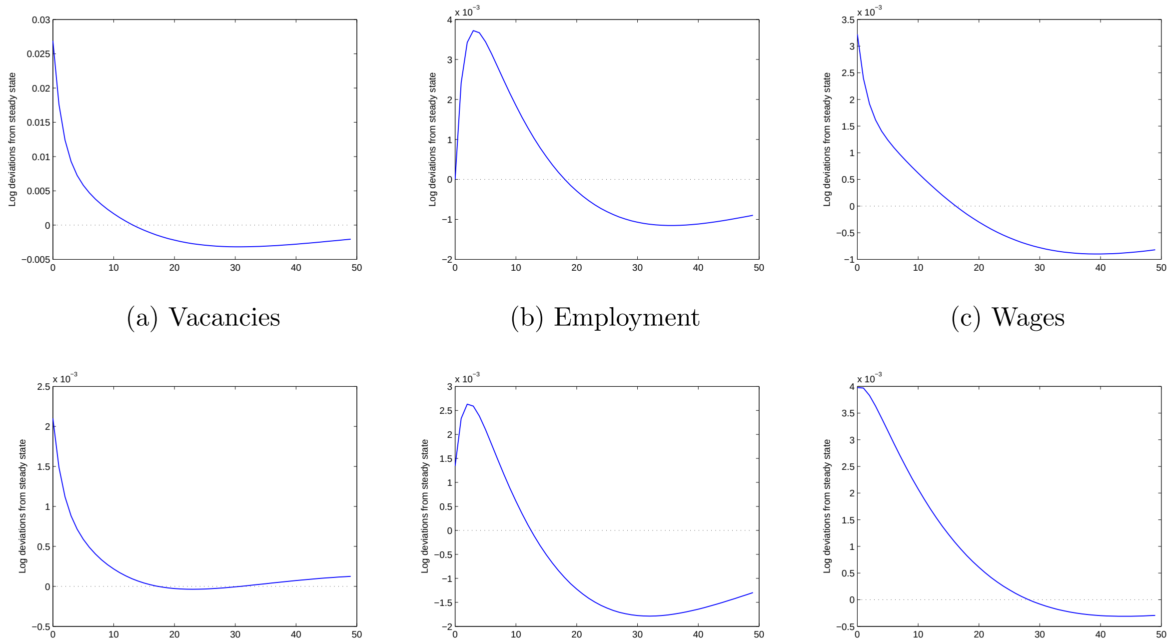
Table 5: Parameters determined using targets

mechanism that the inclusion of the stochastic bargaining makes total hours, employment and hours per workers more volatile in addition to productivity shocks.

3.3 Parameters determined using targets (7)

I choose the remaining 7 parameters using equilibrium conditions in the steady state and targets from the literature and data from US over 1960:Q1-2012:Q1. The targets I used are summarized in Tables 3. First, I set Frisch elasticity of hours for employed to 0.50, the steady state job-filling rate to 0.90 as in Andolfatto (1996), which is common across the literature. According to Silva and Toledo (2009), the average cost of time spent hiring one worker is approximately 3.6%-4.3% of total labor costs. I take the target the mid point of those range, 3.9%, which gives vacancy expenditure to output ratio $\frac{\kappa v}{y} = 0.0218$.[16] I use 40 percent as the value of unemployment benefits following Shimer (2005). In Shimer, this value implicitly includes the value of leisure, but in this paper I explicitly consider the leisure in the utility function, so

[16] This value is calculated based on job-filling rate $\Phi = 0.90$ and labor share $= 0.62$.



(a) Vacancies (b) Employment (c) Wages

(d) Hours per worker (e) Output (f) Labor share

Figure 1: IRFs to the positive one standard deviation bargaining shock

unemployment benefits, b, are purely unemployment benefits as in Nakajima (2012). Targets and parameters determined using these targets are listed in Table 4 and Table 5 respectively. I set the mean value of the bargaining weight of existing workers, $\overline{\gamma}$, to be 0.5697 which is the same as the bargaining weight of a marginal worker, μ, calibrated in the model.[17] Since the parameter $\overline{\gamma}$ is a free parameter and there is no clear way to pin down this parameter, I calibrate it such that $\overline{\gamma} = \mu$ in the steady state. As I will discuss in the robustness section later, lower values of $\overline{\gamma}$ generates more volatile labor market variables. However, I set $\overline{\gamma} = \mu = 0.5697$ which gives almost the least volatility among $\overline{\gamma} \in (0, 1)$ in the baseline model. In this regard, I think the choice of $\overline{\gamma} = 0.5697$ is innocuous and parsimonious. Also, note that calibrated value for the bargaining weight of a marginal worker, μ, is 0.5697, which guarantees quantitative results of the baseline model are not a direct result from a low value of μ as noted in Hagedorn and Manovskii (2008).

[17] The mean of the bargaining weight of existing workers, $\overline{\gamma}$, and the bargaining weight of a new worker, μ, are jointly determined in the steady state.

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho(x, Output)$			$\rho(x_t, x_{t-1})$		
Variable (x)	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto	Data	Baseline	Andolfatto
Output	1.54 (1.00)	1.36 (1.00)	1.31 (1.00)	-	-	-	0.86	0.86	0.82
Total Hours	1.38 (0.90)	0.99 (0.73)	0.70 (0.53)	0.85	0.81	0.92	0.88	0.93	0.91
Employment	1.00 (0.65)	0.91 (0.67)	0.68 (0.52)	0.81	0.77	0.78	0.91	0.92	0.89
Hours per Worker	0.49 (0.32)	0.25 (0.18)	0.19 (0.15)	0.74	0.40	0.55	0.56	0.56	0.58
Wages	0.91 (0.59)	0.68 (0.50)	0.62 (0.47)	0.34	0.91	0.94	0.69	0.69	0.65
Labor Productivity	0.82 (0.53)	0.80 (0.59)	0.72 (0.55)	0.45	0.70	0.92	0.57	0.66	0.62
Labor Share	0.74 (0.48)	0.64 (0.47)	0.12 (0.09)	-0.08	0.09	-0.72	0.78	0.73	0.51
Vacancies	13.23 (8.59)	4.36 (3.21)	3.65 (2.79)	0.90	0.76	0.80	0.91	0.59	0.54

Table 6: Business cycle moments in data and models over 1960:Q1-2012:Q1

4 Quantitative analysis

4.1 Impulse response functions of positive bargaining shocks

Figure 1 shows the impulse response of key labor market variables to the positive one standard deviation bargaining shock. When positive bargaining shocks hit the economy, the bargaining weight of existing workers instantly increases. Since bargaining powers of existing workers are higher than before, a firm has more incentives to hire marginal workers by offering higher wages to forgo the higher cost associated with the failure to hire marginal workers. Therefore, the firm instantly posts more vacancies, and employment increases one period later due to the nature of search frictions. Since the firm pays higher wages, hours per worker increase and higher total hours yield higher outputs in the equilibrium. Higher employment, hours per worker, and wages results in an increase in labor share by offsetting an increase in outputs.

4.2 Business cycle moments

Table 6 summarizes quantitative results of the baseline model. I compare the baseline model to the Andolfatto model to see what gains and what shortcomings the inclusion of stochastic bargaining and bargaining shocks gives. Again, all data are in log and HP filtered. First of all, the baseline model generates a high (relative) volatility of employment, 0.67, which almost close to the actual U.S. data, 0.65. This is a remarkable success and the main contribution in this paper. Since employment is very volatile, total hours is much volatile than the Andolfatto model. Hours per worker and vacancies are slightly more volatile than Andolfatto, but the differences are small. The moments for labor share are almost similar to the actual US data. This result might be a direct result of the identification strategy for bargaining shocks from labor share data. However, the moments for labor share in the model, along with the overshooting property I will discuss shortly, justify the assumption for the identification of bargaining shocks; $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n}\frac{n}{w}$ does not move much around the steady state.

The main mechanism generates more volatile labor market variables is that the impact of productivity shocks is amplified by changes in bargaining powers of existing workers in addition to the impact of each shock. Recall that the estimated parameter for ρ_{zv} is 0.3937, which means that as the productivity shocks today positively affect the bargaining powers tomorrow, and as the bargaining weight of existing workers increases, the firm will have more incentives to hire marginal workers by offering higher wages to forgo the higher cost associated with the failure to hire marginal workers. This dynamic interaction between productivity shocks and bargaining shocks amplifies the volatility of labor market variables, especially employment.

I now consider shortcomings of the baseline model relative to Andolfatto. The baseline model generates the higher volatility of labor productivity, weak pro-cyclicality of total hours, employment and hours per worker. Also, labor productivity is more persistent and vacancies are less persistent than the Andolfatto model and actual US data. Despite of these shortcomings, the baseline model performs better than Andolfatto model in general. This result mainly comes from time-varying bargaining weight of existing workers and firms' incentives to hire workers.

4.3 Implication on labor share

Rios-Rull and Santaeulalia-Llopis (2010) first document the overshooting property of labor share. They showed that labor share overshoots in response to productivity shocks, and the dynamic overshooting response of labor share drastically dampens the role of productivity shocks on labor markets due to huge wealth effects. Figure 3 shows the overshooting of labor

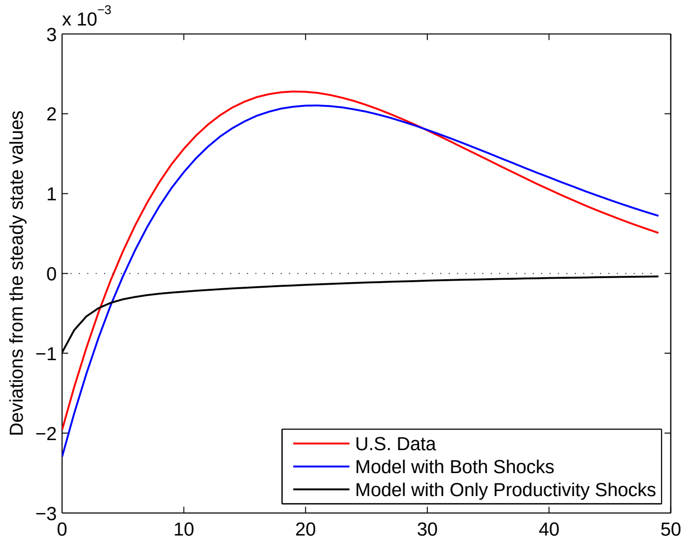


Figure 2: Impulse response function of labor share to productivity innovation

share in the baseline model. If I abstract from bargaining shocks, the model no longer generates the overshooting of labor share. The reason the model with bargaining shocks features the overshooting of labor share might be a direct result of the identification strategy of bargaining shocks. Again, the fact that labor share overshoots in the baseline model, along with other moments for labor share are almost the same as those in actual data, justifies the assumption I pose to identify bargaining shocks; $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n}\frac{n}{w}$ does not move much around the steady state.

More importantly, the baseline model generates the overshooting property of labor share, but the effect of productivity shocks is still significant on labor markets in contrast to the prediction of Rios-Rull and Santaeulalia-Llopis (2010) in which the effect of productivity shocks is dampened when labor share overshoots because of huge wealth effects from the overshooting property. In contrast to their model, the baseline model has a search and matching framework, and the nature of this framework weakens wealth effects resulting from the overshooting of labor share. On top of these differences, more incentives for firms to hire workers offset the huge reduction of total hours in booms. In response to positive productivity shocks output instantly increases, but employment does not increase because of search frictions, which cause an instant drop in labor share. As the productivity shocks today positively affect the bargaining shocks tomorrow, $\rho_{zv} = 0.3937$, and as the bargaining weight of existing workers increases, the

	$\sigma_x\%$ $\left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho\,(x, Output)$			$\rho\,(x_t, x_{t-1})$		
Variable (x)	Both shocks	Only z	Only γ	Both shocks	Only z	Only γ	Both shocks	Only z	Only γ
Output	1.36 (1.00)	1.31 (1.00)	0.43 (1.00)	-	-	-	0.86	0.82	0.91
Total Hours	0.99 (0.73)	0.70 (0.53)	0.68 (1.58)	0.81	0.92	0.99	0.93	0.91	0.91
Employment	0.91 (0.67)	0.69 (0.53)	0.60 (1.40)	0.77	0.78	0.96	0.92	0.88	0.88
Hours per Worker	0.25 (0.18)	0.19 (0.15)	0.21 (0.49)	0.40	0.54	0.45	0.56	0.58	0.57
Wages	0.68 (0.50)	0.63 (0.48)	0.43 (1.00)	0.91	0.94	0.66	0.69	0.65	0.60
Labor Productivity	0.80 (0.59)	0.72 (0.55)	0.25 (0.58)	0.70	0.92	-0.96	0.66	0.62	0.91
Labor Share	0.64 (0.47)	0.10 (0.08)	0.63 (1.47)	0.09	-0.71	0.83	0.73	0.50	0.76
Vacancies	4.36 (3.21)	3.67 (2.80)	3.23 (7.51)	0.76	0.80	0.49	0.59	0.54	0.54

Table 7: Business cycle moments in models with different shocks

firm will have more incentives to hire marginal workers. Consequently, employment, wages, and hours per worker will increase by offsetting an increase in outputs. This increase explains the overshooting of labor share in response to positive productivity shocks.

4.4 The Role of productivity shocks and bargaining shocks

Now I consider how productivity shocks and bargaining shocks differently affect the model predictions. When the economy has only productivity shocks, z, the model predictions are almost the same as the Andolfatto model. Comparing to the baseline model which has both shocks, the volatility of employment, labor share, and vacancies is dampened, but correlations between labor market variables and outputs get close to the actual data. Auto-correlations are almost the same as the baseline case.

When the economy has only bargaining shocks, the volatility of outputs significantly drops, which means bargaining shocks cannot be the main driving source of output fluctuations. On the other hand, the volatility of total hours, employment, hours per worker remarkably increases, which is far beyond the volatility in the baseline model. Also, total hours, employment, hours per worker, and labor shares are strongly pro-cyclical. However, auto-correlations are almost the same as the baseline case.

Table 8 shows the variance decomposition. Bargaining shocks have a substantial impact on the volatility of total hours, employment, hours per worker, vacancies, and labor share. While

Variable	productivity shocks (z)	bargaining shocks (γ)
Output	89.69	10.31
Total Hours	48.15	51.85
Employment	58.35	41.65
Hours per Worker	9.86	90.14
Wages	68.22	31.78
Labor Productivity	90.58	9.42
Labor Share	27.44	72.56
Vacancies	52.42	47.58

Table 8: Variance decomposition (in percent)

bargaining shocks play a remarkable role in the labor markets, productivity shocks seem to be still the main driving force of business cycles given productivity shocks account for about 90% of output fluctuations. This result is also consistent with the finding in moments in Table 7.

5 Robustness

5.1 Stochastic bargaining weight of a marginal worker, μ_t

Now I assume the bargaining weight of a marginal worker varies stochastically while the bargaining weight of existing workers is fixed at $\gamma = \overline{\gamma} = \overline{\mu}$. Again, I identify series of μ_t by using the solution to the first order differential equation, and series of the labor share data from U.S.

$$(labor\,share)_t = \frac{\mu_t \alpha (1-\alpha)}{\mu_t \bar{\gamma} \alpha} \frac{1}{(-\overline{\epsilon_{w,n}})} \tag{42}$$

$$\mu_t = \frac{1}{\bar{\gamma}\alpha + \frac{\alpha(1-\alpha)}{(labor\,share)_t(-\overline{\epsilon_{w,n}})}} \tag{43}$$

where $\epsilon_{w,n} \equiv \frac{\partial w}{\partial n}\frac{n}{w}$. Again, I assume the elasticity $\epsilon_{w,n}$ does not move much around the steady-state value $\overline{\epsilon_{w,n}} \equiv \overline{\frac{\partial w}{\partial n}\frac{n}{w}} = -\frac{\mu\alpha(1-\alpha)}{1-\mu\bar{\gamma}\alpha}\frac{1}{\overline{labor\,share}}$. Table 9 shows the comparison of business cycle moments. Stochastic bargaining power of a marginal worker cannot quantitatively improve the Andolfatto model, even moments for labor share which is used for identifying shock series μ_t.[18]

[18]This results do not change with different values of $\bar{\gamma}$

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$		$\rho\,(x, Output)$		$\rho\,(x_t, x_{t-1})$	
Variable (x)	Shock on μ	Andolfatto	Shock on μ	Andolfatto	Shock on μ	Andolfatto
Output	1.40 (1.00)	1.31 (1.00)	-	-	0.85	0.82
Total Hours	0.79 (0.57)	0.70 (0.53)	0.93	0.92	0.92	0.91
Employment	0.76 (0.54)	0.68 (0.52)	0.82	0.78	0.89	0.89
Hours per Worker	0.19 (0.14)	0.19 (0.15)	0.57	0.55	0.59	0.58
Wages	0.62 (0.44)	0.62 (0.47)	0.94	0.94	0.68	0.65
Labor Productivity	0.73 (0.52)	0.72 (0.55)	0.91	0.92	0.63	0.62
Labor Share	0.15 (0.11)	0.12 (0.09)	-0.56	-0.72	0.56	0.51
Vacancies	4.00 (2.86)	3.65 (2.79)	0.76	0.80	0.55	0.54

Table 9: Business cycle moments in model: shocks on μ

5.2 Different calibrations for $\overline{\gamma}$

I now simulate the baseline model with different values for $\overline{\gamma} = 0.3$ (an example of low values[19]), 0.5697 (a middle value and the calibrated value for the baseline model such that $\mu = \overline{\gamma}$), and 0.9 (an example of high values[20]). Table 10 shows business cycle moments for each case. If I set $\overline{\gamma} = 0.3$, then volatility of employment and hours per workers significantly increases than the baseline calibration case, $\overline{\gamma} = 0.5697$. However, if I set $\overline{\gamma} = 0.9$, then moments are almost the same as those of the baseline calibration case, $\overline{\gamma} = 0.5697$. Mechanically, low values of $\overline{\gamma}$ increase the volatility of total hours, employment, hours per worker. $\overline{\gamma}$ is a free parameter in this paper and there is no clear way to pin down this parameter. However, the choice of $\overline{\gamma} = 0.5697$ in the baseline model seems innocuous and parsimonious in the sense that setting the same values for the mean of bargaining weights of existing workers and bargaining weights of new workers, $\overline{\gamma} = \mu$, is a reasonable given there is no information on $\overline{\gamma}$, and $\overline{\gamma} = 0.5697$ yields the least volatility of labor market variables among $\overline{\gamma} \in (0, 1)$.

6 Conclusion

This paper studies an alternative mechanism of wage negotiations in multi-worker firms that face diminishing MPL. When Nash bargaining with a marginal worker breaks down, a firm

[19] In this case, the calibrated value of μ is 0.5512

[20] In this case, the calibrated value of μ is 0.5956

	$\sigma_x\%\left(\frac{\sigma_x}{\sigma_{Output}}\right)$			$\rho\,(x, Output)$			$\rho\,(x_t, x_{t-1})$		
Variable (x)	$\overline{\gamma} = 0.3$	$\overline{\gamma} = 0.5697$	$\overline{\gamma} = 0.9$	$\overline{\gamma} = 0.3$	$\overline{\gamma} = 0.5697$	$\overline{\gamma} = 0.9$	$\overline{\gamma} = 0.3$	$\overline{\gamma} = 5697$	$\overline{\gamma} = 0.9$
Output	1.39 (1.00)	1.36 (1.00)	1.36 (1.00)	-	-	-	0.86	0.86	0.86
Total Hours	1.13 (0.81)	0.99 (0.73)	0.98 (072)	0.80	0.81	0.81	0.93	0.93	0.93
Employment	1.02 (0.73)	0.91 (0.67)	0.90 (0.66)	0.77	0.77	0.77	0.91	0.92	0.92
Hours per Worker	0.32 (0.23)	0.25 (0.18)	0.24 (0.18)	0.39	0.40	0.41	0.55	0.56	0.56
Wages	0.75 (0.54)	0.68 (0.50)	0.67 (0.49)	0.87	0.91	0.91	0.67	0.69	0.70
Labor Productivity	0.83 (0.60)	0.80 (0.59)	0.80 (0.59)	0.58	0.70	0.70	0.67	0.66	0.66
Labor Share	0.83 (0.60)	0.64 (0.47)	0.63 (0.46)	0.21	0.09	0.08	0.73	0.73	0.73
Vacancies	5.03 (3.62)	4.36 (3.21)	4.30 (3.16)	0.70	0.76	0.77	0.56	0.59	0.59

Table 10: Business cycle moments in the model with different values for $\overline{\gamma}$

negotiates wages with existing workers collectively and produces with them. The bargaining powers of existing workers are stochastic. Due to diminishing MPL, the breakdown of the negotiation with the marginal worker negatively affects the bargaining position of the firm with existing workers (one fewer workers) since MPL is higher with one fewer workers. How much the firm internalizes this negative effect depends on the stochastic bargaining powers of existing workers which can be identified through labor share data. During expansions, it is relatively difficult for the firm to hire workers, so existing workers might have higher bargaining powers. If the firm fails to hire a marginal worker due to a breakdown of negotiations, the firm has to pay higher wages to existing workers in order to produce goods with them. Since the failure to hire marginal workers is more costly during expansions, the firm has more incentives to hire marginal workers by offering higher wages to forgo the higher cost associated with the breakdown. During recessions, the opposite happens. Through this mechanism, the stochastic bargaining powers of existing workers provide an additional margin to increase the volatility of labor market variables. The calibrated model generates more volatile total hours, employment, hours per worker while labor share overshoots in response to productivity shocks as documented in Ríos-Rull and Santaeulalia-Llopis (2010). In particular, the volatility of employment in the model is similar to the actual US data. In contrast to the prediction of Ríos-Rull and Santaeulalia-Llopis (2010), in which the effect of productivity shocks is dampened when labor share overshoots due to huge wealth effects from the overshooting property of labor share, this paper presents a model in which the labor share overshoots in response to productivity shocks and the volatility of employment closely matches that of US data.

In this paper, I assume the bargaining weight of existing workers to be exogenous. The quantitative results show that the time-varying bargaining weight of existing workers is an important margin to understand the fluctuations of total hours, employment, hours per workers, and labor share, and the overshooting property. However, this paper abstracts from the endogenous mechanism for the time-varying bargaining weight of existing workers. Therefore, coming up with an endogenous mechanism for bargaining shocks would be worthwhile for future research. One possible theory could be related to the entry and exit of firms over business cycles. In booms, several firms compete with a specific firm because of higher entry rates of new firms and lower exit rates of existing firms, which reduce the monopolistic or bargaining power of firms over existing workers. However, during recessions, the opposite happens. By incorporating the entry and exit decision of firms, I might be able to explain the endogenous movements of the bargaining weight of existing workers. Moreover, this paper does not focus on unemployment because the baseline model treats the unemployed and the non-employed who are out of the labor force similarly, and the measure of unemployment is inconsistent with the data. In this regard, I could extend the baseline model by distinguishing between the unemployed and the non-employed to obtain a proper measure of unemployment.

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Appendix

Derivation of the equilibrium conditions

Household solves the following dynamic programming problem.

$$\Omega(S, s_H) = \max_{c,a'} u(c) + n\widetilde{u}(1 - h(S, s_H)) + (1 - n)\widetilde{u}(1) + \beta E[\Omega(S', s'_H)]$$

$$s.t.$$

$$c + a' + T(S) = w(S, s_H) h(S, s_H) n + (1 - n) b + (1 + r(S)) a + \Pi(S)$$

$$n' = (1 - \chi) n + p(S)(1 - n)$$

$$S' = G(S)$$

Let λ_H and μ_H be the Lagrange multiplier on budget constraint, and law of motion for employment respectively. Then we have the following first order conditions.

$$\begin{aligned} u_c &= \lambda_H \\ E[\beta\Omega'_a] &= \lambda_H \end{aligned}$$

From the envelope condition with respect to a, we get

$$\Omega_a = (1 + r)\lambda_H$$

Taking a derivative with respect to n', we have

$$\mu_H = E[\beta\Omega'_n]$$

By combining equations above, we get the standard Euler equation.

$$E\left[\beta\frac{u'_c}{u_c}\left(1 + r'\right)\right] = 1$$

Now, firms solve the following problem.

$$\begin{aligned} J(S, s_F) &= \max_{v,k} \Pi(S) + E\left[\widetilde{\beta}(S, S') J(S', s_F')\right] \\ &= \max_{v,k} F(z, k, nh) - w(S, s_F) h(S, s_F) n - (r(S) + \delta) k - \kappa v + E\left[\widetilde{\beta}(S, S') J(S', s_F')\right] \\ s.t. & \\ n' &= (1 - \chi) n + q(S) v \\ S' &= G(S) \end{aligned}$$

where $\widetilde{\beta}(S, S') = \beta u_c(c(S')) / u_c(c(S))$ is the stochastic discount factor and $q(S) = M/V$ is the job-filling rate.

Let μ_F be the Lagrange multipliers on law of motion of employment. Then, we have the following first order conditions for firms.

$$\begin{aligned} \kappa &= \mu_F q(S) \\ r + \delta &= F_k \end{aligned}$$

From the definition of the marginal value of an additional employee to the firm, $J^m \equiv \frac{\partial J}{\partial n}$, the following condition should hold.

$$E\left[\widetilde{\beta} J^{m'}\right] = \mu_F$$

By combining equations above, we have an equation for the rental rate and a job creation condition.

$$\begin{aligned} r &= F_k - \delta \\ \kappa &= qE\left[\widetilde{\beta} J^{m'}\right] \end{aligned}$$

Derivation of the marginal value of an additional employee to the firm

$$\begin{aligned}
J^m &= \lim_{\Delta\to 0}\frac{1}{\Delta}\left[\left(F(z,k,(n+\Delta)h) - w[n+\Delta](n+\Delta)h - (r+\delta)k - \kappa v + \beta E\left[\frac{u_c^{'}}{u_c}J\left[(n+\Delta)^{'}\right]\right]\right)\right.\\
&\left.-\left(F(z,k,nh) - w^e[n]nh - (r+\delta)k - \kappa v + \beta E\left[\frac{u_c^{'}}{u_c}J^B\left[n^{'}\right]\right]\right)\right]\\
&= \lim_{\Delta\to 0}\frac{F(z,k,(n+\Delta)h) - F(z,k,nh)}{\Delta} - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)h - w^e[n]nh}{\Delta}\\
&+E\left[\beta\frac{u_c^{'}}{u_c}\lim_{\Delta\to 0}\frac{J[(1-\chi)(n+\Delta)] - J^B[(1-\chi)n]}{\Delta}\right]\\
&= \frac{\partial F(z,k,nh)}{\partial n} - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)h - w^e[n]nh}{\Delta}\\
&+(1-\chi)E\left[\beta\frac{u_c^{'}}{u_c}\lim_{(1-\chi)\Delta\to 0}\frac{J[(1-\chi)n+(1-\chi)\Delta] - J^B[(1-\chi)n]}{(1-\chi)\Delta}\right]\\
&= F_n - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)h - w^e[n]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right]
\end{aligned}$$

Proofs of Propositions

Proof of Proposition 1

Under $w^e[n] = w[n+\triangle]$, the equation (10) can be rewritten as

$$\begin{aligned}
J^m &= F_n - \lim_{\Delta\to 0}\frac{w[n+\Delta](n+\Delta)h - w[n+\Delta]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right]\\
&= F_n - \lim_{\Delta\to 0}\frac{w[n+\Delta]nh + w[n+\Delta]\Delta h - w[n+\triangle]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right]\\
&= F_n - \lim_{\Delta\to 0} w[n+\Delta]h + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right]\\
&= F_n - w[n]h + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right]
\end{aligned}$$

Proof of Proposition 2

Under $w^e[n] = w[n]$, the equation (10) can be rewritten as

$$\begin{aligned}
J^m &= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta](n+\Delta)h - w[n]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta]nh + w[n+\Delta]\Delta h - w[n]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta] - w[n]}{\Delta}nh - \lim_{\Delta\to 0} w[n+\Delta]h + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right] \\
&= F_n - \frac{\partial w[n]}{\partial n}nh - w[n]h + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right] \\
&= F_n - w[n]h - \frac{\partial w[n]}{\partial n}nh + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right]
\end{aligned}$$

Proof of Proposition 3

Under $w^e[n] = \gamma w[n] + (1-\gamma)w[n+\Delta]$, the equation (10) can be rewritten as

$$\begin{aligned}
J^m &= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta](n+\Delta)h - (\gamma w[n] + (1-\gamma)w[n+\Delta])nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta]nh + w[n+\Delta]\Delta h - \gamma w[n]nh - (1-\gamma)w[n+\Delta]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0} \frac{w[n+\Delta]\Delta h - \gamma w[n]nh + \gamma w[n+\Delta]nh}{\Delta} + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right] \\
&= F_n - \lim_{\Delta\to 0} w[n+\Delta]h - \lim_{\Delta\to 0} \gamma\frac{w[n+\Delta] - w[n]}{\Delta}nh + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right] \\
&= F_n - w[n]h - \gamma\frac{\partial w[n]}{\partial n}nh + (1-\chi)\beta E\left[\frac{u_c^{'}}{u_c}J^{m'}\right]
\end{aligned}$$

Solutions to the bargaining problem with a marginal worker

Now, we turn to the bargaining problem which is the same as standard Nash bargaining given the marginal value of employment for the worker and the marginal value of an additional employee to the firm.

$$\Omega^m = wh - b - \frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right]$$
$$J^m = F_n - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]$$

Given the bargaining power of the marginal worker, $\mu \in [0,1]$, and the bargaining powers of existing workers , $\gamma \in [0,1]$, wages and hours per worker are determined via the following standard bargaining problem.

$$\begin{aligned}(w,h) &= arg\max_{w,h}(\Omega^m)^{\mu}(J^m)^{1-\mu} \\ &= arg\max_{w,h}\left(wh - b - \frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right]\right)^{\mu} \\ &\times\left(F_n - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right)^{1-\mu}\end{aligned}$$

The first order condition with respect to w gives the following sharing rule.

$$\mu J^m = (1-\mu)\Omega^m$$

By plugging the definitions of Ω^m and J^m, we have

$$\mu\left(F_n - wh - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) = (1-\mu)\left(wh - b - \frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right]\right)$$

It can be rewritten as

$$\begin{aligned}wh &= \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b - (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}\Omega^{m'}\right]\right) \\ &= \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + (1-\chi)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right] - (1-\chi-p)\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \\ &= \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + p\beta E\left[\frac{u'_c}{u_c}J^{m'}\right]\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \\ &= \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + p\frac{\kappa}{q}\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right) \\ &= \mu\left(F_n - \gamma\frac{\partial w}{\partial n}nh + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{\widetilde{u}(1) - \widetilde{u}(1-h)}{u_c} + b\right)\end{aligned}$$

The sharing rule, $\mu J^m = (1-\mu)\,\Omega^m$ is used in the second line and the optimal contdition for vacancies, $\kappa = qE\left[\widetilde{\beta} J^{m'}\right] = q\beta E\left[\frac{u_c'}{u_c} J^{m'}\right]$is used in the third line. The definitions of p and q are used in the last line.

The first order condition with respect to h gives the following intra-temporal condition for hours per worker.

$$\mu J^m\left(w - \frac{\widetilde{u}_l(1-h)}{u_c}\right) = (1-\mu)\Omega^m\left(-\frac{\partial F_n}{\partial h} + w + \gamma\frac{\partial w}{\partial n}n\right)$$

Since $\mu J^m = (1-\mu)\,\Omega^m$ holds from the first order condition with respect to w,

$$\begin{aligned}
w - \frac{\widetilde{u}_l(1-h)}{u_c} &= -\frac{\partial F_n}{\partial h} + w + \gamma\frac{\partial w}{\partial n}n \\
\frac{\widetilde{u}_l(1-h)}{u_c} &= \frac{\partial F_n}{\partial h} - \gamma\frac{\partial w}{\partial n}n \\
&= \frac{\partial}{\partial h}\left(\frac{\partial F(z,k,nh)}{\partial n}\right) - \gamma\frac{\partial w}{\partial n}n \\
&= \frac{\partial}{\partial h}\left(F_{nh}h\right) - \gamma\frac{\partial w}{\partial n}n \\
&= F_{nh} - \gamma\frac{\partial w}{\partial n}n
\end{aligned}$$

Following Andolfatto (1996) and Cheron and Langot (2004), it is assumed that a weight of each worker is small so that F_{nh} is taken as given by both the worker and the firm during the wage bargaining.

Solutions to the first order differential equation w.r.t. wages

Given the Cobb-Douglas production function, the sharing rule, and the intra-temporal condition, and the wage bill can be written as

$$\begin{aligned}
\mu J^m &= (1-\mu)\,\Omega^m \\
\frac{\widetilde{u}_l(1-h)}{u_c} &= (1-\alpha)\,zk^\alpha\,(nh)^{-\alpha} - \gamma\frac{\partial w}{\partial n}n \\
wh &= \mu\left((1-\alpha)\,zk^\alpha h^{1-\alpha}n^{-\alpha} - \gamma\frac{\partial w}{\partial n}nh + \frac{V}{1-N}\kappa\right) + (1-\mu)\left(\frac{\widetilde{u}\,(1) - \widetilde{u}\,(1-h)}{u_c} + b\right)
\end{aligned}$$

We can rewrite the wage bill as the first order differential equation as follows

$$\begin{aligned} \mu\gamma nh\frac{\partial w}{\partial n}+hw &= \mu\left(\left(1-\alpha\right)zk^{\alpha}h^{1-\alpha}n^{-\alpha}+\frac{V}{1-N}\kappa\right)+\left(1-\mu\right)\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right) \\ \frac{\partial w}{\partial n}+\frac{1}{\mu\gamma n}w &= \frac{1}{\mu\gamma nh}\left(\mu\left(\left(1-\alpha\right)zk^{\alpha}h^{1-\alpha}n^{-\alpha}+\frac{V}{1-N}\kappa\right)+\left(1-\mu\right)\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right) \quad (44) \end{aligned}$$

So, the integrating factor is

$$e^{\int\left(\frac{1}{\mu\gamma n}\right)dn}=e^{\frac{1}{\mu\gamma}ln(n)}=n^{\frac{1}{\mu\gamma}}$$

By multiplying both sides of the equation (44) by $n^{\frac{1}{\mu\gamma}}$ and integrating both sides with respect to n, we have

$$\begin{aligned} w &= n^{-\frac{1}{\mu\gamma}}\int n^{\frac{1}{\mu\gamma}}\frac{1}{\mu\gamma nh}\left[\mu\left(\left(1-\alpha\right)zk^{\alpha}h^{1-\alpha}n^{-\alpha}+\frac{V}{1-N}\kappa\right)+\left(1-\mu\right)\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right]dn+Dn^{-\frac{1}{\mu\gamma}} \\ &= n^{-\frac{1}{\mu\gamma}}\int n^{\frac{1}{\mu\gamma}}\frac{1}{\mu\gamma nh}\left[\mu\left(\left(1-\alpha\right)zk^{\alpha}h^{-\alpha}n^{-\alpha}\right)+\mu\left(\frac{V}{1-N}\kappa+\frac{1-\mu}{\mu}\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right)\right]dn+Dn^{-\frac{1}{\mu\gamma}} \\ &= n^{-\frac{1}{\mu\gamma}}\left[\int\frac{(1-\alpha)}{\gamma}zk^{\alpha}h^{-\alpha-1}n^{-\alpha-1+\frac{1}{\mu\gamma}}dn+\int n^{\frac{1}{\mu\gamma}-1}\frac{1}{\gamma h}\left(\frac{V}{1-N}\kappa+\frac{1-\mu}{\mu}\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right)dn\right]+Dn^{-\frac{1}{\mu\gamma}} \\ &= n^{-\frac{1}{\mu\gamma}}\left[\frac{\mu\left(1-\alpha\right)}{1-\mu\gamma\alpha}zk^{\alpha}h^{-\alpha}n^{-\alpha+\frac{1}{\mu\gamma}}+n^{\frac{1}{\mu\gamma}}\frac{\mu}{h}\left(\frac{V}{1-N}\kappa+\frac{1-\mu}{\mu}\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right)\right]+Dn^{-\frac{1}{\mu\gamma}} \\ &= \frac{\mu\left(1-\alpha\right)}{1-\mu\gamma\alpha}zk^{\alpha}h^{-\alpha}n^{-\alpha}+\frac{\mu}{h}\left(\frac{V}{1-N}\kappa+\frac{1-\mu}{\mu}\left(\frac{\widetilde{u}(1)-\widetilde{u}(1-h)}{u_c}+b\right)\right)+Dn^{-\frac{1}{\mu\gamma}} \end{aligned}$$

where D is a constant of the integration of the homogeneous equation. By assuming the total wage bill wnh has to remain finite as employment becomes small as in Hawkins (2011) or alternatively by assuming $\lim_{n\to 0} wnh = 0$ as in Cahuc, Maroque, and Wasmer (2008), we have $D = 0$. From the equation above, we also have

$$\frac{\partial w}{\partial n}=-\frac{\mu\alpha\left(1-\alpha\right)}{1-\mu\gamma\alpha}zk^{\alpha}h^{-\alpha}n^{-\alpha-1}<0$$

Equilibrium conditions

The equilibrium of the model is characterized by the following conditions under functional forms specified in the calibration section.

$$
\begin{aligned}
E\left[\beta\frac{C}{C'}\left(1+r'\right)\right] &= 1 \\
r &= \alpha\frac{Y}{K}-\delta \\
N^{'} &= (1-\chi)N+\omega V^{\psi}\left(1-N\right)^{1-\psi} \\
q &= \omega V^{\psi-1}\left(1-N\right)^{1-\psi} \\
Y &= C+I+\kappa V \\
I &= K^{'}-\left(1-\delta\right)K \\
Y &= e^{z}K^{\alpha}\left(Nh\right)^{1-\alpha} \\
\frac{\kappa}{q} &= E\left[\beta\frac{C}{C'}\left[\frac{1-\alpha}{1-\mu\gamma\alpha}\frac{Y^{'}}{N'}-w^{'}h^{'}+\left(1-\chi\right)\frac{\kappa}{q'}\right]\right] \\
\phi_{e}\left(1-h\right)^{-\eta}C &= \frac{\left(1-\alpha\right)}{1-\mu\gamma\alpha}\frac{Y}{Nh} \\
wh &= \mu\left(\frac{1-\alpha}{1-\mu\gamma\alpha}\frac{Y}{N}+\frac{V}{1-N}\kappa\right)+\left(1-\mu\right)\left(\left(\phi_{e}\frac{1}{1-\eta}-\phi_{e}\frac{\left(1-h\right)^{1-\eta}}{1-\eta}\right)C+b\right)
\end{aligned}
$$

Data Appendix

Raw data

1. Employment, Average Weekly Hours Worked, Population: Bureau of Labor Statistics (BLS).

2. Real GDP, GDP, Compensation of Employees, Proprietors Income, GDP deflator: National Income and Product Accounts (NIPA) published by the Bureau of Economics Analysis. (BEA)

3. Vacancies: Conference Board's Help Wanted Index and the Composite Help Wanted Index by Barnichon (2010)

4. Consumption of Fixed Capital, Capital Expenditure in non-financial non-corporate business: Flow of Funds

Constructed data

1. Employment Rate $= \frac{\text{Employment}}{\text{Population}}$

2. Hours per Worker $= \frac{\text{Average Weekly Hours Worked}}{20\times5}$

3. Total Hours = Employment × Hours per Worker

4. Labor Share $= \frac{\text{Compensation of Employees}}{\text{GDP-Proprietors Income}}$

5. Real Wage $= \frac{\text{Labor Share}\times\text{Real GDP}}{\text{Tolal Hours}}$

6. Labor Productivity $= \frac{\text{Real GDP}}{\text{Tolal Hours}}$

7. Vacancies = Conference Board's Help Wanted Index and the Composite Help Wanted Index by Barnichon (2010)

8. Investment = Capital Expenditure deflated by GDP deflator

9. Depreciation = Consumption of Fixed Capital deflated by GDP deflator

10. Capital Stock is constructed by the perpetual inventory method using follow law of motion

$$k_{t+1} = k_t + \text{Investment} - \text{Depreciation}$$

Initial capital stock is chosen so that the capital-output ratio does not display any trend over the period 1960Q1-2012Q1.