The Rise of Software and Skill Demand Reversal*

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Abstract

This paper analyzes a bi-directional relation between technology and occupational structure (job allocation). Jobs have polarized in the U.S. since at least the 1980s but the growth of high-skill jobs has been stagnated since 2000s (skill demand reversal). I document that software innovation has risen compared to equipment, and relate this change in the direction of innovation to the skill demand reversal based on a novel empirical observation: The intensity of software / equipment use by occupation is tied to cognitive / routine intensity by occupation. I then propose a general equilibrium model that endogenously explains both employment share and software innovation trends through a bi-directional interaction between them. In the model, equilment (machine) innovation replaces the routine (middle) jobs. Then the demand for machine decreases and software innovation becomes more profitable, leading to a rise of software innovation. This, in turn, reduces the employment in high-skill jobs by making the cognitive task more productive. Quantitative analysis shows that the model explains 70∼80% of the rise in software and skill demand reversal observed in the data.

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1 Introduction

The occupational structure – distribution of labor across occupations – has been changed dramatically in the United States: The employment share of middle-skill jobs has declined since 1980s (job polarization), and the growth of high-skill jobs has been stagnated since 2000s (skill demand reversal, Beaudry et al., 2016). Many suggest a recent technological change as a reason for the changes in the job polarization: a technological change replaces the routine tasks and those jobs in the middle are intensive in the routine tasks. This paper suggests that the relation between technological change and occupational structure is bi-directional: Technological change shapes an occupational structure but changes in the occupational structure also affects a direction of the following technological change. Notably, the bi-directional relation can provide an explanation for both job polarization and skill demand reversal.

We first document that skill demand reversal was accompanied higher growth in software innovation relative to other types of innovation (figure 2.4), and argue that this change in the direction of innovation was closely related to changes in the occupational structure of the U.S. economy. This is done by combining two datasets—the National Income and Product Account (NIPA) and O*NET Tools and Technology Database. The newly merged dataset shows that the average amount of investments in software and/or equipment by occupation are strongly linked to the tasks of each occupation. Namely, software is used intensively by cognitive (high-skill) occupations, while equipment is used intensively by routine (middle-skill) occupations (figure 2.6).

We then provide a unified framework that can explain both skill demand reversal *and* the rise of software relative to equipment endogenously. The model has three novel features. First, the model features workers of heterogeneous skill sorting into heterogeneous tasks, and also different types of capital (software and equipment). Second, technological changes in the model are embodied into different types of capital, and innovators endogenously choose which type of technology (or capital) to improve. These two features enable us to simultaneously analyze the static and dynamic implications of interactions between technology and the labor market. Last but not least, technological changes (embodied in different types of capital) alter job allocations because all workers use both types of capital, but with different intensities depending on their occupations. This departs from the typical assumption that only particular kinds of occupations are affected by a specific type of technical change, and also implies that impact of one type of technological change on the labor market can change the subsequent direction of technological change.

In the model, the intensities at which each occupation use software and equipment can be measured directly from the newly merged dataset mentioned above. Equipment and software are modeled as a composite of infinitesimal varieties provided by innovators who are free to choose a type of capital to innovate, so the amount of innovation toward each type of capital can also be directly mapped into capital investment data in the National Accounts, facilitating quantification of the model.

After characterizing the equilibrium, we prove a series of comparative statics in response to one exogenous change: an increase in the productivity of the equipment-producing sector.[1] Increased productivity in equipment production lowers the price of equipment, which leads to job polarization if different occupations are complementary in production. Since middle-skill occupations use equipment most intensively, labor flows out from these jobs and into high- and low-skill jobs. But the decline in middle-skill employment also means that the demand for equipment declines, inducing innovators to shift their focus away from equipment and more toward enhancing software. In turn, the rise of software leads to skill demand reversal if jobs are complementary: Middle-skills jobs were already declining (job-polarization), the employment share of high-skill jobs decelerates since they use software most intensively, and consequently skill demand becomes concentrated in low-skill jobs.

We verify the empirical validity of the model's mechanism using the fact that the decline in the relative price of equipment to software varies across industries. The model predicts: i) A negative relationship between the speed of decline in the relative price of equipment to software and the growth of middle-skill employment relative to high-skill employment; and ii) A positive correlation between the speed of decline in the relative price of equipment to software and the relative growth of software innovation to R&D other than software. We confirm significant correlations in both cases.

Confident of the mechanism, we use the model directly to quantify its importance. Our quantitative analysis shows that the channel of directed technical change can account for more than two-thirds of the rise of software and skill demand reversal. The former is measured by the relative size of software investment to equipment investment, and the latter is measured by a gap between the actual series and the level implied by the linear trend of the 1980s.

The results have two important implications. First, software and equipment capital measured in the National Accounts is a good proxy for the technological changes

[1] We also document a faster increase in the productivity of the equipment-producing sector in the data.

shaping the structure of the labor market. Since technological changes have significant impacts on many economic variables, careful investigation of the composition of capital investment can be fruitful in understanding economic phenomena other than job polarization as well.

Second, a technological change that directly affects a particular group of occupations could lead to other types of technological change that eventually affect other occupations. Hence, innovation policy targeting a specific group of products may have to consider this dynamic general equilibrium effect. This paper also shows that recent technical changes reduce cognitive intensive occupations as long as those occupations use software intensively. Moreover, while not explicitly analyzed here, changes in the demand for high-skill occupations will also change the expected returns to skill acquisition, consequently altering individuals' education decisions and labor supply.

Related Literature The relationship between polarization and increases in the productivity of middle-skilled occupations, which are intensive with respect to routine tasks, is well documented in the literature (e.g., Autor et al., 2006; Autor and Dorn, 2013; Goos et al., 2014, among others). Though fewer, there are also studies that have discussed the flattening of the demand for high-skilled workers around 2000, such as Beaudry et al. (2016) and Valletta (2016). This paper contributes to this literature by analyzing both polarization and skill demand reversal in a unified framework, and extends it by linking labor market phenomena to changes in the composition of capital investment.

Several papers analyze the consequences of task-specific technological change on the labor market with an assignment model (Costinot and Vogel, 2010; Lee and Shin, 2017; Michelacci and Pijoan-Mas, 2016; Stokey, 2016; Cheng, 2017, among others). We include a similar assignment feature, but characterize tasks by their different uses of two types of capital, and also introduce endogenous task-specific technological change generated from innovations on each type of capital. By doing so, we obtain a direct mapping of two distinctive task-specific technological changes to observed data.[2] Also, we explain why a particular type of technology may or may not change.

[2] Cheng (2017) also obtains the routine-biased technological change from the data by measuring different capital intensities across occupations. Different from ours, Cheng (2017) measures the capital intensities across occupations from industry level capital share and the variations in the composition of occupations across industries, and confirms that the middle-skill occupations are capital intensive. Summing the equipment and software, our dataset also shows that the total capital is intensively used in the middle jobs. We show, however, that the distinction between equipment and software is important as the software is not used intensively by the middle jobs.

Recent studies by Bárány and Siegel (forthcoming) and Lee and Shin (2017) show that either task-specific technological change or sector-specific technological change can lead to both job polarization and structural change. Since a single type of technological change can result in both phenomena, it is not easy to conclude whether the source of technological change has been task- or sector-specific. Our paper implies that the technological change embodied in a particular type of capital could be a source of task-specific technological change that can generate both phenomena.

Acemoglu and Restrepo (2016) and Hémous and Olsen (2016) also analyze the interaction between technological change and the labor market with the directed technical change framework of Acemoglu (2002). While they provide new insights on how automated technology evolves and affects labor market outcomes, the interpretation of the technology with respect to the observable data is not straightforward. Our technological changes are directly measured from investment in software and equipment in the National Accounts, so the changes are easy to interpret. Our tasks also have a clear interpretation as they are mapped directly to different occupations in the data.

A seminal paper by Krusell et al. (2000) links changes in the price of equipment capital to skill-biased technical change to analyze the effects of technological change on labor market outcomes. They emphasize that skill-capital complementarity (capital substitutes low-skill labor *more* than high-skill labor) is key to understanding how a rise in the productivity of capital leads to higher demand for high-skill workers. In contrast to Krusell et al. (2000), in our model, the substitutability between labor and capital is same across occupations. Instead, we assume that occupations vary in how intensively they use different types of capital, and that the occupations are complementary to one another.

The work by Krusell et al. (2000) and our paper do not contradict each other, as the worker classifications are essentially different.[3] They classify workers by education, and we classify workers by occupation. Low-educated workers may well be able to do what high-educated workers usually do (though less efficiently), whereas workers in certain occupations may not be able to do what workers of other occupations usually do. Indeed, recent papers such as Goos et al. (2014) and Lee and Shin (2017) highlight complementarity between tasks as a key to understanding task-level employment changes (i.e., polarization). In this regard, our paper complements Krusell et al. (2000) by linking capital to task-based employment.

Another important feature of this paper is distinguishing software capital from

[3]They also classify workers into two types, while we consider more.

equipment capital. Software investment is becoming increasingly important, as evidenced by its rapid rise as a share of total investment. Aum et al. (2017) analyzes the role of computer capital (hardware and software) in shaping the dynamics of aggregate productivity. Koh et al. (2016) emphasizes the importance of software capital (more broadly, intellectual property products capital) in accounting for the declining labor share in the US. Whereas their analysis focuses on the relation between total labor and capital, we emphasize the separate roles of software and other types of investment in shaping the distribution of occupational demand. Though not a primary focus of this paper, our model also generates a decline in the labor share caused by higher software investment, and we also show that there is a significant correlation between labor share declines and software intensity at the industry-level.

The rest of the paper is organized as follows. In section 2, we summarize the relevant empirical facts. In section 3, we present the model and characterize its equilibrium. In section 4, we conduct analytical comparative statics and in section 5, verify that the model's predictions hold empirically across industries. In section 6, we calibrate the model to quantify how important its mechanism is for accounting for the rise of software and skill demand reversal. Section 7 concludes.

2 Key Facts

We document several data observations in this section. First, equipment-producing industries have experienced much faster TFP growth than that of software-producing industries. Second, the pattern of polarization shows that the rise of high-skill occupations slowed with a greater increase in low-skill occupations since the late 1990s. Third, software development expenditures have increased relative to other R&D expenditures. Meanwhile, a share of software investment in total investment has also increased whereas that of equipment investment has decreased. Fourth, most importantly, we show that middle-skill occupations use equipment intensively, whereas high-skill occupations use software intensively. Moreover, the intensity of equipment and software across tasks is closely correlated with routine task intensity and cognitive task intensity. Again, our main hypothesis is that the first observation – together with the fourth observation – can generate both the second and the third observations.

2.1 Productivity of Equipment / Software Production

The input-output table published by BEA reports the industrial composition of equipment and intellectual property products (IPP) investment, where the IPP investment consists of software, R&D, and others. From the table, we can obtain the weights on detailed industries producing equipment and software investment goods. On the basis of these weights, we compute the total factor productivity (TFP) of equipment- and software-producing industries according to the Törnqvist index.

Using Industry Accounts from BEA, we first compute an industry i's TFP growth between time u and t as

$$\log(TFP_{i,t}/TFP_{i,u}) = \log(y_{i,t}/y_{i,u}) - \frac{\alpha_{i,t}+\alpha_{i,u}}{2}\log(k_{i,t}/k_{i,u}),$$

where y is the real value added per employment, k is the real non-residential capital divided by the number of employment, and α is one minus the labor share.

From the input-output table of each year (t), we obtain the share of each industry commodity in equipment investment ($\omega^e_{i,t}$) and software investment ($\omega^s_{i,t}$). Then, the TFPs of the equipment- and software-producing industries are computed by

$$\log(TFP_{e,t}/TFP_{e,t-1}) = \sum_i \frac{\omega^e_{i,t}+\omega^e_{i,t-1}}{2}\log(TFP_{i,t}/TFP_{i,t-1}),$$
$$\log(TFP_{s,t}/TFP_{s,t-1}) = \sum_i \frac{\omega^s_{i,t}+\omega^s_{i,t-1}}{2}\log(TFP_{i,t}/TFP_{i,t-1}).$$

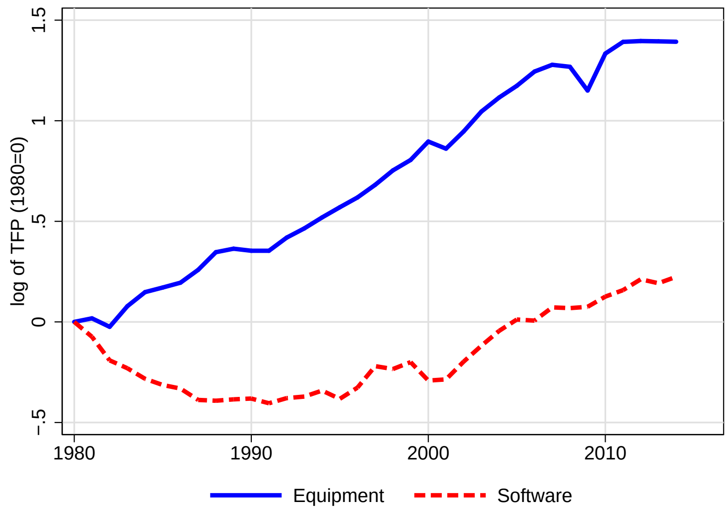
The results are presented in figure 2.1, which shows that equipment-producing sector has experienced much faster increase in the productivity than software-producing sector.

2.2 The Pattern of Job Polarization

Figure 2.2 shows the changes in employment share across skill percentile by decade from 1980, computed from Census/ACS data. Each point in the skill percentile represents a group of occupations representing 1% of the labor supply in 1980, sorted by average log hourly wage in 1979.

The figure shows clear U-shaped changes in employment share from 1980 to 2010. By assessing the three lines separately, however, we see that the rise in high-skill occupations is strongest in the first two decades while that of low-skill occupations acceler-

Fig. 2.1: TFP of equipment / software producing industries



ates during 2000-2010. Moreover, the range of shrinking occupations shifts toward the right across decades.

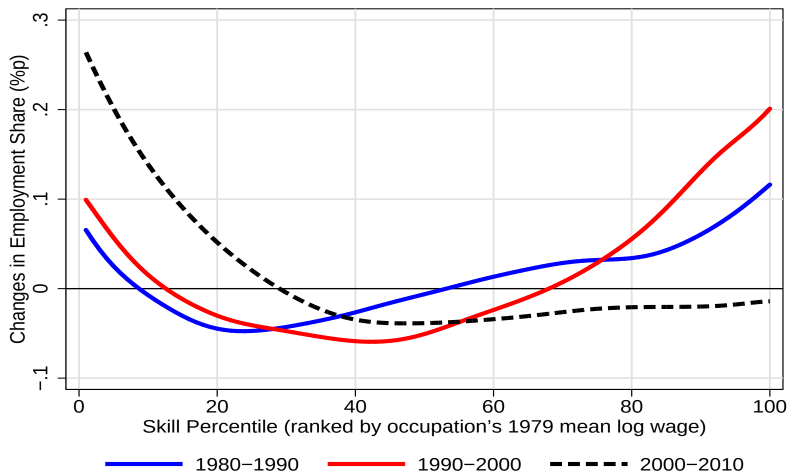
Similar observations are also in the annual data from CPS when occupations are classified into three groups: cognitive (high-skill), routine (middle-skill), and manual (low-skill)[4]. We compare two different trends – a linear trend from 1980 to 1995 and an HP trend including all data points – of the employment share of each occupational group. Figure 2.3 confirms that there are breaks in the trends of employment shares of cognitive (high-skill) occupations and manual (low-skill) occupations in the late 1990s. Interestingly, the decline in routine (middle-skill) occupations follows similar trends before and after 1995.

2.3 Rising Software Innovation and Investment

We now turn to the R&D composition in the US. NIPA does not report expenditures on software development directly but the series can be obtained from Crawford et al. (2014) or from differences between R&D in NIPA excluding software development and R&D recorded in the innovation satellite account which includes software development. Figure 2.4 shows the size of software development relative to other R&D ex-

[4]The classification of occupations is based on one-digit SOC. The cognitive occupations are management, professionals, and technicians. The routine occupations are machine operators, transportation, sales and office, mechanics, and miners and production.

Fig. 2.2: Changes in employment structure in the US by decade



Note: Each point on the horizontal axis is a group of occupations composing 1% of total employment in 1980, sorted by 1979 average log wage.

penditures, with and without chemical-related R&D's [5], across years. Both show an increasing pattern, especially during the late 1990s, suggesting that the changes in the pattern of polarization could be related to increasing software innovation.

Another observation to note is that software investment, as well as software innovation, has also increased faster than other types of investment. From NIPA, we compute the share of software investment and equipment investment of total non-residential investment and plot the results in figure 2.5. Figure 2.5a shows an increasing trend of software investment while figure 2.5b shows a decreasing trend of equipment investment. Moreover, the downward trend of equipment share has accelerated since the mid-1990s.

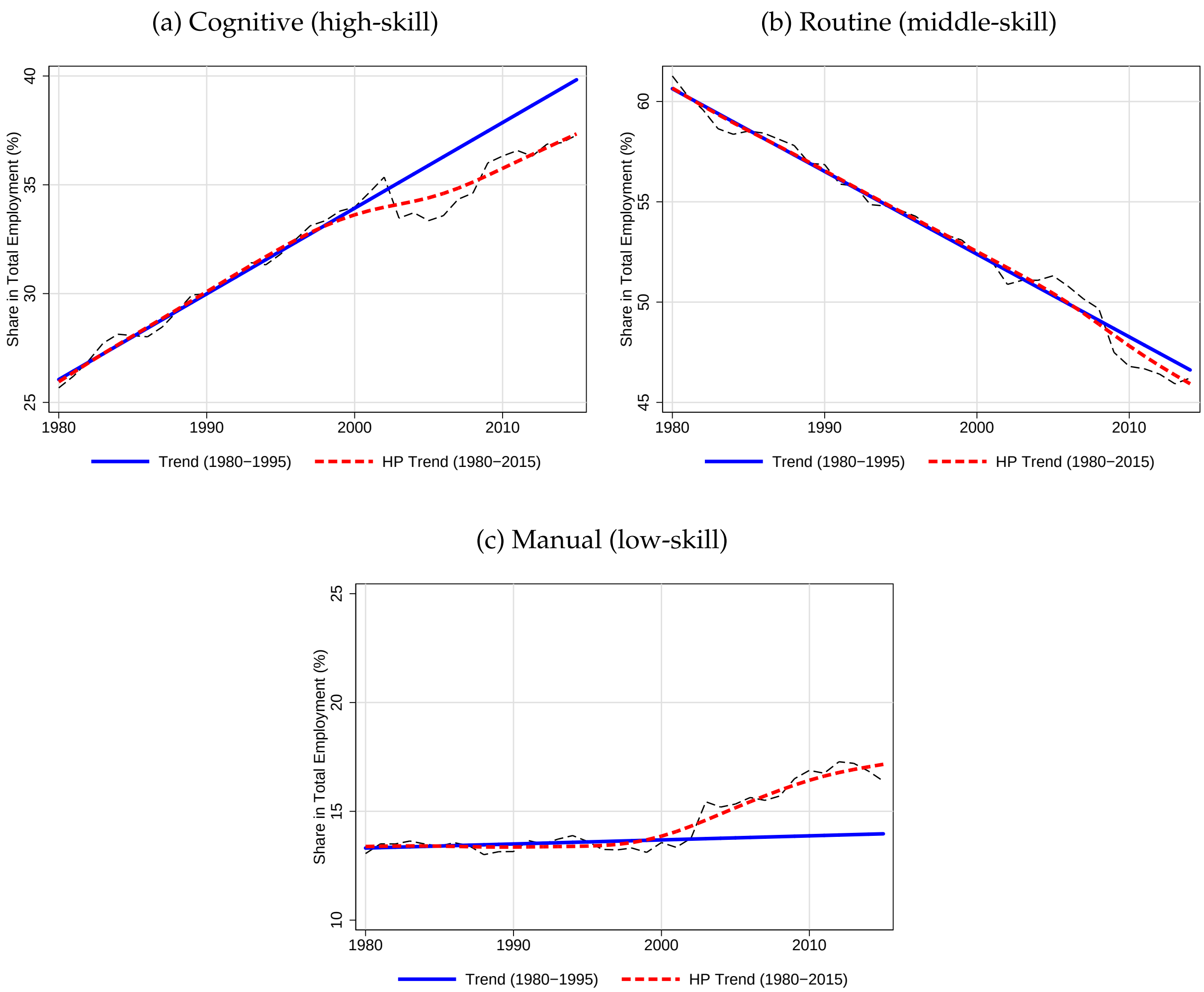
2.4 Capital Use by Occupation

We provide data evidence that documents strong connections between the use of different types of capital across occupations. Specifically, we construct capital use by occupation data by combining two data sources – NIPA and O*NET Tools and Technology Database.

The O*NET Tools and Technology database provides information about the types of tools and technology (software) used by each occupation. One caveat of this dataset

[5]We view R&D expenditures funded by chemicals-related industries as the expenditures least closely related to R&Ds on capital.

Fig. 2.3: Employment share of cognitive, routine, and low-skill services occupations

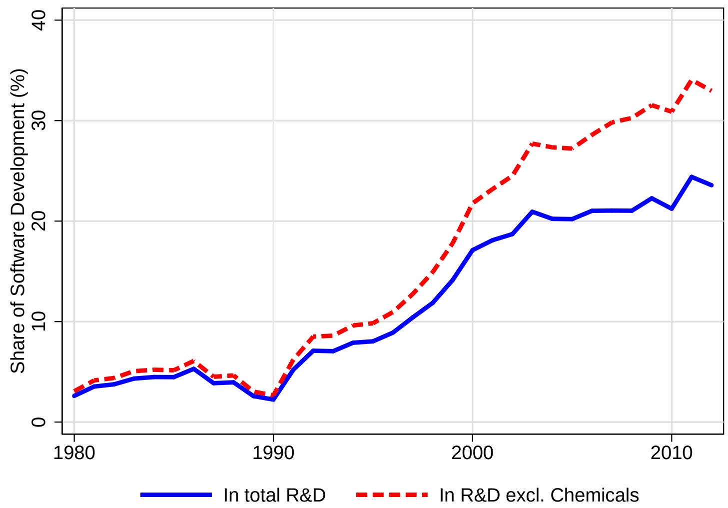


Note: 1) Cognitive occupations are management, professionals, and technicians. Routine occupations include office and sales, transportation, machine operators, mechanics, construction and production workers.
2) The blue line is the linear trend of 1990 to 1995, and the red (dash) line is the HP trend with smoothing parameter 100. All vertical axes represent 15%p of the range.

is that it does not provide information about the value of each item. To address this shortcoming, we attempt to link capital items in O*NET Tools and Technology to NIPA data obtained from the Bureau of Economic Analysis (BEA).

Specifically, we make a naive concordance between the UN Standard Product and Services Code (UNSPSC), a product classification system used in the O*NET database, and 25 categories of non-residential equipment in NIPA table 5.5 (details can be found in the appendix A). Then, we distribute the total amount of a particular type of equip-

Fig. 2.4: Software innovation compared to other R&D excluding chemicals



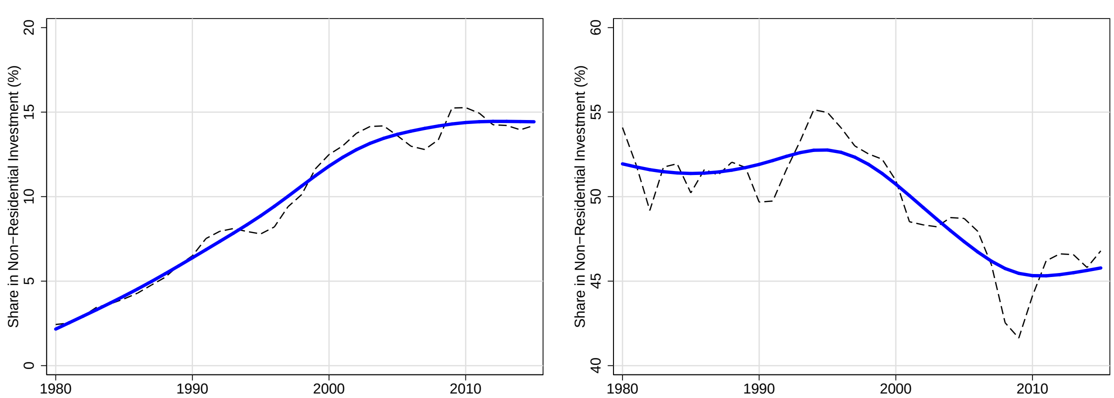
ment investment to each occupation by means of the number of tools included in the investment category according to the concordance.

For example, suppose that firms have invested USD 20 billion in metalworking machinery in the NIPA table. According to the constructed concordance, metalworking machinery includes a total of 139 commodities in UNSPSC. Some occupations use none of the 139 commodities, and other occupations use various numbers of the commodi-

Fig. 2.5: Investment share in private non-residential investment

(a) Software

(b) Equipment



Note: The blue line shows the HP trend with smoothing parameter 100. All vertical lines represent 20 % of the range.

ties in the category. Because we know the number of employment by occupation, we can calculate the total number of metalworking machinery items used by all workers in a given year. Then, we can approximate the amount attributed to an individual occupation by distributing the total USD 20 billion investment according to the number of items used by the occupation. Subsequently, dividing by the number of employees provides an estimate of the per capita investment in metalworking machinery by occupation.

The per capita investment in equipment by occupational skill group is shown in figure 2.6a, where an occupational skill group is defined as a group representing 1% of total employment among all occupations ranked by mean hourly wages. We also plot the routine-intensive task share – a share of routine-intensive employment out of total employment within the skill group – in the same figure. Here, routine-intensive employment is defined as employment in occupations with the highest one-third routine task index of all occupations, where the routine task index is computed using the O*NET task database following Acemoglu and Autor (2011).

In figure 2.6b, we plot software investment per capital across the same wage percentile, and the cognitive-intensive task share defined similarly to the routine-intensive task share. Again, the cognitive task index is computed following Acemoglu and Autor (2011).

We can see from the figures that middle-skill workers use equipment more intensively, whereas high-skill workers use software more intensively. Moreover, the use of equipment closely follows the routine task share while the use of software is closely related to the cognitive task share. We further illustrate the use of equipment subitem by occupation in figures 2.6c (industrial equipment) and 2.6d (industrial and information processing equipment). Among the equipment subitems, industrial equipment is most strongly correlated with routine task intensity.

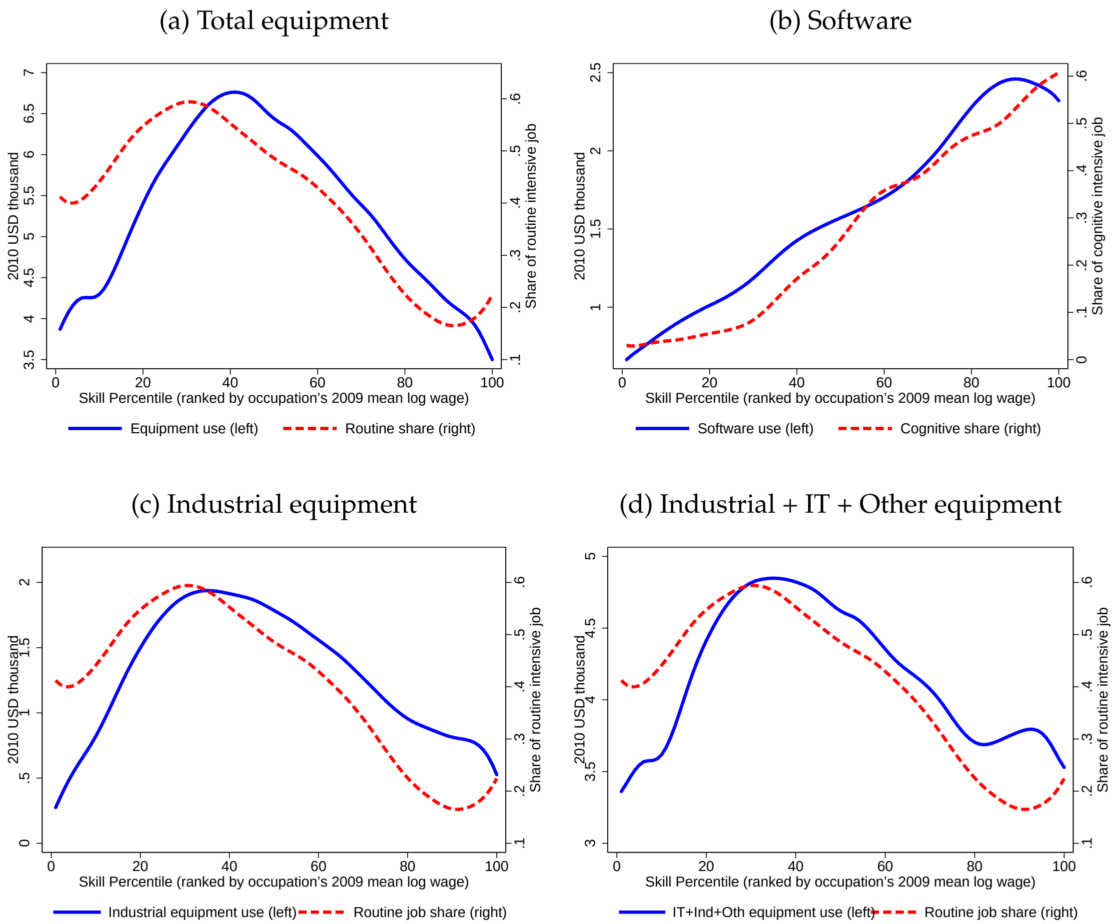
3 Model

There is a continuum of individuals endowed with human capital $h \in [1, \bar{h}]$ drawn from a measure $\mathcal{M}(h)$. Specifically, we assume that:

Assumption 1 (distribution) *The measure of skill, $\mathcal{M} : [1, \bar{h}] \mapsto [0, 1]$ is a cumulative distribution function with a differentiable probability distribution function, $\mu : [1, \bar{h}] \mapsto \mathbb{R}+$.*

There is a continuum of tasks $\tau \in [0, \bar{\tau}]$, and final goods are produced by combining

Fig. 2.6: Use of equipment and software across skill percentile



Note: Detailed information on the data is provided in appendix A.

task output $T(\tau)$ according to:

$$Y = \left(\int_\tau \gamma(\tau)^{\frac{1}{\epsilon}} T(\tau)^{\frac{\epsilon-1}{\epsilon}} d\tau \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{1}$$

The task output is produced by integrating human capital specific task production $y(h,\tau)$ across all skill levels used for the production of task τ:

$$T(\tau) = \int_{h\in\mathcal{L}(\ll)} y(h,\tau) dh. \tag{2}$$

The human capital specific task production, $y(h,\tau)$, depends not only on worker human capital h but also on task τ that the worker is performing. Specifically, the functional form of $y(h,\tau)$ is given by

$$y(h,\tau) = \left[\left\{ \alpha_h(\tau) \left(b(h,\tau) l \right)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_e(\tau) E^{\frac{\sigma_e - 1}{\sigma_e}} \right\}^{\frac{\sigma_e(\sigma_S - 1)}{(\sigma_e - 1)\sigma_S}} + \alpha_s(\tau) S^{\frac{\sigma_S - 1}{\sigma_S}} \right]^{\frac{\sigma_S}{\sigma_S - 1}}, \tag{3}$$

where $l(h)$ represents the level of employment of workers with human capital h, S and E represent software and equipment, respectively.

The function $b(h,\tau)$ captures the productivity of a worker with human capital h when she performs task τ. We assume that $b(h,\tau)$ is strictly log supermodular.

Assumption 2 *The function $b(h,\tau) : [1,\bar{h}] \times [0,\bar{\tau}] \mapsto \mathbb{R}^+$ is differentiable and strictly log supermodular. That is,*

$$\log b(h',\tau') + \log b(h,\tau) > \log b(h,\tau') + \log b(h',\tau),$$

for all $h' > h$ and $\tau' > \tau$.

As shown in Costinot and Vogel (2010), assumption 2 helps to ensure positive assortative matching (PAM). In other words, the higher human capital h is, the higher τ task she will perform in equilibrium. Not only how each occupation utilizes a worker's human skill, but tasks are also different in to which intensities they use two types of capital. This second feature is essential to understanding the differential effects of capital-embodied technical change on various occupations. [6]

The software and equipment available for workers are given by

$$S = \left(\int_0^{N_s} s(k)^{\nu_s} dk \right)^{\frac{1}{\nu_S}} \text{ and } E = \left(\int_0^{N_e} e(k)^{\nu_e} dk \right)^{\frac{1}{\nu_e}}, \tag{4}$$

where each variety of capital ($s(k)$ and $e(k)$) is provided by a permanent patent owner under monopolistic competition.

The production technology of software or equipment is

$$s(k) = A_s x, \quad e(k) = A_e x,$$

[6] Many studies following Krusell et al. (2000) consider a production technology in which capital substitute a certain group of workers more or less than others. In our model, the substitutability between capital and human skill is same across occupations. We still have differential effects of capital-embodied technical change on various occupations for three reasons. First, each occupation utilizes human skill differently. Second, occupations rely on capital with various intensities. Third, any changes in the relative productivity in occupation-level alter relative demand for occupations through the final production, combining all tasks.

where x is the amount of final goods used to produce software or equipment. The production technology implies that the marginal costs of producing software and equipment are given by the inverse of productivity, $q_s := 1/A_s$ and $q_e := 1/A_e$.

New software and equipment are created from R&D expenditures Z_s and Z_e, and the laws of motion for total varieties follow

$$\dot{N}_s = Z_s/\eta_s \text{ and } \dot{N}_e = Z_e/\eta_e. \tag{5}$$

Finally, the representative household has CRRA preference given by

$$\int_s^\infty e^{-\rho t} \frac{C(t)^{1-\theta} - 1}{1-\theta} dt,$$

and the resource constraint in the economy is

$$C + q_e \int_0^{N_s} s(k)dk + q_s \int_0^{N_e} e(k)dk + Z_e + Z_s \leq Y. \tag{6}$$

3.1 Static Equilibrium

To characterize the static equilibrium, we take the total varieties of software and equipment, N_e and N_s, as given. We first define the equilibrium.

Definition 1 (Static equilibrium) *The static equilibrium consists of the price function $p(\tau)$, $w(h)$, $p_s(k)$, and $p_e(k)$, the quantity function $T(\tau)$, $l(h,\tau)$, $s(k,\tau)$, $e(k,\tau)$, and the quantity Y such that:*

1. *Given $p(\tau)$, final goods producer solves*

$$\max\ Y - \int_\tau p(\tau)T(\tau)d\tau,$$

   given equation (1).

2. *For each task, the task output is produced to solve*

$$\max\ p(\tau)T(\tau) - \int_h w(h)l(h,\tau)dh - \int_0^{N_s} p_s(k)s(k,\tau)dk - \int_0^{N_e} p_e(k)e(k,\tau)dk,$$

   given equation (2), $w(h)$, $p_s(k)$, and $p_e(k)$.

3. *A capital provider solves*

$$\max \ \pi_s(k) = \int_\tau \left[p_s(k)s(k,\tau) - q_s s(k,\tau)\right] d\tau,$$
$$\max \ \pi_e(k) = \int_\tau \left[p_e(k)e(k,\tau) - q_e e(k,\tau)\right] d\tau,$$

given the marginal cost q_s and q_e.

4. *All workers choose the highest-paying occupation (task).*

5. *The labor market clears $\mu(h) = \int_\tau l(h,\tau)d\tau$.*

From the final goods production, the demand for task output $T(\tau)$ is given by

$$p(\tau) = \left(\frac{\gamma(\tau)Y}{T(\tau)}\right)^{\frac{1}{\epsilon}}, \tag{7}$$

and the price function $p(\tau)$ satisfies $\int_\tau \gamma(\tau)p(\tau)^{1-\epsilon}d\tau = 1$.

Since we assume that the capital producer maximizes profit under monopolistic competition, we obtain the price of the software and equipment as

$$p_s(k) = \frac{1}{A_s\nu_s} \text{ and } p_e(k) = \frac{1}{A_e\nu_e}, \text{ for all } k.$$

By substituting this result into the first-order conditions from task output production, we can show that the wage function $w(h)$ satisfies

$$w(h) \geq \underbrace{\left[\left\{p(\tau)^{1-\sigma_s} - \left(\frac{\alpha_s(\tau)^{\frac{\sigma_s}{1-\sigma_s}}}{A_s N_s^{\varphi_s}\nu_s}\right)^{1-\sigma_s}\right\}^{\frac{1-\sigma_e}{1-\sigma_s}} - \left(\frac{\alpha_e(\tau)^{\frac{\sigma_e}{1-\sigma_e}}}{A_e N_e^{\varphi_e}\nu_e}\right)^{1-\sigma_e}\right]^{\frac{1}{1-\sigma_e}} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}}}_{:=\omega(\tau)}$$
$$\times b(h,\tau), \tag{8}$$

with equality when $l(h,\tau) > 0$.

Equation (8) shows that the wage function $w(h)$ can be expressed as a product of terms depending only on τ ($\omega(\tau)$) and human capital task-specific productivity $b(h,\tau)$. The existence of PAM between h and τ follows.

Lemma 1 (Positive assortative matching) *Under assumptions 1 and 2, there exists a continuous and strictly increasing assignment function $\hat{h} : [0,\bar{\tau}] \mapsto [1,\bar{h}]$ such that $\hat{h}(0) = 1$ and $\hat{h}(\bar{\tau}) = \bar{h}$.*

The proof is same as the proof of Lemma 1 in Costinot and Vogel (2010) and is omitted.

The equilibrium assignment $\hat{h}$ is characterized by:

Lemma 2 (Equilibrium assignment function) *The equilibrium assignment function $\hat{h}(\tau)$, price function $p(\tau)$, and the wage rate $\omega(\tau)$ satisfy the following system of differential equations.*

$$\frac{d\log\omega(\tau)}{d\tau} = -\frac{\partial \log b(\hat{h}(\tau),\tau)}{\partial\tau}, \tag{9}$$

$$\hat{h}'(\tau) = \frac{\gamma(\tau)p(\tau)^{\sigma_s-\epsilon}\alpha_h(\tau)^{\sigma_s}\psi(\tau)^{\sigma_e-\sigma_s}Y}{\omega(\tau)^{\sigma_e}b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))}, \tag{10}$$

$$p(\tau) = \left[\psi(\tau)^{1-\sigma_s} + \alpha_s(\tau)^{\sigma_s}\left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1}\right]^{\frac{1}{1-\sigma_s}}, \tag{11}$$

with $\hat{h}(0)=1$, $\hat{h}(\bar{\tau})=\bar{h}$, *and* $\int\gamma(\tau)p(\tau)^{1-\epsilon}d\tau = 1$,
$\psi(\tau) := \left[\alpha_h(\tau)^{\sigma_e}\omega(\tau)^{1-\sigma_e} + \alpha_e(\tau)^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right]^{\frac{1}{1-\sigma_e}}$, $\varphi_e := \frac{1-\nu_e}{\nu_e}$, *and* $\varphi_s := \frac{1-\nu_s}{\nu_s}$.

Proof *In appendix C.* ∎

After the assignment function $\hat{h}$ is obtained, all the equilibrium quantities and prices can be computed.

3.2 Dynamic Equilibrium

Now consider a dynamic equilibrium where technology evolves endogenously. The HJB equations for innovators are given by

$$r(t)V_s(k,t) - \dot{V}_s(k,t) = \pi_s(k,t), \tag{12}$$

$$r(t)V_e(k,t) - \dot{V}_e(k,t) = \pi_e(k,t), \tag{13}$$

with profit functions,

$$\pi_s(k) = \int_\tau [p_s(k)s(k,\tau) - q_s s(k,\tau)]d\tau = \frac{1-\nu_s}{\nu_s A_s}\int_\tau s(k,\tau)d\tau, \tag{14}$$

$$\pi_e(k) = \int_\tau [p_e(k)e(k,\tau) - q_e e(k,\tau)]d\tau = \frac{1-\nu_e}{\nu_e A_e}\int_\tau e(k,\tau)d\tau. \tag{15}$$

The free entry condition ensures that

$$V_e \le \eta_e, \text{ with equality if } Z_e > 0, \quad \text{and} \quad V_s \le \eta_s, \text{ with equality if } Z_s > 0.$$

If both R&D's are positive, we have $\eta_e V_e = \eta_s V_s$, and from equations (12) and (13),

$$r(t) = \pi_e(t)/\eta_e = \pi_s(t)/\eta_s. \tag{16}$$

Finally, from the household's problem, we have a standard Euler equation:

$$\frac{\dot{C}(t)}{C(t)} = \frac{r(t) - \rho}{\theta}, \tag{17}$$

and the transversality condition:

$$\lim_{t \to \infty} \left[e^{-\int_0^t r(s)ds} \left(N_e(t)V_e(t) + N_s(t)V_s(t) \right) \right] = 0.$$

Now, we have a characterization of the steady state equilibrium in the following lemma.

Lemma 3 (Steady state equilibrium) *There exist $\nu_e < 1$ and $\nu_s < 1$ sufficiently large that are compatible with the unique steady state equilibrium, i.e.,*

$$\pi_e/\eta_e = \pi_s/\eta_s = \rho, \tag{18}$$

and every variable remains constant. Moreover, when $\sigma_s = \sigma_e = 1$, $\max \left\{ \frac{1-\nu_s}{\nu_s} \frac{\alpha_s(\tau)}{\alpha_h(\tau)} + \frac{1-\nu_e}{\nu_e} \frac{\alpha_e(\tau)}{\alpha_h(\tau)} \right\} < 1$ *ensures the existence of the steady state equilibrium.*

Proof *In appendix C.* ∎

Intuitively, high enough ν_e and ν_s ensure profits by providing additional variety not too large, which makes the rate of return on increasing variety strictly decreasing on the total varieties. As the rate of return is strictly decreasing in the size of varieties, we have a certain level of varieties that equates the rate of return and time preference (ρ), leading to the existence of the steady state.

We consider only a case with no growth steady state as no standard balanced growth path exists when the task production is a general CES function. Note that the source of growth (increasing variety) is a capital-augmented technological change in our model. It is well-known that no balanced growth path would exist for a capital-augmented technical change if the production function is not of the Cobb-Douglas form (e.g. Grossman et al., 2017). [7]

[7] In the Cobb-Douglas task production case ($\sigma_s = \sigma_e = 1$); however, a sustained growth can be obtained

A detailed analysis of the transitional dynamics is not the primary focus of this paper. Instead, we focus on the differences between the static equilibrium (where N_s and N_e are fixed) and the steady state throughout the paper. We confirm numerically that the obtained steady state is saddle-path stable in the quantitative analysis. When the steady state is saddle-path stable, the transitional dynamics will be similar to that of the Neo-classical growth model, as a key is that the production function is strictly concave in the varieties.

Exogenous vs Endogenous Productivity Our model has both exogenous and endogenous productivity for equipment and software. Exogenous productivity is augmented in capital production, A_e or A_s, and captures how well one can produce equipment or software that has already been introduced into the economy. For example, when equipment production became faster as a result of using a faster computer in the production process, this shift would be captured in the increase in A_e. Instead, endogenous productivity, N_e or N_s, captures an introduction of new types of capital to the economy. For example, the development of the Uber application supporting drivers would be captured by an increase in N_s.

4 Comparative Statics

In this section, we restrict our attention to the case with $\sigma_e = \sigma_s = 1$, $\eta_e = \eta_s$ and $\nu_e = \nu_s$ to obtain analytical comparative statics. Specifically, we assume:

Assumption 3 *The elasticities of substitution between labor and equipment/software are one, i.e., $\sigma_s = \sigma_e = 1$. The individual task production function is then*

$$y(h,\tau) = (b(h,\tau)l(h))^{\alpha_h(\tau)} E^{\alpha_e(\tau)} S^{\alpha_s(\tau)}.$$

Additionally, we put some structures on the intensity functions $\alpha_h(\tau)$, $\alpha_e(\tau)$, and $\alpha_s(\tau)$ to reflect the fact that high-skill workers use software intensively and middle-skill workers use equipment intensively, i.e.,

Assumption 4 (intensities) *The functions $\alpha_h(\tau) : [0,\bar{\tau}] \mapsto (0,1]$, $\alpha_s(\tau) : [0,\bar{\tau}] \mapsto (0,1]$ and $\alpha_e(\tau) : [0,\bar{\tau}] \mapsto (0,1]$ satisfy the following.*

by assuming strictly positive population growth, as in Jones (1995). We still have every task growing at a different rate, so the most labor intensive task (the slowest-growing task) would dominate the economy in the limit under complementarity between tasks ($\epsilon < 1$), which is similar to the results in Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008).

2.1 $\alpha_s(\tau)$ *is differentiable and increasing on* $[0, \bar{\tau}]$.

2.2 $\alpha_e(\tau)$ *is differentiable, increasing on* $[0, \tau_e]$ *and decreasing on* $[\tau_e, \bar{\tau}]$.

2.3 $\alpha_e(\tau_e) > \alpha_s(\tau_e)$, $\alpha_s(\bar{\tau}) > \alpha_e(\bar{\tau})$, *and* $\alpha_e(0) = \alpha_s(0)$.

Now, we show that an increase in the productivity of equipment production ($A_e \uparrow$) leads to polarization and the rise of software and skill demand reversal when the tasks are complementary. Specifically, we focus on three main predictions of the model: (1) the polarization induced by the rise of equipment-producing productivity in the static equilibrium, and (2) the subsequent rise of software innovation, and (3) the decreasing demand for high-skilled employment in the steady state.

Job Polarization

First, we show the impact of an increase in the equipment productivity (A_e) on equilibrium assignment function $\hat{h}(\tau)$ in the static equilibrium (i.e., when N_e and N_s are fixed). We consider $A_{1e} < A_{2e}$ and denote the equilibrium assignment functions corresponding to A_{1e} and A_{2e} as $\hat{h}_1$ and $\hat{h}_2$, respectively.

Proposition 1 (Polarization) *Consider* $A_{1e} < A_{2e}$. *Suppose* $\epsilon < 1$ *and assumptions 1 to 4. For sufficiently small* $\alpha'_h(\tau)$, *we have* $\tau^* \in (0, \bar{\tau})$ *such that* $\hat{h}_1(\tau^*) = \hat{h}_2(\tau^*)$, $\hat{h}_1(\tau) < \hat{h}_2(\tau)$ *for* $\tau \in (0, \tau^*)$, *and* $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ *for* $\tau \in (\tau^*, \bar{\tau})$.

Proof *In appendix C.* ∎

Proposition 1 states that there will be a shrinking employment of task around τ^* where corresponding equipment intensity $\alpha_e(\tau^*)$ is relatively higher than $\alpha_e(0)$ and $\alpha_e(\bar{\tau})$. Figure 4.1 illustrates the change in the assignment function with A_{1e} (blue solid line) and $A_{2e} > A_{1e}$ (red dashed line). For a given task $\tau \in [\tau^* - \epsilon, \tau^* + \epsilon]$, we can see that employment decreases because we have higher $\hat{h}_2(\tau)$ on the left side of τ^* and lower $\hat{h}_2(\tau)$ on the right side of τ^*.

As shown in section 2, tasks with higher equipment intensities are consistent with routine-intensive tasks; hence, the proposition states that decreasing routine employment can be caused by a decrease in the price of equipment.

The condition of sufficiently small $\alpha'_h(\tau)$ is assumed because the impact of the change in equipment price on human capital depends on the relative size of α_e to α_h, not α_e alone. The condition is sufficient but not necessary. As we show via numerical examples in appendix D, $\alpha'_h(\tau)$ need not be too small.

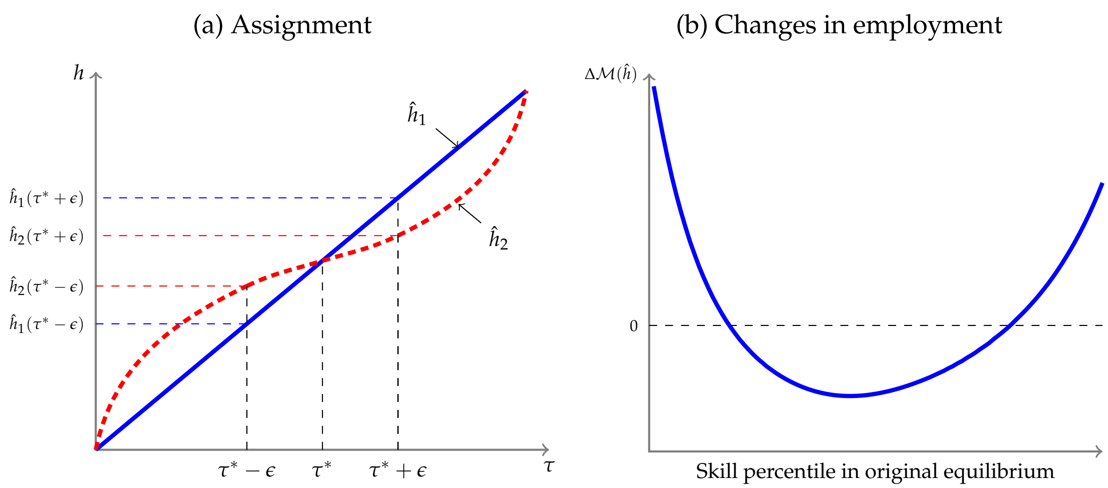
The intuition under the proposition is as follows. An increase in the equipment productivity (A_e) leads to a decrease in the price of equipment (q_e). This increases the productivity of all tasks but to a greater extent for tasks with higher equipment intensities. When the production is complementary in the tasks ($\epsilon < 1$), the rise of relative productivity causes factors to flow out to other tasks, which results in polarization.

The intuition is similar to other papers in the literature, for example, Lee and Shin (2017), Goos et al. (2014), and Cheng (2017): The technological change making the middle-skill tasks more productive reduces demand for the middle jobs when the tasks are relatively more complement than the relation between workers and technology. Our model enables a more direct mapping to the data due to the intensity function $\alpha_e(\tau)$ and $\alpha_s(\tau)$, meaning that it is not the technology affects a certain occupational group only but affects occupations to different extents. Cheng (2017) also introduces heterogeneous intensities across occupations. Ours differs from Cheng (2017) that we focus on the equipment among capital, which is a specific component used by middle-skill occupations intensively. We also highlight that the changes in the occupational structure itself can lead to another type of task-specific technological change, which we explore in the following propositions.

The Rise of Software

The profits from providing software and equipment are proportional to the demand, which in turn, is proportional to the task output times the factor intensity of the task.

Fig. 4.1: Equilibrium comparison: A_{1e} vs $A_{2e} > A_{1e}$



Hence, changes in the relative size of task production result in changes in the profit from providing each type of capital according to the corresponding factor intensity.

We know from proposition 1 that the employment share around τ^* (in the middle) shrinks. As long as $\alpha'_h(\tau)$ is small, the share of task production around τ^* also has to decrease. Meanwhile, $\alpha_e(\tau^*) > \alpha_s(\tau^*)$, together with $\alpha_e(\bar{\tau}) < \alpha_s(\bar{\tau})$ and $\alpha_e(0) = \alpha_s(0)$ (assumption 4), imply that a decrease in production share around τ^* actually decreases e more than s, and an increase in production share around $\bar{\tau}$ increases s more than e. Therefore, providing software becomes more profitable for innovators. Innovators then focus innovation toward software, resulting in higher N_s/N_e in the new steady state.

Although this prediction is valid for most reasonable quantifications, we have to impose tight restrictions on the structures of the intensities over the entire range of $\tau \in [0, \bar{\tau}]$ to prove the analytical proposition as we are comparing the ratio of two integrations over all τ ($\pi_e/\pi_s \propto \int \alpha_e(\tau)p(\tau)T(\tau)d\tau / \int \alpha_s(\tau)p(\tau)T(\tau)d\tau$). To express the analytical proposition in a simple way, we consider an approximation with three discrete tasks ($j = 0, 1, 2$ for low, middle, and high) in this subsection. Specifically, consider a production technology given by

$$Y = \left(\sum_j \gamma_j^{\frac{1}{\epsilon}} T_j^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}} \quad \text{for } j = 0, 1, 2, \tag{19}$$

with $T_j = (b(h,j)l(h))^{\alpha_{h,j}} E^{\alpha_{e,j}} S^{\alpha_{s,j}}$. The detailed derivation of the equilibrium conditions for this approximation can be found in appendix B.

With this approximation, assumptions 1 and 4 are replaced by the following.

Assumption 5 (distribution-II) *The measure $\mathcal{M} : [1, \bar{h}] \to [0, 1]$ has a differentiable p.d.f. $\mu(h)$, where $\mu(h)$ is sufficiently small everywhere.*

Assumption 6 (intensities-II) *The discrete intensities satisfy the following.*

6.1 $\frac{\alpha_{e,1}}{\alpha_{h,1}} > \frac{\alpha_{e,0}}{\alpha_{h,0}} \approx \frac{\alpha_{e,2}}{\alpha_{h,2}}$.

6.2 $\alpha_{e,0} \approx \alpha_{s,0}$, $\alpha_{e,1} > \alpha_{e,2}$, *and* $\alpha_{s,2} > \alpha_{s,1}$.

In assumption 5, we add the requirement for $\mu(h)$ to be sufficiently small to consider the discretization as an approximation of continuous tasks matched with a continuum of skills[8].

[8]For discretized tasks, the assignment function $\hat{h}(\tau)$ becomes a sequence of threshold human capital $\hat{h}_j$.

Assumption 6.1 states that task 1 is the most equipment intensive, relative to labor, compared to task 0 and task 2. Assumption 6.2 states that middle-skill tasks use equipment more than software, high-skill tasks use software more than equipment, and low-skill tasks use software and equipment similarly.

Again, consider an exogenous increase in the productivity of equipment, $A_{1e} < A_{2e}$. Denote the total varieties in the previous steady state as N_{s1} and N_{e1} and those in the new steady state as N_{s2} and N_{e2}. Then, we have:

Proposition 2 (Rise of software) *Consider $A_{1e} < A_{2e}$ with discretized tasks (19), where equipment variety is at least as large as software variety in the original equilibrium ($N_{e1} \geq N_{s1}$). Suppose $\epsilon < 1$, $\nu_e = \nu_s$, assumptions 2, 5, and 6. In the new steady state, software variety increases more than equipment variety, i.e., $N_{s2}/N_{e2} > N_{s1}/N_{e1}$.*

Proof *In appendix C.* ∎

Skill Demand Reversal

We now show that an increase in N_s results in skill demand reversal (i.e., a decrease in the demand for high-skilled labor). We consider $N_{s2} > N_{s1}$, and denote $\hat{h}_1$ and $\hat{h}_2$ as the equilibrium assignment corresponding to N_{s1} and N_{s2}, respectively.

Proposition 3 (Skill demand reversal) *Consider $N_{s2} > N_{s1}$ and suppose $\epsilon < 1$ and assumptions 1 to 4. With sufficiently small $\alpha'_h(\tau)$, the matching function shifts upward everywhere, i.e., $\hat{h}_2(\tau) > \hat{h}_1(\tau)$ for all $\tau \in (0, \bar{\tau})$.*

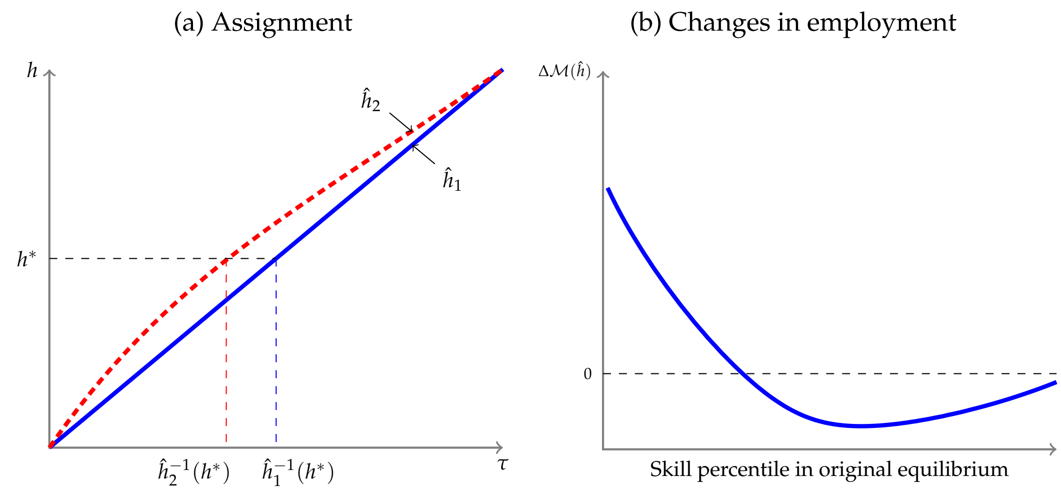
Proof *In appendix C.* ∎

Note that an increase in variety increases the productivity of software-intensive tasks more than that of other tasks (equation (11)). Following the same intuition as in the case of polarization, this would lead to a reallocation of labor from high-skill tasks to lower-skilled tasks under complementarity ($\epsilon < 1$). The change in assignment function is depicted in figure 4.2, which shows that all workers downgrade their tasks.

This proposition, together with proposition 2, impies that skill demand reversal results from the increase in software innovation induced by the increase in A_e. Note

When the $\hat{h}_j$'s change, it not only affects demand for labor around the threshold level but also the total labor supply given to each task, $\int_{\hat{h}_{j-1}}^{\hat{h}_j} \mu(h)dh$. We ignore indirect effects resulting from changes in $\int_{\hat{h}_{j-1}}^{\hat{h}_j} \mu(h)dh$ by assuming $\mu(\hat{h}_j)$ and $\mu(\hat{h}_{j-1})$ are sufficiently small.

Fig. 4.2: Equilibrium comparison: N_{s1} and $N_{s2} > N_{s1}$



that, when A_e increases, N_e should also increase; otherwise, $\pi_e/\eta_e > \rho$. The proposition 2, however, confirms that the variety of software would increase *more* than that of equipment. Accordingly, we have the following transition dynamics.

First, increases in A_e lead to immediate polarization according to proposition 1. Second, N_s jumps to equate π_e and π_s. N_s and N_e rise from then on until N_e and N_s reach the new steady state. Since increasing N_e will lead to polarization, the resulting steady state equilibrium itself would be a mix of polarization *and* skill demand reversal. As N_s increases more quickly, the skill demand reversal effect becomes stronger.

Again, the technical assumption of sufficiently small $\alpha_h'(\tau)$ is required for proof, but it does not need be that small quantitatively. We provide numerical examples, including the case with a general CES task production, in appendix D to illustrate the comparative statics.

5 Empirical Evidence

This section checks a validity of the model's predictions using industry data. Specifically, we test two predictions. First, the model predicts a negative relationship between changes in the relative price of software to equipment and changes in middle-skill employment relative to high-skill employment. Second, the model implies a positive correlation between changes in the relative price of software to equipment and changes in software innovation relative to other innovation. Note that, in our model, the prices of equipment and software are inversely related to productivity in the equipment- and software-producing sectors, respectively.

We measure the relative price of equipment to software by dividing nominal investment by real investment, provided by BEA. They are different by industry as each industry uses a different combination of subitems within the category of equipment or software. For the relative employment of middle-skill to high-skill occupations, we use the employment of routine occupations divided by the employment of cognitive occupations by industry, computed from Census data. Finally, the relative size of software innovation to other innovation is measured by own account software investment (in-house software investment by firms) divided by R&D excluding software. Details of the data construction are presented in appendix E.

Figure 5.1a shows the differences in the growth of middle-skill and high-skill employment against changes in software price relative to equipment price. Figure 5.1b shows the changes in software innovation net of R&D expenditures excluding software against changes in the relative price. The first has a negative relation, and the second has a positive relation, consistent with the model's predictions.

To determine whether these relations are statistically significant, we estimate the following regression:

$$\Delta \log y_{i,t} = a + c_t + \Delta \log(q_{s,i,t}/q_{e,i,t}) + \varepsilon_{i,t},$$

where $y_{i,t}$ is either the ratio of routine (middle-skill) employment to cognitive (high-skill) employment or the ratio of in-house software investment to R&D expenditures excluding software. The estimation results, which show significant relations between the two variables, are given in table 5.1.

Tab. 5.1: Estimation results

	Routine/Cognitive		Sft/R&D (excl. sft.)	
Sft price/ Eqp price	-0.220*** (0.000)	-0.152** (0.014)	+0.747** (0.016)	+0.717*** (0.001)
Fixed Effects	Yes	No	Yes	No
R^2	0.172	0.054	0.117	0.064

p-values in parentheses.

6 Quantitative Analysis

In this section, we use the discretized model (appendix B) to map the tasks to ten occupational groups consistent with one-digit SOC code (as in table 6.2). Specifically, the following production technology is used for the quantitative analysis.

$$Y = \left(\sum_j \gamma_j^{\frac{1}{\epsilon}} T_j^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}, \text{ and} \tag{20}$$

$$T_j = M_j \left[\left(\alpha_{h,j} \left(\int_{\hat{h}_{j-1}}^{\hat{h}_j} b(h,j)\mu(h)dh \right)^{\frac{\sigma_e-1}{\sigma_e}} + \alpha_{e,j} E_j^{\frac{\sigma_e-1}{\sigma_e}} \right)^{\frac{\sigma_e(\sigma_S-1)}{(\sigma_e-1)\sigma_S}} + \alpha_{s,j} S_j^{\frac{\sigma_S-1}{\sigma_S}} \right]^{\frac{\sigma_S}{\sigma_S-1}}.$$

Sources of Exogenous Variation Note that we have two types of exogenous productivity (A_i's and M_j's), as well as endogenous productivity (N_i's). The changes in exogenous productivities (A_i's and M_j's) are sources of exogenous variation in this section. What do these different productivities represent?

First, A_e and A_s capture how well a given technology produces already-introduced equipment or software. In other words, increases in A_i capture improvements in the production process, not the varieties of tools that occupations can utilize. To map A_i's as data, we use the TFP of equipment- or software-producing industries.

Second, M_j's are additional task-specific productivities introduced to match changes in the employment share of routine occupations in the model with data exactly. Recall that the changes in A_e or A_s also act as task-specific productivity in our model. A natural question is how much of the changes in the employment share between occupations can be captured only through A_e and A_s. We answer this question in one of the exercises in this section.

Third, N_e and N_s are endogenous productivities that capture varieties of software or equipment that workers can use to do their tasks. We view changes in N_e or N_s roughly correspond to R&D expenditures on each kind of capital in the data.

One may wonder how we can distinguish the productivity embodied in capital (A_i's) from dis-embodied technical change (N_i's). Regarding identification, increases in A_e and A_s result in the decline in the price of equipment and software, whereas changes in N_e and N_s do not alter the price of capital. Indeed, the price of equipment decreases more quickly than that of software, and the TFP of equipment-producing industries increases faster than that of software. To the contrary, software development expenditures rise more quickly than other types of R&D. These observations are consistent with our distinction between A_i's and N_i's.

Scenarios Given the changes in exogenous productivities, we perform two main exercises. The first is to investigate the extent to which endogenous innovation of software explains the *changes in the pattern* of polarization. For this exercise, we match the changes in the employment share of middle-skill occupations with A_i's and M_j's and look at the employment dynamics of high- and low-skill occupations generated from the model with innovation and without innovation.

The next exercise aims to determine the extent to which changes in the productivity of the equipment- and software-producing sectors only account for the shifts in the employment share between occupations. To address this question, we repeat the simulation with all the other parameters fixed, assuming constant M_j for all middle-skill occupations.

Finally, as a sensitivity analysis, we test how the simulation results change by varying the elasticity of substitution between tasks, mark-ups, and alternative measures for the productivity of the equipment- and software-producing sectors. Importantly, we confirm that observed changes in the price of equipment and software are consistent with the changes in the TFPs.

Following the literature, we label high-, middle-, and low-skill occupations as cognitive, routine, and manual occupations, respectively. Cognitive occupations include management, professionals, and technicians. Routine occupations are administrative, machine operators, transportation, sales, mechanics, and production workers. Finally, manual occupations are low-skilled services occupations. The reason we use ten occupational groups rather than three occupational groups is that we use the changes in the payroll share of each occupational group to calibrate the elasticity of substitution between tasks, which is the most crucial parameter driving the mechanism.

6.1 Calibration

We calibrate most of the parameters according to the 1980 data assuming a steady state. For the functional forms, we set the productivity function $b(h, j)$ as

$$b(h,j) = \begin{cases} \bar{h} & \text{if } j = 0 \\ h - \chi_j & \text{if } j \geq 1 \end{cases}$$

and the skill distribution $\mathcal{M}(h)$ as

$$\mathcal{M}(h) = 1 - h^{-a}.$$

The weight parameters in the final production (γ_j's) are taken from the employment share by occupation in 1980. The χ_j's and a are determined to match income share across occupational groups in 1980. Between-factor intensities by task (α_h, α_e, α_s) are matched to equipment and software investment by occupational group[9] and labor share in 1980. For the benchmark analysis, we map the equipment in the model to industrial equipment in the data as it has the closest relation with the routineness of occupations (figure 2.5).

There are two categories of parameters that are difficult to identify from only 1980 data: (1) the elasticity of substitution (ϵ, σ_e and σ_s), and (2) markup-related parameters (ν_e and ν_s). We use various methods to identify these parameters.

For the elasticity of substitution between tasks, we set ϵ to minimize the root-mean-squared error of the changes in payroll share between 1980 and 2010 by occupation. That is, we set ϵ to minimize $\left[\sum_{\tau=1}^{J}\left[(w^m_{\tau,2010} - w^m_{\tau,1980}) - (w^d_{\tau,2010} - w^d_{\tau,1980})\right]^2 / J\right]^{\frac{1}{2}}$, where ω_τ is a payroll share of occupational group τ. Intuitively, occupations are complementary when changes in quantity share (employment share) and changes in the relative price (relative wage) move in the same direction. Figure 6.1 shows that this is the case as relative wages of cognitive and manual occupations to routine occupations both have increased while the employment share of routine occupations has decreased (figure 2.3). The resulting parameter value for the elasticity of substitution between tasks is 0.301, which confirms the complementarity between tasks. We also perform a robustness check by varying the value of ϵ in subsection 6.3.

For the elasticity of substitution between factors in task production (σ_e and σ_s), we match linear trends of aggregate labor share and labor share only with equipment capital. To illustrate the identification process, note that factor share in a given task τ can be derived as follows:

[9] We assume that the number of commodities used by each occupation is the same and attribute the capital investment in 1980 to each occupation to get occupational use of equipment and software in 1980.

$$LS_{-s} = \frac{wL}{wL + p_e \tilde{E}} = \frac{1}{1 + \left(\frac{\alpha_e}{\alpha_h}\right)^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e} \omega\right)^{\sigma_e - 1}}, \tag{21}$$

$$\begin{aligned} LS &= \frac{wL}{wL + p_e \tilde{E} + p_s \tilde{S}} \\ &= \frac{1}{1 + \left(\frac{\alpha_e}{\alpha_h}\right)^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e} \omega\right)^{\sigma_e - 1} + \frac{\alpha_s^{\sigma_s} \left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s - 1}}{\omega^{1-\sigma_e} \alpha_h^{\sigma_e} \left(\alpha_h^{\sigma_e} \omega^{1-\sigma_e} + \alpha_e^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e - 1}\right)^{\frac{\sigma_e - \sigma_s}{1 - \sigma_e}}}}, \end{aligned} \tag{22}$$

where LS_{-s} is the labor share without software and LS is the standard labor share. From equation (21), it is straightforward to see that the labor share without software does not directly depend on the elasticity of substitution between labor and software, σ_s.

The fact that the aggregate labor share and labor share with equipment capital only show different trends from 1980 to 2010 makes this strategy even more useful (figure 6.5a). The labor share with equipment capital only has an increasing trend, whereas the aggregate labor share has a declining trend. [10] It is easy to predict $\sigma_e < 1$ and $\sigma_s > 1$ on the basis in the trends of total labor share and increasing productivity (both exogenous and endogenous) of capital.

We estimate the markup-related parameters ν_e and ν_s separately using the Industry Account and Fixed Asset Table from BEA, following Domowitz et al. (1988). Specifically, we estimate

$$\Delta \log q_{it} - \alpha_{Lit} \Delta \log l_{it} - \alpha_{mit} \Delta \log m_{it} = c_i + b \Delta \log q_{it} + \varepsilon_{it},$$

where q is gross output/capital, l is employment/capital, m is intermediate input/capital, and α_{Lit} and α_{Mit} are the labor and intermediate shares, respectively. We estimate this relation for the equipment-producing industry (industry 3 in the BEA industry codes) and software-producing industry (industry 511). To control for endogeneity, GDP growth is used as an instrumental variable. Once estimated, ν_e and ν_s can be obtained by calculating $1 - b$. The estimation results are presented in table 6.1.

[10] We compute the labor share with equipment capital only following Koh et al. (2016). To be specific, a standard asset pricing formula gives $R_i = (1 + r) q_i - q_i'(1 - \delta_i)$, where R_i is the gross return on capital type i, q_i is the relative price of capital type i, δ_i is a depreciation rate of capital type i, and r is the net rate of return. The no arbitrage condition implies that the net rate of return, r, is common across i. Using the fact that one minus labor share is equal to $\sum_i R_i K_i / Y$ under the CRS production technology, we can impute the gross rate of return on equipment, R_e. The labor share with equipment capital only then can be computed

Tab. 6.1: Estimation results: markup

	Equipment[1)]	Software[2)]
b	.228	.473
	(.000)	(.113)
N	333	37

Note: 1) Industries 331, 332, 333, 334, 335, 3361MV, 3364TO, 337, and 339.
2) Industry 511. 3) p-values in parentheses.

Tab. 6.2: Parameters by occupation

	α_e	α_s	α_h	γ	χ
Low-skilled services	0.190	0.009	0.801	0.004	
Administrative	0.060	0.186	0.754	0.711	0.000
Machine operators	0.644	0.017	0.339	0.077	0.002
Transportation	0.551	0.016	0.433	0.037	0.027
Sales	0.084	0.019	0.898	0.004	0.029
Technicians	0.265	0.020	0.714	0.002	0.071
Mechanics	0.696	0.020	0.285	0.135	0.071
Production	0.528	0.022	0.450	0.022	0.096
Professionals	0.133	0.012	0.854	0.005	0.097
Management	0.019	0.013	0.969	0.004	0.097
Target	Equipment, software, and labor share			Employment	Income share

Table 6.2 and 6.3 summarizes all the calibration results. Detailed calibration procedure is in appendix F.

6.2 Simulation Results

6.2.1 Changes in A_i's and M_j's

We assume the economy was in a steady state in 1980 and compute a new steady state corresponding to the exogenous changes (A_e, A_s, M_j's). We then assess how well the model explains the shifts in the trends of high-skilled and low-skilled employment with and without endogenous software innovation.

The Pattern of Occupational Employment Figure 6.2a displays the annualized changes in employment during the first two decades (blue bar) and last decade (light blue bar) by occupational group. Figures 6.2b and 6.2c show the same series gener-

by $CE/(CE + R_eK_e)$, where CE is the compensation of employees and K_e is the equipment capital stock.

Tab. 6.3: Remaining parameters

	Value	Obtained from
σ_s	1.425	Labor share with and without software in 2010
σ_e	0.974	
ν_e	0.772	Estimation (table 6.1)
ν_s	0.527	
ϵ	0.301	Changes in average wage by occupation

ated from the model with endogenous innovation (varying N_e and N_s) and without innovation (no changes in N_e and N_s), respectively.

The blue bar (changes during 1980-2000) is higher than the light blue bar (changes during 2000-2010) for cognitive occupations and lower for manual occupations, as highlighted in section 2. As shown in Figures 6.2b and 6.2c, these changes in the pattern appear only in the simulation with endogenous innovation, i.e., with increases in N_s/N_e. The increase in cognitive occupation during the last decade in the data is 0.31%p lower than the average of the first two decades, whereas in the model, it is 0.26%p lower with endogenous innovation and only 0.05%p lower without endogenous innovation. For manual occupations, the change in the increases between 2000-2010 and 1980-2000 is +0.26%p in the data and +0.22%p in the full model. By contrast, the model without innovation shows a change of +0.01%p only.

Figure 6.3 shows a decadal pattern from 1980 to 2010. The deviation from the initial trend in cognitive occupation in the model captures 75% of the actual deviation in the data (figure 6.3a and 6.3b), where the deviation from the initial trend in manual employment in the model is 70% of that in the data (figure 6.3c and 6.4a). The model captures not only the magnitude but also the timing of changes in the trends, as it produces much larger changes during 2000-2010 than during the first two decades. Without endogenous innovation, the simulation generates almost no variation in the trends of high- and low-skill employment.

The Rise of Software The ratio of software investment to industrial equipment investment increases from 0.16 to 1.7 in the data, a more than tenfold increase. Since we match the initial level of relative investment 0.16 exactly by calibration, we compare the level of the ratio in 2010 to determine how well the model explains the rise of software. The full model with innovation explains 63% of the rise of software investment relative to that of equipment (figure 6.4a). If we remove the endogenous innovation channel (i.e., no changes in N_s and N_e), the model generates only 19% of the change in the software to equipment ratio (green dashed line).

In figure 6.4b, we plot the ratio of software variety to equipment variety (N_s/N_e). There is no clear counterpart for the varieties in the data as we do not have data of R&D on equipment. As a crude measure, we compare the varieties N_e/N_s to the cumulated software development and the cumulated R&D funded by manufacturing industries, excluding chemicals. Both show an increasing pattern, and the ratio between varieties in the model increases faster in the last decade.

The Decline of Labor Share Although the labor share dynamics are not a goal of this exercise, they merit further discussion. Note that we use labor share trend as a target variable to calibrate the elasticity of substitution (σ_e and σ_s); therefore, it is not surprising that the labor share in the model exactly matches the labor share trend in the data. What is new is that the simulation without endogenous software innovation produces an almost flat labor share (figure 6.5b).

This occurs because an elasticity of substitution between equipment and labor (σ_e) is close to one, and hence exogenous variation does not generate declining labor share without software innovation. Therefore, the declining labor share in our model is mostly a result of endogenously increasing software investment. We highlight a negative correlation between software investment and labor share not only in the time series (figure 6.5a) but also in the industrial variation, especially during 2000-2010 (figures 6.5c and 6.5d). We believe that a detailed investigation of the relation between labor share and software capital is meaningful future research.

6.2.2 Changes in A_i's only with constant M_j's

Recall that we the use exogenous variation in the middle-skill specific technical change (changes in M_j) in addition to the evolution of the productivities of capital to match the changes in the employment share of the middle-skill occupations exactly. The natural question is how much of the observed changes in the productivities alone explain the variation in the share of employment by occupation.

The results are shown in figure 6.6. The changes in equipment price explain 78%, 75%, and 69% of the changes in cognitive, routine and low-skilled services employment. Two characteristics are noteworthy.

First, *all* occupational groups move in the same direction as the data, meaning that the differential growth of sectoral productivities – together with differences in the use of capital – captures routine-biased technical change quite well. The analysis suggests that a differential productivity growth on the sector level (among capital-producing sectors) could be an underlying source of routine biased technological change.

Second, the decadal pattern of changes in occupational employment is also similar to that of the data, even without additional task-specific technical change (figure 6.6a and 6.6b). Moreover, changes in TFP generates 75% of job polarization (declines in routine occupations) in magnitude. We conclude that the evolution of the productivities embodied in equipment and software has been crucial in generating a pattern consistent with the data.

The analysis suggests that two further studies would be helpful in understanding the changes in the occupational structure caused by capital-embodied technical change. The first is to look at heterogeneity in sector-level production more closely. The second is an attempt to obtain a better productivity measure for various capital items.

6.3 Sensitivity

We assess how the results vary by the elasticity of substitution between tasks (ϵ), markups (ν_e and ν_s), and measures for A's.

The Elasticity of Substitution between Tasks Regarding the elasticity of substitution between tasks, we expect the model's explanatory power to increase as ϵ decreases as the model mechanism is amplified when the tasks are more complementary. Table 6.4 confirms this intuition.

Markups As can be seen in figure 6.7, the price-to-cost margins of equipment- and software-producing industries exhibit different trends. The changes in market structure also affect innovational incentives. To examine the importance of the time-varying markups, we map the variations in the price-to-cost margin into changes in ν_e and ν_s. With time-varying markups, explanation for the pattern of cognitive employment share decreases and that of manual employment share increases.

Alternative Measures for A_e and A_s Another way to measure the capital-embodied productivity is to compute the inverse of the price of equipment and software. We compare the simulation results with the inverse of the price of equipment and software as

Tab. 6.4: Sensitivity

The elasticity of substitution between tasks (ϵ)

		Data	Innov.		No innov.	
Cognitive (dev. from trend)	benchmark	-4.16	-3.11	(.75)	-0.61	(.15)
	$\epsilon = .1$	-4.16	-3.63	(.87)	-0.60	(.14)
	$\epsilon = .5$	-4.16	-2.97	(.72)	-0.64	(.15)
	$\epsilon = .7$	-4.16	-2.57	(.62)	-0.63	(.15)
Low skilled (dev. from trend)	benchmark	3.50	2.45	(.70)	-0.05	(-.01)
	$\epsilon = .1$	3.50	2.97	(.85)	-0.06	(-.02)
	$\epsilon = .5$	3.50	2.31	(.66)	-0.02	(-.01)
	$\epsilon = .7$	3.50	1.90	(.54)	-0.03	(-.01)
Soft/eqp (lev. in 2010)	benchmark	1.68	1.06	(.63)	0.32	(.19)
	$\epsilon = .1$	1.68	1.08	(.65)	0.31	(.19)
	$\epsilon = .5$	1.68	1.21	(.72)	0.34	(.21)
	$\epsilon = .7$	1.68	1.30	(.78)	0.37	(.22)

Markup-related parameters (ν_e and ν_s)

		Data	Innov.		No innov.	
Cognitive (dev. from trend)	benchmark	-4.16	-3.11	(.75)	-0.61	(.15)
	time-varying	-4.16	-2.72	(.65)	-0.55	(.13)
Low skilled (dev. from trend)	benchmark	3.50	2.45	(.70)	-0.05	(-.01)
	time-varying	3.50	2.64	(.76)	-0.44	(-.13)
Soft/eqp (lev. in 2010)	benchmark	1.68	1.06	(.63)	0.32	(.19)
	time-varying	1.68	0.97	(.58)	0.33	(.20)

Alternative measure for A_e and A_s

		Data	Innov.		No innov.	
Cognitive (dev. from trend)	Ind eqp	-4.16	-2.05	(.49)	-0.39	(.09)
	Total eqp	-4.16	-1.75	(.42)	0.33	(-.08)
	Ind+IT	-4.16	-2.24	(.54)	-0.11	(.03)
Low skilled (dev. from trend)	Ind eqp	3.50	1.39	(.40)	-0.27	(-.08)
	Total eqp	3.50	1.09	(.31)	-0.99	(-.28)
	Ind+IT	3.50	1.57	(.45)	-0.55	(-.16)
Soft/eqp (lev. in 2010)	Ind eqp	1.68	1.06	(.63)	0.42	(.25)
	Total eqp	0.35	0.63	(1.83)	0.12	(.35)
	Ind+IT	0.59	0.76	(1.27)	0.19	(.33)

A_e and A_s. The sensitivity analysis shows that the price series give a bit lower explanatory power for changes in employment share but higher explanation for the increase in software investment than a case with the benchmark.

7 Conclusion

We provided a model with heterogenous tasks and two types of capital whose varieties are determined endogenously through a firm's innovation. We showed both analytically and quantitatively that the mechanism in the model is important in understanding the impact of capital-augmented technical change on the structure of the labor market.

One important implication is that two types of capital – software and equipment– measured in National Accounts, provide a good proxy for recent technological changes. Understanding the impact of a technical change on the economy has always been an important topic. One of the main difficulties is that technological change is not easy to measure, especially in aggregate analyses. This paper shows that the investigation of different types of capital can be a meaningful process to capture recent technological changes.

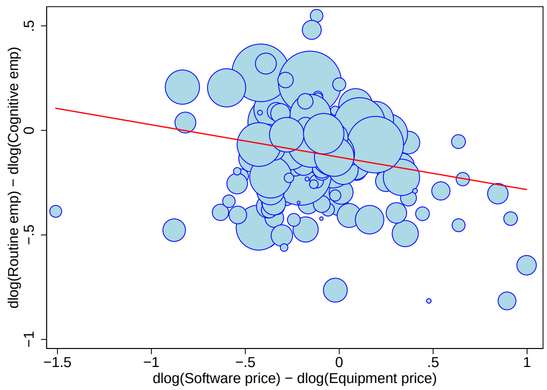
Our paper also implies that a technological change affecting a small group of occupations leads to other types of innovation, eventually affecting a broader set of occupations. Note that the same intuition applies to sectoral technical changes. This paper analyzes the technical change in the context of task-biased technological change, but a task-biased technical change is strongly linked to a sector-biased technical change, as emphasized in Lee and Shin (2017) and Bárány and Siegel (forthcoming).

Our model has many useful extensions that can be implemented easily. For example, further decomposition of equipment capital into subcategories will be helpful in understanding more detailed changes in occupational structure through technological changes embodied in capital. Further, integrating a multi-sector structure would provide additional interesting implications with respect to the relation between polarization and structural changes and the evolution of task-specific and sector-specific productivity, as in Aum et al. (2017).

Though not as straightforward, the analysis herein could also lead to many interesting future research topics. For example, by using firm-level software and equipment investment data, we may generate interesting implications on the impact of technological change on firm-level heterogeneity and occupation-level heterogeneity. Many countries are attempting to broaden the types of capital measured in National Accounts (System of National Account 2008). A multi-country extension would also be meaningful, enabling the analysis of trade or offshoring in addition to technological changes.

Fig. 5.1: Changes in the relative price and employment / innovation

(a) Relative employment vs Relative price



(b) Relative innovation vs Relative price

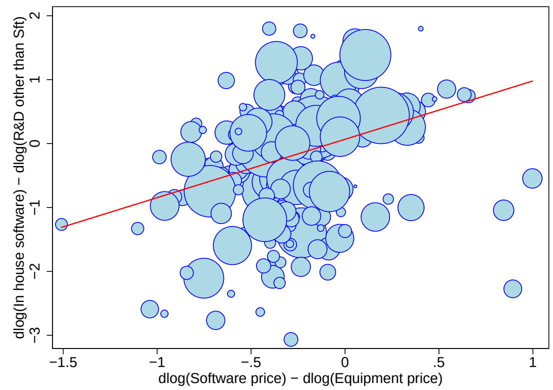
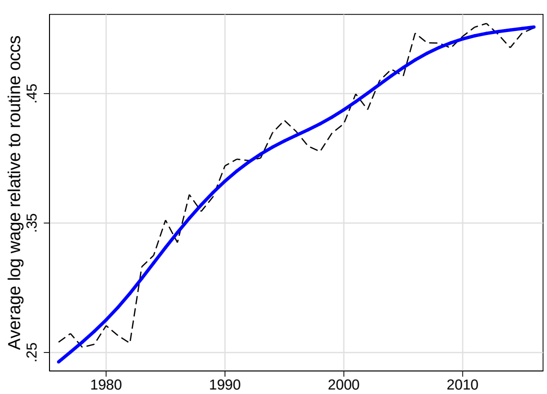


Fig. 6.1: Log of relative wage

(a) Cognitive to Routine



(b) Manual to Routine

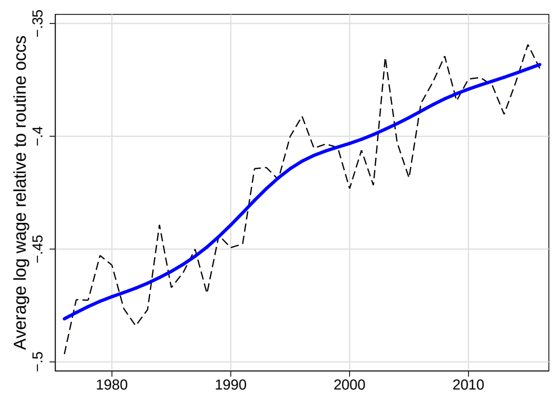


Fig. 6.2: Simulation results – changes in employment shares

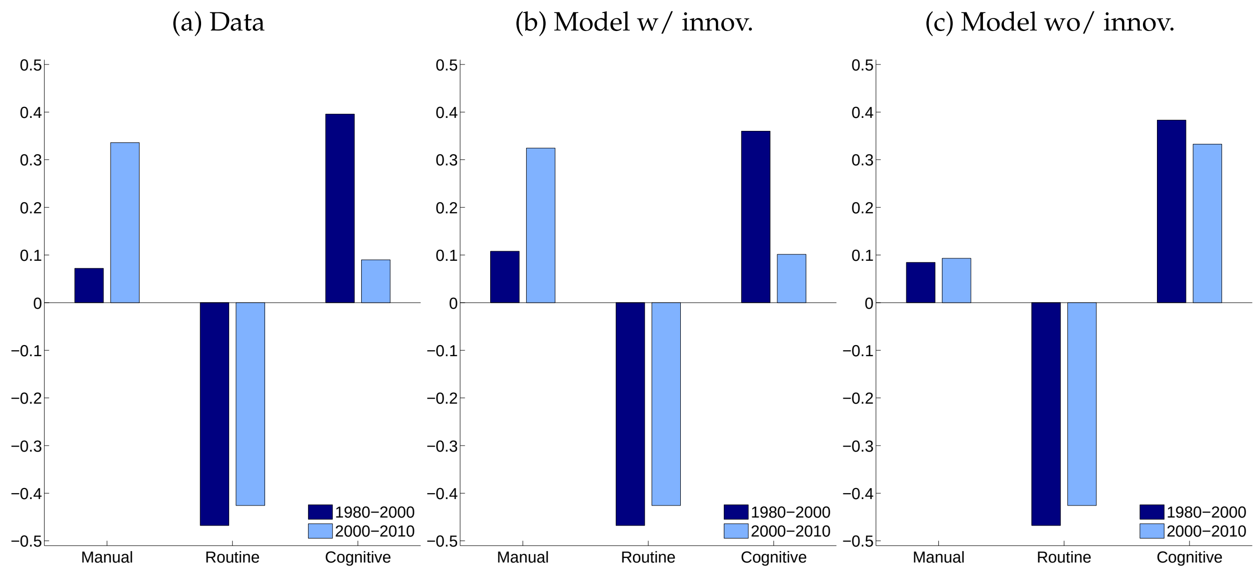


Fig. 6.3: Simulation results – employment shares by decade

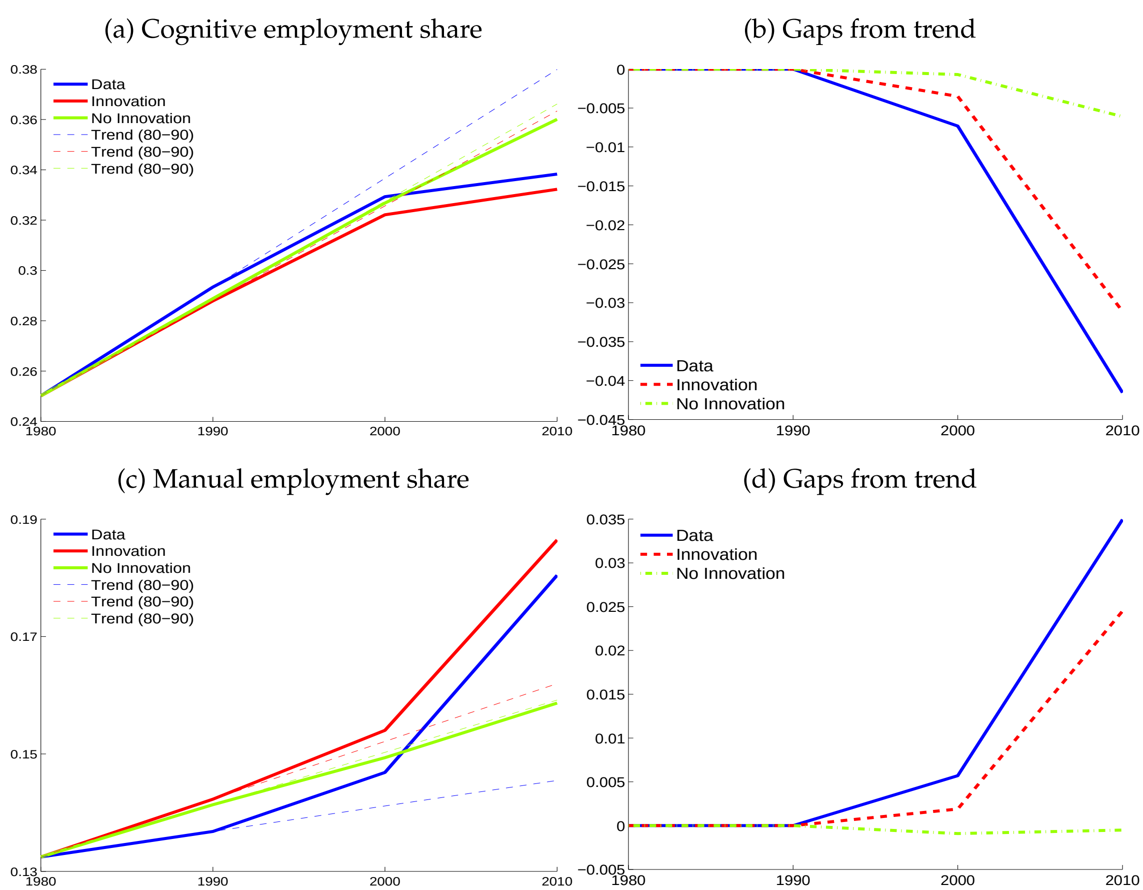
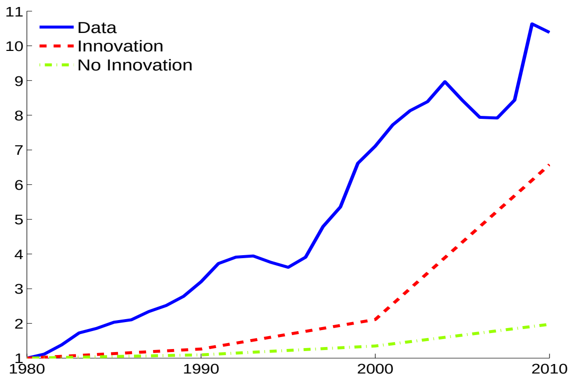


Fig. 6.4: Simulation results – relative investment and labor share

(a) Software/Equipment investment



(b) Software/Equipment varieties

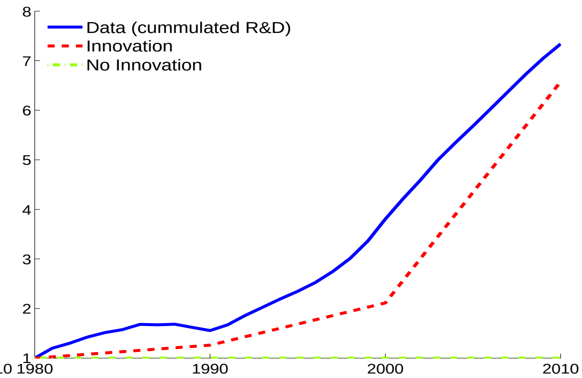
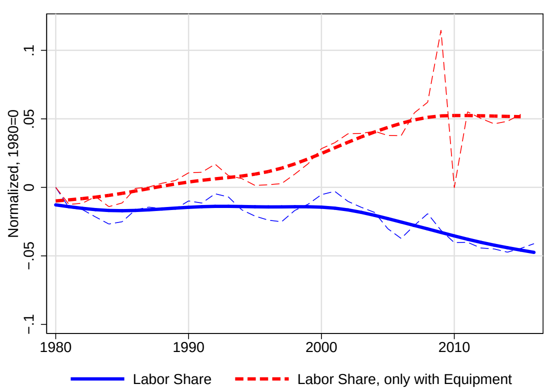
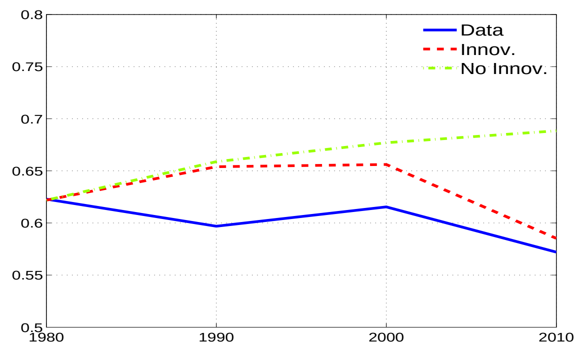


Fig. 6.5: Labor share and software

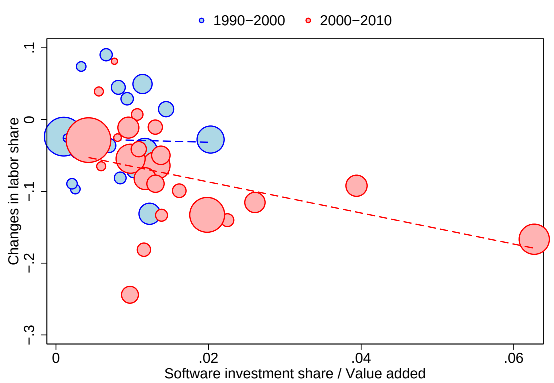
(a) Labor share trend[1)]



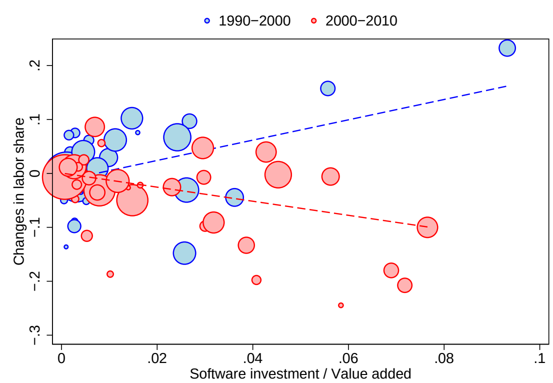
(b) Labor share simulation



(c) By industry – Manufacturing



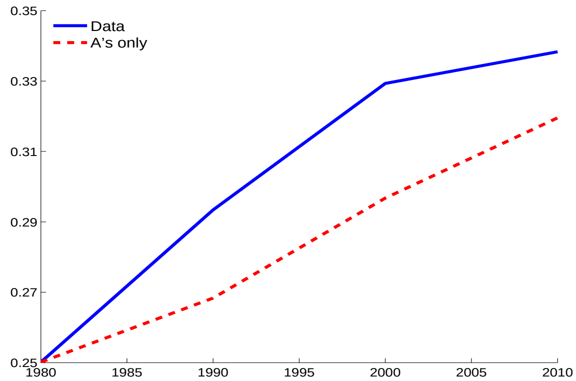
(d) By industry – Services[2)]



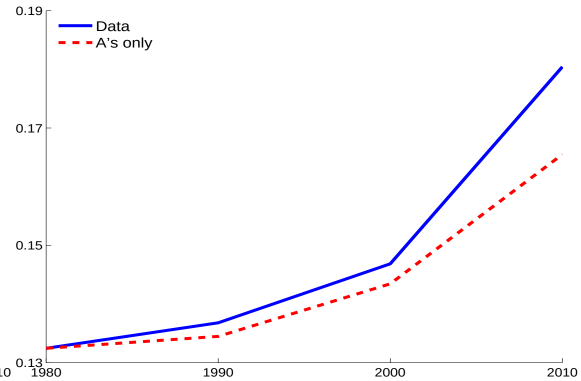
Note: 1) The labor share only with equipment capital is constructed following Koh et al. (2016). The solid lines are HP trend with smoothing parameter 100. They are normalized to 0 in 1980. 2) Industry 514 (with changes in labor share greater than 1 in both periods) has been excluded from this figure.

Fig. 6.6: Simulation results: no task-specific technological changes (constant M_j's)

(a) Cognitive employment



(b) Manual employment



(c) By occupation, 1980-2010

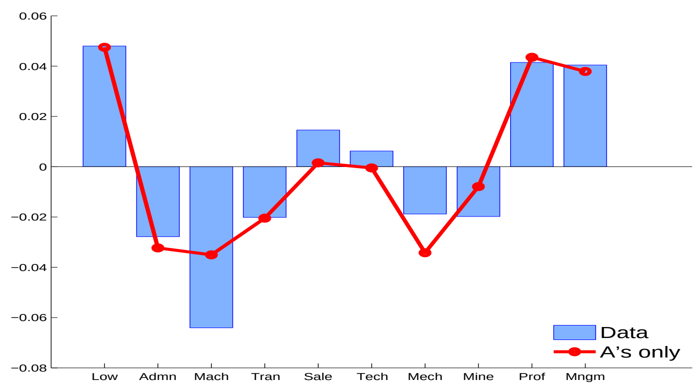
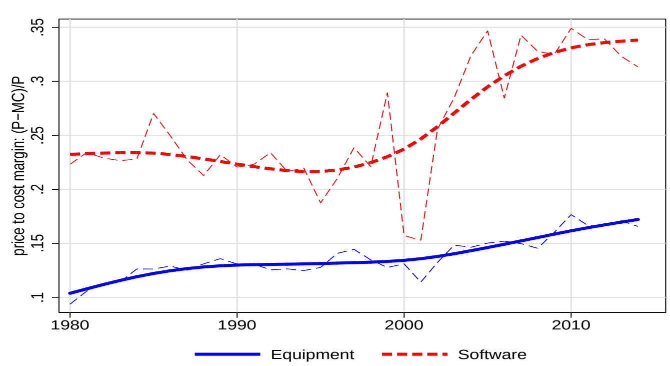


Fig. 6.7: Price-to-cost margin[1)] of the equipment- and software-producing industries[2)]



Notes: 1) (Gross output-intermediates-compensation of employees)/gross output. 2) The equipment-producing industries are 331, 332, 333, 334, 335, 3361MV, 3364TO, 337, and 339. The software-producing industry is 511.

References

Acemoglu, D. (2002). Directed technical change. *The Review of Economic Studies 69*(4), 781–809.

Acemoglu, D. and D. Autor (2011). Chapter 12 - skills, tasks and technologies: Implications for employment and earnings. Volume 4, Part B of *Handbook of Labor Economics*, pp. 1043 – 1171. Elsevier.

Acemoglu, D. and V. Guerrieri (2008, 06). Capital Deepening and Nonbalanced Economic Growth. *Journal of Political Economy 116*(3), 467–498.

Acemoglu, D. and P. Restrepo (2016, May). The Race Between Machine and Man: Implications of Technology for Growth, Factor Shares and Employment. NBER Working Papers 22252, National Bureau of Economic Research, Inc.

Aum, S., S. Y. T. Lee, and Y. Shin (2017, October). Computerizing industries and routinizing jobs: Explaining trends in aggregate productivity. Working papers.

Autor, D. H. and D. Dorn (2013, August). The Growth of Low-Skill Service Jobs and the Polarization of the US Labor Market. *American Economic Review 103*(5), 1553–97.

Autor, D. H., L. F. Katz, and M. S. Kearney (2006). The polarization of the U.S. labor market. *American Economic Review 96*(2), 189–194.

Bárány, Z. L. and C. Siegel (forthcoming). Job polarization and structural change. *American Economic Journal: Macroeconomics*.

Beaudry, P., D. A. Green, and B. M. Sand (2016). The great reversal in the demand for skill and cognitive tasks. *Journal of Labor Economics 34*(S1), S199–S247.

Cheng, W.-J. (2017, December). Explaining Job Polarization: The Role of Heterogeneity in Capital Intensity. IEAS Working Paper : academic research 17-A015, Institute of Economics, Academia Sinica, Taipei, Taiwan.

Costinot, A. and J. Vogel (2010, 08). Matching and Inequality in the World Economy. *Journal of Political Economy 118*(4), 747–786.

Crawford, M. J., J. Lee, J. E. Jankowski, and F. A. Moris (2014). Measuring R&D in the national economic accounting system. Survey of Current Business 11, Bureau of Economic Analysis.

Domowitz, I., R. G. Hubbard, and B. C. Petersen (1988, February). Market Structure and Cyclical Fluctuations in U.S. Manufacturing. *The Review of Economics and Statistics 70*(1), 55–66.

Goos, M., A. Manning, and A. Salomons (2014). Explaining job polarization: Routine-biased technological change and offshoring. *American Economic Review 104*(8), 2509–26.

Grossman, G. M., E. Helpman, E. Oberfield, and T. Sampson (2017, April). Balanced growth despite Uzawa. *American Economic Review 107*(4), 1293–1312.

Hémous, D. and M. Olsen (2016). The rise of the machines: Automation, horizontal innovation and income inequality. https://ssrn.com/abstract=2328774.

Jones, C. I. (1995, August). R&D-Based Models of Economic Growth. *Journal of Political Economy 103*(4), 759–784.

Koh, D., R. Santaeulàlia-Llopis, and Y. Zheng (2016, September). Labor Share Decline and Intellectual Property Products Capital. Working Papers 927, Barcelona Graduate School of Economics.

Krusell, P., L. E. Ohanian, J.-V. Ríos-Rull, and G. L. Violante (2000). Capital-skill complementarity and inequality: A macroeconomic analysis. *Econometrica 68*(5), 1029–1053.

Lee, S. Y. T. and Y. Shin (2017). Horizontal and vertical polarization: Task-specific technological change in a multi-sector economy. http://lee.vwl.uni-mannheim.de/materials/hvpol.pdf.

Michelacci, C. and J. Pijoan-Mas (2016). Labor supply with job assignment under balanced growth. *Journal of Economic Theory 163*(C), 110–140.

Ngai, L. R. and C. A. Pissarides (2007, March). Structural Change in a Multisector Model of Growth. *American Economic Review 97*(1), 429–443.

Stokey, N. L. (2016, April). Technology, skill and the wage structure. Working Paper 22176, National Bureau of Economic Research.

Valletta, R. G. (2016, December). Recent flattening in the higher education wage premium: Polarization, skill downgrading, or both? Working Paper 22935, National Bureau of Economic Research.

Appendices

A Use of Equipment and Software by Occupation

The capital use by occupation data is constructed by combining BEA NIPA and O*NET Tools and Technology Database. In NIPA table 5.5, the investment on non-residential equipment are categorized by 25 types. In UNSPSC, the classification system used in O*NET Tools and Technology database, there are 4,300 commodities, which are in 825 classes, in 173 families, and in 36 segments.

To construct a mapping between two, we firstly assign one of NIPA investment types to the relevant segment in UNSPSC. Often, it is apparent that a segment includes several types of equipment investment in NIPA. In this case, we use the family categories in the assignment procedure. Again, if a family apparently includes several types in NIPA, we use classes. Through this procedure, we could make a rough concordance between a subset of UNSPSC and the types of equipment investment in NIPA. The constructed concordance is shown in table A1.

Next, we assume that two tools have same price if they are classified in the same category. For example, the "metal cutting machines" in UNSPSC is assigned to "metalworking machinery" in NIPA investment type. The value of using the metal cutting machines are then the amount of investment in metalworking machinery divided by total use of all the commodities in the metalworking machinery category, where the total use of all the tools in the metalworking machinery is defined as sum of a number of total employment of each occupation times a number of UNSPSC commodities assigned to the metalworking machinery that each occupation uses.

The method is assuming that the number of tools above well represent the value of them, only within the NIPA investment category. Across the NIPA investment categories, each number of tools used would get different weights, according to the average amount of investment given to each tool. The procedure may make a big difference from average number of tools if a category with many commodities had small values compared to a category with few commodities. However, as more differentiated categories are usually advanced (and hence have expensive items), we expect not much difference from the adjustment.

Tab. A1: Concordance between NIPA equipment investment types and UNSPSC

NIPA		UNSPSC	
Line	Title	Code	Title
3	**Information processing equipment**		
4	Computers and peripheral equipement	43210000	Computer Equipment and Accessories
5	Communication equipment	43190000, 45110000	Communications Devices and Accessories, Audio and visual presentation and composing equipment
6	Medical equipment and instruments	42000000	Medical Equipment and Accessories and Supplies
9	Nonmedical instruments	41000000	Laboratory and Measuring and Observing and Testing Equipment
10	Photocopy and related equipment	45100000, 45120000	Printing and publishing equipment, Photographic or filming or video equipment
11	Office and accounting equipment	44100000, 31240000	Office machines and their supplies and accessories, Industrial optics
12	**Industrial equipment**		
13	Fabricated metal products	27000000, 31150000, 31160000, 31170000, 40140000, 40170000	Tools and General Machinery, Rope and chain and cable and wire and strap, Hardware, Bearings and bushings and wheels and gears, Fluid and gas distribution, Pipe piping and pipe fittings
14	Engines and turbines	26101500, 26101700	Engines, Engine components and accessories
17	Metalworking machinery	23240000, 23250000, 23260000, 23270000, 23280000	Metal cutting machinery and accessories, Metal forming machinery and accessories, Rapid prototyping machinery and accessories, Welding and soldering and brazing machinery and accessories and supplies, Metal treatment machinery
18 + 19	Special industry machinery, n.e.c. + General industrial, including materials handling, equipment	23100000, 23110000, 23120000, 23130000, 23140000, 23150000, 23160000, 23180000, 23190000, 23200000, 23210000, 23220000, 23230000, 23290000, 24100000, 24110000, 31140000, 40000000	Raw materials processing machinery, Petroleum processing machinery, Textile and fabric machinery and accessories, Lapidary machinery and equipment, Leatherworking repairing machinery and equipment, Industrial process machinery and equipment and supplies, Foundry machines and equipment and supplies, Industrial food and beverage equipment, Mixers and their parts and accessories, Mass transfer equipment, Electronic manufacturing machinery and equipment and accessories, Chicken processing machinery and equipment, Sawmilling and lumber processing machinery and equipment, Industrial machine tools, Material handling machinery and equipment, Containers and storage, Moldings, Distribution and Conditioning Systems and Equipment and Components
20 + 41	Electrical transmission, distribution, and industrial apparatus + Electrical equipment, n.e.c.	26101100, 26101200, 26101300, 26110000, 26120000, 26130000, 26140000, 39000000	Electric alternating current AC motors, Electric direct current DC motors, Non electric motors, Batteries and generators and kinetic power transmission, Electrical wire and cable and harness, Power generation, Atomic and nuclear energy machinery and equipment, Electrical Systems and Lighting and Components and Accessories and Supplies
21	**Transportation equipment**		
22 + 25	Trucks, buses, and truck trailers + Autos	25100000	Motor vehicles
26	Aircraft	25130000	Aircraft
27	Ships and boats	25110000	Marine transport
28	Railroad equipment	25120000	Railway and tramway machinery and equipment
29	**Other equipment**		
30	Furniture and fixtures	56000000	Furniture and Furnishings
33	Agricultural machinery	21000000	Farming and Fishing and Forestry and Wildlife Machinery and Accessories
36	Construction machinery	22000000	Building and Construction Machinery and Accessories
39	Mining and oilfield machinery	20000000	Mining and Well Drilling Machinery and Accessories
40	Service industry machinery	48000000	Service Industry Machinery and Equipment and Supplies

B Discrete Approximation of the Model

This section discusses equilibrium conditions with discrete approximation of the model. For the approximation, assumption 1 and 4 are replaced by assumption 5 and 6 in section 3 and 4.

The task production is given by equation (19) with tasks discretized into $j = 0, 1, \cdots, J$. Now the tasks are discrete, so workers are sorted into each task according to cut-off level of human capital $\hat{h}_j$. More preciesely, we have a sequence of human capital $\{\hat{h}_j\}_{j=0,\cdots,J+1}$ such that a worker with $h \in [\hat{h}_j, \hat{h}_{j+1})$ are sorted into task j with $\hat{h}_0 = \underline{h}$ and $\hat{h}_{J+1} = \bar{h}$.

For a worker with exactly the threshold level of human capital should be indifferent between tasks so that

$$\omega_j b(\hat{h}_j, j) = \omega_{j-1} b(\hat{h}_j, j-1), \text{ for all } j, \text{ for }, j = 1, \cdots, J \tag{B.1}$$

replacing the original equilibrium condition (9).

The task production is solving

$$\max p_j T_j - \int_h w(h) l(h) dh - \int_{k=0}^{N_e} p_e(k) e(k) dk - \int_{k=0}^{N_s} p_s(k) s(k) dk,$$

which gives the FOCs,

$$\begin{aligned}
w(h) = \omega_j b(h,j) &= p_j T_j^{\frac{1}{\sigma_s}} H_j^{\frac{1}{\sigma_e}-\frac{1}{\sigma_s}} \left(\int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h,j)\mu(h) dh \right)^{-\frac{1}{\sigma_e}} b(h,j), \\
\frac{1}{A_e \nu_e} &= p_j T_j^{\frac{1}{\sigma_s}} H_j^{\frac{1}{\sigma_e}-\frac{1}{\sigma_s}} (N_e)^{\frac{\sigma_e-1}{\sigma_e \nu_e}-1} e_j^{-\frac{1}{\sigma_e}}, \\
\frac{1}{A_s \nu_s} &= p_j T_j^{\frac{1}{\sigma_s}} (N_s)^{\frac{\sigma_s-1}{\sigma_s \nu_s}-1} s_j^{-\frac{1}{\sigma_s}},
\end{aligned}$$

using the fact that $p_e = 1/(A_e \nu_e)$, $p_s = 1/(A_s \nu_s)$, $e_j(k) = e_j$, and $s_j(k) = s_j$ in equilibrium, and $H_j := \left[\alpha_{h,j} (\int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h,j)\mu(h) dh)^{\frac{\sigma_e-1}{\sigma_e}} + \alpha_{e,j} (\int_{k=0}^{N_e} e(k)^{\nu_e} dk)^{\frac{\sigma_e-1}{\sigma_e \nu_e}} \right]^{\frac{\sigma_e}{\sigma_e-1}}$.

Combining the FOCs, we get

$$p_j = \left[\left(\alpha_{h,j}^{\sigma_e} \omega_j^{1-\sigma_e} + \alpha_{e,j}^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e} \right)^{\sigma_e - 1} \right)^{\frac{1-\sigma_s}{1-\sigma_e}} + \alpha_{s,j}^{\sigma_s} \left(\nu_s A_s N_s^{\varphi_s} \right)^{\sigma_s-1} \right]^{\frac{1}{1-\sigma_s}}, \text{ for } j = 0, \cdots, J \tag{B.2}$$

which replaces equation (11).

The demand for each task is from

$$\max Y - \sum_j p_j T_j,$$

which gives

$$p_j = \left(\frac{\gamma_j Y}{T_j}\right)^{\frac{1}{\epsilon}}.$$

Combining this with FOCs, we obtain

$$p_j^{\epsilon-\sigma_s} = \frac{\gamma_j \alpha_{h,j}^{\sigma_s}\left(\alpha_{h,j}^{\sigma_e}\omega_j^{1-\sigma_e} + \alpha_{e,j}^{\sigma_e}\left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1}\right)^{\frac{\sigma_e-\sigma_s}{1-\sigma_e}} Y}{\omega_j^{\sigma_e}\int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h,j)\mu(h)dh}, \text{ for } j = 0, \cdots, J, \tag{B.3}$$

replacing equation (10).

Now the equilibrium thresholds $\hat{h}_j$'s, wage rate ω_j's and prices p_j's are obtained by solving equation (B.1) to (B.3), which are $3J+1$ equations with the same number of unknowns.

C Proof

***Proof of lemma* 2** *Since assignment function $\hat{h}(\tau)$ is strictly increasing, its inverse $\hat{\tau}(h)$ is well-defined. From the demand for task, equation (7), we know that there will be strictly positive task output $T(\tau) > 0$ (and hence $l(h, \hat{\tau}(h)) > 0$) for all $\tau \in [0, \bar{\tau}]$. The equation (8) and lemma 1 then implies*

$$w(h) = \omega(\hat{\tau}(h))b(h, \hat{\tau}(h)) \geq \omega(\hat{\tau}(h'))b(h, \hat{\tau}(h')), \text{ and}$$
$$w(h') = \omega(\hat{\tau}(h'))b(h', \hat{\tau}(h')) \geq \omega(\hat{\tau}(h))b(h', \hat{\tau}(h)).$$

Combining these two inequalities, we have

$$\frac{b(h, \hat{\tau}(h'))}{b(h, \hat{\tau}(h))} \leq \frac{\omega(\hat{\tau}(h))}{\omega(\hat{\tau}(h'))} \leq \frac{b(h', \hat{\tau}(h'))}{b(h', \hat{\tau}(h))}$$

Let $\tau = \hat{\tau}(h)$ and $\tau' = \hat{\tau}(h')$. Since $\hat{\tau}$ has an inverse function $\hat{h}$, above inequality is equivalent to

$$\frac{b(\hat{h}(\tau),\tau')}{b(\hat{h}(\tau),\tau)} \leq \frac{\omega(\tau)}{\omega(\tau')} \leq \frac{b(\hat{h}(\tau'),\tau')}{b(\hat{h}(\tau'),\tau)}$$

By taking log on both sides and dividing by $\tau' - \tau$,

$$\frac{\log b(\hat{h}(\tau),\tau') - \log b(\hat{h}(\tau),\tau)}{\tau' - \tau} \leq \frac{-(\log \omega(\tau') - \log \omega(\tau))}{\tau' - \tau} \leq \frac{\log b(\hat{h}(\tau'),\tau') - \log b(\hat{h}(\tau'),\tau)}{\tau' - \tau}$$

As $\tau' - \tau \to 0$, we have

$$\frac{d \log \omega(\tau)}{d\tau} = -\frac{\partial \log b(\hat{h}(\tau),\tau)}{\partial \tau},$$

which is the equation (9).

Now consider the task production. For notational convenience, we introduce

$$H(h,\tau) = \left[\alpha_h(\tau)(b(h,\tau)l(h))^{\frac{\sigma_e-1}{\sigma_e}} + \alpha_e(\tau)\left(\int_0^{N_e} e(k,\tau)^{\nu_e} dk\right)^{\frac{\sigma_e-1}{\sigma_e \nu_e}}\right]^{\frac{\sigma_e}{\sigma_e-1}}$$

From

$$\max \ p(\tau)T(\tau) - \int_h w(h)l(h,\tau)dh - \int_0^{N_s} p_s(k)s(k,\tau)dk - \int_0^{N_e} p_e(k)e(k,\tau)dk,$$

we have the following first order conditions:

$$w(h) \geq \alpha_h(\tau)p(\tau)T(\tau)^{\frac{1}{\sigma_s}} H(h,\tau)^{\frac{\sigma_s-\sigma_e}{\sigma_e\sigma_s}} l(h)^{-\frac{1}{\sigma_e}} b(h,\tau), \tag{C.1}$$

$$p_e(k) = \alpha_e(\tau)p(\tau)T(\tau)^{\frac{1}{\sigma_s}} H(h,\tau)^{\frac{\sigma_s-\sigma_e}{\sigma_e\sigma_s}} \left(\int_0^{N_e} e(k,\tau)^{\nu_e}\right)^{\frac{\sigma_e-1-\nu_e\sigma_e}{\nu_e\sigma_e}} e(k,\tau)^{\nu_e-1}, \tag{C.2}$$

$$p_s(k) = \alpha_s(\tau)p(\tau)T(\tau)^{\frac{1}{\sigma_s}} \int_0^{N_s} s(k,\tau)^{\nu_s} dk^{\frac{\sigma_s-1-\nu_s\sigma_s}{\nu_s\sigma_s}} s(k,\tau)^{\nu_s-1}, \tag{C.3}$$

In equipment- and software-producing sector, we solve

$$\max p_e(k)e(k) - e(k)/A_e, \quad \max p_s(k)s(k) - s(k)/A_s$$

subject to (C.2) and (C.3). The solution gives

$$p_e = 1/(\nu_e A_e), \quad p_s = 1/(\nu_s A_s) \quad \textit{for all } k. \tag{C.4}$$

Substituting (C.4) into the FOCs, we get

$$p(\tau) = \left[\left\{ \alpha_h(\tau)^{\sigma_e} \omega(\tau)^{1-\sigma_e} + \alpha_e(\tau)^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e} \right)^{\sigma_e - 1} \right\}^{\frac{1-\sigma_s}{1-\sigma_e}} + \alpha_s(\tau)^{\sigma_s} \left(\nu_s A_s N_s^{\varphi_s} \right)^{\sigma_s - 1} \right]^{\frac{1}{1-\sigma_s}},$$

by combining the FOCs, which is the equation (11).

Again from equation (C.1) to (C.3), the task production $T(\tau)$ can be expressed by

$$T(\tau) = p(\tau)^{-\sigma_s} \omega(\tau)^{\sigma_e} \alpha_h(\tau)^{-\sigma_e} \left(\alpha_h(\tau)^{\sigma_e} \omega(\tau)^{1-\sigma_e} + \alpha_e(\tau)^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e} \right)^{\sigma_e - 1} \right)^{\frac{\sigma_s - \sigma_e}{1-\sigma_e}} \int_h b(h,\tau) l(h,\tau) dh \tag{C.5}$$

From the labor market clearing condition and lemma 1, we have

$$l(h,\tau) = \mu(h) \delta[\tau - \hat{\tau}(h)],$$

where δ is a Dirac delta function. Then we have

$$\int_h b(h,\tau) l(h,\tau) dh = \int_{\tau'} b(\hat{h}(\tau'),\tau) \mu(\hat{h}(\tau)) \delta[\tau - \tau'] \hat{h}'(\tau') d\tau' = b(\hat{h}(\tau),\tau) \mu(\hat{h}(\tau)) \hat{h}'(\tau).$$

Combining this with equation (7) and (C.5), we have

$$\hat{h}'(\tau) = \frac{\gamma(\tau) p(\tau)^{\sigma_s - \epsilon} \alpha_h(\tau)^{\sigma_s} \left(\alpha_h(\tau)^{\sigma_e} \omega(\tau)^{1-\sigma_e} + \alpha_e(\tau)^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e} \right)^{\sigma_e - 1} \right)^{\frac{\sigma_e - \sigma_s}{1-\sigma_e}} Y}{\omega(\tau)^{\sigma_e} b(\hat{h}(\tau),\tau) \mu(\hat{h}(\tau))},$$

which is the equation (10). ∎

Proof of lemma 3 *In steady state, if it exists, $r = \pi_s/\eta_s = \pi_e/\eta_e = \rho$ from the Euler equation (17). Then $\dot{X}/X = 0$ for $X = C, E, S, N_e$, and N_s follow from usual argument. What we need to show is that there exist N_s and N_e that satisfy $\pi_s/\eta_s = \pi_e/\eta_e = \rho$.*

We start with the following lemma.

Lemma 4 *Fix $p(\tau)$ and $\hat{h}(\tau)$. There exists a pair $(\nu_s, \nu_e) \in (0,1) \times (0,1)$ such that $s(\tau)$ is strictly decreasing in N_s and $e(\tau)$ is strictly decreasing in N_e.*

Proof Combining equation (C.1) to (C.3) (FOCs), we have

$$s(\tau) = N_s^{-1} N_s^{\varphi_s(\sigma_s-1)} \left(\nu_s A_s\right)^{\sigma_s} \alpha_s(\tau)^{\sigma_s} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}} b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)$$
$$\times \left[\left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} \left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1} \right)^{\frac{1-\sigma_e}{1-\sigma_s}} - \alpha_e(\tau)^{\sigma_e} \left(\nu_e A_e (N_s n_{es})^{\varphi_e}\right)^{\sigma_e-1} \right]^{\frac{\sigma_e}{1-\sigma_e}}$$
$$\times \left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} \left(\nu_s A_s N_s^{\varphi_s}\right)^{\sigma_s-1} \right)^{\frac{\sigma_s-\sigma_e}{1-\sigma_s}}, \tag{C.6}$$

and

$$e(\tau) = N_e^{-1} N_e^{\varphi_e(\sigma_e-1)} \left(\nu_e A_e\right)^{\sigma_e} \alpha_e(\tau)^{\sigma_e} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}} b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)$$
$$\times \left[\left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} \left(\nu_s A_s (N_e/n_{es})^{\varphi_s}\right)^{\sigma_s-1} \right)^{\frac{1-\sigma_e}{1-\sigma_s}} - \alpha_e(\tau)^{\sigma_e} \left(\nu_e A_e N_e^{\varphi_e}\right)^{\sigma_e-1} \right]^{\frac{\sigma_e}{1-\sigma_e}}, \tag{C.7}$$

where $n_{es} := N_e/N_s$.

From equation (C.6) and (C.7), we can express

$$\frac{\partial \log s(\tau)}{\partial N_s} = -\frac{1}{N_s} + s_1(\tau;\varphi_s), \tag{C.8}$$
$$\frac{\partial \log e(\tau)}{\partial N_e} = -\frac{1}{N_e} + e_1(\tau;\varphi_e), \tag{C.9}$$

and it's straightforward to check that $\lim_{\varphi_s \downarrow 0} |s_1(\tau;\varphi_s)| = 0$, $\lim_{\varphi_e \downarrow 0} |e_1(\tau;\varphi_e)| = 0$, and $\partial s_1/\partial \varphi_s > 0$, $\partial e_1/\partial \varphi_e > 0$. This implies that there should be $0 < \nu_s < 1$ and $0 < \nu_e < 1$ which make $s(\tau)$ strictly decreasing in N_s and $e(\tau)$ strictly decreasing in N_e. ∎

Lemma 5 *Fix $p(\tau)$ and $\hat{h}(\tau)$. With ν_e and ν_s close to one, we have the following:*

$$\lim_{N_s \to 0} s(\tau) = \infty, \quad \lim_{N_e \to 0} e(\tau) = \infty, \quad \lim_{N_s \to \infty} s(\tau) = 0, \quad \lim_{N_e \to \infty} e(\tau) = 0.$$

Proof By substituting $\nu_e = 1$ and $\nu_s = 1$ (and hence $\varphi_e = \frac{1-\nu_e}{\nu_e} = 0$ and $\varphi_s = \frac{1-\nu_s}{\nu_s} = 0$) into equation (C.6) and (C.7), we have

$$s(\tau) = N_s^{-1} \left(\nu_s A_s\right)^{\sigma_s} \alpha_s(\tau)^{\sigma_s} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}} b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)$$
$$\times \left[\left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} \left(\nu_s A_s\right)^{\sigma_s-1} \right)^{\frac{1-\sigma_e}{1-\sigma_s}} - \alpha_e(\tau)^{\sigma_e} \left(\nu_e A_e\right)^{\sigma_e-1} \right]^{\frac{\sigma_e}{1-\sigma_e}}$$
$$\times \left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} \left(\nu_s A_s\right)^{\sigma_s-1} \right)^{\frac{\sigma_s-\sigma_e}{1-\sigma_s}}, \tag{C.10}$$

and

$$e(\tau) = N_e^{-1} \left(\nu_e A_e\right)^{\sigma_e} \alpha_e(\tau)^{\sigma_e} \alpha_h(\tau)^{-\frac{\sigma_e}{1-\sigma_e}} b(\hat{h}(\tau), \tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)$$
$$\times \left[\left(p(\tau)^{1-\sigma_s} - \alpha_s(\tau)^{\sigma_s} \left(\nu_s A_s\right)^{\sigma_s - 1} \right)^{\frac{1-\sigma_e}{\cdot}} 1 - \sigma_s - \alpha_e(\tau)^{\sigma_e} \left(\nu_e A_e\right)^{1-\sigma_e} \right]^{\frac{\sigma_e}{1-\sigma_e}} \quad \text{(C.11)}$$

The result is straightforward from equation (C.10) and (C.11). ∎

Since π_e and π_s are proportional to integration of $s(\tau)$ and $e(\tau)$, lemma 4 and 5 imply the existence of unique steady state under some ν_e and ν_s large enough, fixing static equilibrium.

Note that both $\hat{h}$ and $\mu(h)dh$ are bounded above by assumtion and boundary conditions, and $p(\tau)$ is also bounded as $\int_\tau \gamma(\tau)p(\tau)^{1-\epsilon}d\tau = 1$. Hence, the existence follows when π_e and π_s are continuous in N_e and N_s even when considering changes in static equilibrium. Recall that $p(\tau)$ and $\hat{h}(\tau)$ could be obtained from the system of differential equations (9) to (11). Since all functions in equation (9) to (11) are differentiable, π_e and π_s are also continuous in N_e and N_s and the desired result follows.

Intuitively, large ν_e and ν_s mean small returns to introducing additional variety, in turn, meaning decreasing rete of return. To see this intuition more clearly, recall that the task production function is given by

$$T(\tau) = \left[\left\{ \alpha_h(\tau) \left(b(\hat{h}(\tau), \tau)\mu(\hat{h}(\tau))\hat{h}'(\tau) \right)^{\frac{\sigma_e - 1}{\sigma_e}} + \alpha_e(\tau) N_e^{\frac{\sigma_e - 1}{\sigma_e \nu_e}} e(\tau)^{\frac{\sigma_e - 1}{\sigma_e}} \right\}^{\frac{\sigma_e(\sigma_s - 1)}{(\sigma_e - 1)\sigma_s}} \right.$$
$$\left. + \alpha_s(\tau) N_s^{\frac{\sigma_s - 1}{\sigma_s \nu_s}} s(\tau)^{\frac{\sigma_s - 1}{\sigma_s}} \right]^{\frac{\sigma_s}{\sigma_s - 1}}, \quad \text{(C.12)}$$

as $s(k, \tau) = s(\tau)$ and $e(k, \tau) = e(\tau)$ in equilibrium. The production is homogeneous of degree one in labor, N_e and N_s when $\nu_e \to 1$ and $\nu_s \to 1$. Since labor is fixed component, the production features strict concavity along N_e and N_s, meaning decreasing returns to scale in terms of total varieties.

The second part of lemma (3) is when $\sigma_e = \sigma_s = 1$. In this case,

$$p(\tau)T(\tau) = \frac{\omega(\tau)b(\hat{h}(\tau), \tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)}{\alpha_h(\tau)}, \quad \text{(C.13)}$$

$$s(\tau) = \frac{\nu_s A_s \alpha_s(\tau) p(\tau) T(\tau)}{N_s}, \quad \text{(C.14)}$$

$$e(\tau) = \frac{\nu_e A_e \alpha_e(\tau) p(\tau) T(\tau)}{N_e}. \quad \text{(C.15)}$$

Combining the FOCs, $T(\tau)$ satisfies

$$p(\tau)T(\tau) = p(\tau)^{\frac{1}{\alpha_h(\tau)}}\kappa(\tau)N_s^{\Psi_{es}(\tau)}\left(\frac{N_e}{N_s}\right)^{\Psi_e(\tau)}B(\tau), \tag{C.16}$$

where $\kappa(\tau) := (\alpha_s(\tau)\nu_s A_s)^{\frac{\alpha_s(\tau)}{\alpha_h(\tau)}}(\alpha_e(\tau)\nu_e A_e)^{\frac{\alpha_e(\tau)}{\alpha_h(\tau)}}$, $\Psi_{es}(\tau) := \frac{1-\nu_s}{\nu_s}\frac{\alpha_s(\tau)}{\alpha_h(\tau)} + \frac{1-\nu_e}{\nu_e}\frac{\alpha_e(\tau)}{\alpha_h(\tau)}$, $\Psi_e(\tau) := \frac{1-\nu_e}{\nu_e}\frac{\alpha_e(\tau)}{\alpha_h(\tau)}$, *and* $B(\tau) := b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\hat{h}'(\tau)$ *are introduced to simplify notation.*

From equation (C.14) and (C.15), it is apparent that $s(\tau)$ and $e(\tau)$ are dcreasing in N_s and N_e respectivley when $\Psi_{es}(\tau) < 1$, which is a condition given in lemma 3. ∎

Proof of proposition 1 (job polarization) *Substituting $p(\tau)$ out from equation (9) to (11), we have*

$$\hat{h}'(\tau) = \frac{\gamma(\tau)\alpha_h(\tau)^{1-\alpha_h(\tau)(1-\epsilon)}Y}{b(\hat{h}(\tau),\tau)\mu(\hat{h}(\tau))\omega(\tau)^{1-\alpha_h(\tau)(1-\epsilon)}}\times$$
$$\left[\left(\alpha_s(\tau)\nu_s A_s N_s^{(1-\nu_s)/\nu_s}\right)^{\alpha_s(\tau)}\left(\alpha_e(\tau)\nu_e A_e N_e^{(1-\nu_e)/\nu_e}\right)^{\alpha_e(\tau)}\right]^{\epsilon-1} \tag{C.17}$$

$$\frac{d\log\omega(\tau)}{d\tau} = -\frac{\partial\log b(\hat{h}(\tau),\tau)}{\partial\tau} \tag{C.18}$$

First, we show $\hat{h}_1$ and $\hat{h}_2$ has to cross at least once. Suppose there is no crossing. Since $\hat{h}_1(0) = \hat{h}_2(0)$ and $\hat{h}_1(\bar{\tau}) = \hat{h}_2(\bar{\tau})$, we have

$$\left(\frac{\omega_1(0)}{\omega_2(0)}\right)^{1-\alpha_h(0)(1-\epsilon)} = \frac{\hat{h}_2'(0)}{\hat{h}_1'(0)}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)\alpha_e(0)}, \tag{C.19}$$

$$\left(\frac{\omega_1(\bar{\tau})}{\omega_2(\bar{\tau})}\right)^{1-\alpha_h(\bar{\tau})(1-\epsilon)} = \frac{\hat{h}_2'(\bar{\tau})}{\hat{h}_1'(\bar{\tau})}\left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)\alpha_e(\bar{\tau})}, \tag{C.20}$$

from equation (C.17). Combining,

$$\left(\frac{\omega_1(\bar{\tau})/\omega_1(0)}{\omega_2(\bar{\tau})/\omega_2(0)}\right)^{1-\alpha_h(0)(1-\epsilon)}\left(\frac{\omega_1(\bar{\tau})}{\omega_2(\bar{\tau})}\right)^{(\alpha_h(0)-\alpha_h(\bar{\tau}))(1-\epsilon)} = \frac{\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0)}{\hat{h}_1'(\bar{\tau})/\hat{h}1'(0)} \tag{C.21}$$

Since $\hat{h}(\tau)$ is strictly monotone and continous, with no crossing on entire $(0,\bar{\tau})$, we have to have either (i) $\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0) < \hat{h}_1'(\bar{\tau})/\hat{h}_1'(0)$ and $\hat{h}_1(\tau) < \hat{h}_2(\tau)$ for $\tau \in (0,\bar{\tau})$, or (ii) $\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0) > \hat{h}_1'(\bar{\tau})/\hat{h}_1'(0)$ and $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ for $\tau \in (0,\bar{\tau})$. However, from equation (C.18) and log supermodularity of $b(h,\tau)$, we have $\omega_1(\bar{\tau})/\omega_1(0) > \omega_2(\bar{\tau})/\omega_2(0)$ with $\hat{h}_1(\tau) < \hat{h}_2(\tau)$. With small enough $\alpha_s(\bar{\tau})$, $(\omega_1(\bar{\tau})/\omega_2(\bar{\tau}))^{(\alpha_h(0)-\alpha_h(\bar{\tau}))(1-\epsilon)}$ goes close to one, and hence equation (C.21) contradicts log supermodularity of $b(h,\tau)$.

Second, we show that when $\hat{h}_1(\tau)$ and $\hat{h}_2(\tau)$ cross at any three points $\tau_a < \tau_b < \tau_c$, we have

$\hat{h}_1'(\tau_a)/\hat{h}_1'(\tau_b) < \hat{h}_2'(\tau_a)/\hat{h}_2'(\tau_b)$ with $\hat{h}_2(\tau) > \hat{h}_1(\tau)$ for $\tau \in (\tau_a, \tau_b)$ and $\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b) < \hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)$ with $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ for $\tau \in (\tau_b, \tau_c)$.

From equilibrium condition (C.17),

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)} \left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_a)-\alpha_h(\tau_b))(1-\epsilon)} = \frac{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)}{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)} \left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_b)-\alpha_e(\tau_a))} \tag{C.22}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)} \left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_b)-\alpha_h(\tau_c))(1-\epsilon)} = \frac{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)}{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)} \left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_c)-\alpha_e(\tau_b))} \tag{C.23}$$

With small enough $\alpha_h'(\tau)$, these equations are approximated to

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)} \approx \frac{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)}{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)} \left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_b)-\alpha_e(\tau_a))} \tag{C.24}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)} \approx \frac{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)}{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)} \left(\frac{A_{e2}}{A_{e1}}\right)^{(1-\epsilon)(\alpha_e(\tau_c)-\alpha_e(\tau_b))} \tag{C.25}$$

The only possibility that this can hold at the same time is when $\alpha_e(\tau_b) > \alpha_e(\tau_a)$ and $\alpha_e(\tau_b) > \alpha_e(\tau_c)$ so that the signs of exponent term with respect to (A_{e2}/A_{e1}) are different. Recall that $\omega_1(\tau_b)/\omega_1(\tau_a) < \omega_2(\tau_b)/\omega_2(\tau_a)$ implies $\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a) > \hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)$ from equilibrium condition (C.18) and log supermodularity of $b(h,\tau)$. Since $q_{e1} > q_{e2}$, $\alpha_e(\tau_b) > \alpha_e(\tau_a)$, and $\alpha_e(\tau_b) > \alpha_e(\tau_c)$, we must have $\omega_1(\tau_b)/\omega_1(\tau_a) > \omega_2(\tau_b)/\omega_2(\tau_a)$ and $\omega_1(\tau_c)/\omega_1(\tau_b) < \omega_2(\tau_c)/\omega_2(\tau_b)$, which implies $\hat{h}_1(\tau) < \hat{h}_2(\tau)$ for $\tau \in (\tau_a, \tau_b)$ and $\hat{h}_1(\tau) > \hat{h}_2(\tau)$ for $\tau \in (\tau_b, \tau_c)$.

The proof in the first part rules out any even number of crossings and no crossing. The second part implies they have to cross only a single time on $\tau \in (0, \bar{\tau})$ as they already meet at 0 and $\bar{\tau}$. Then the result follows from the second part of proof. ∎

Proof of proposition 2 (the rise of software) *We firstly show that the production share of middle skill task (task 1) falls and that of high skill task (task 2) rises in response to the decline of price of equipment in a discretized model as well. To be specific, we prove the following lemma first.*

Lemma 6 *Fix N_e and N_s. Consider a decline of the price of equipment; $d\log A_e > 0$ and suppose $\epsilon < 1$ and assumption 2, 5, and 6. Then we have $d\log p_1 < 0$ and $d\log p_2 > 0$.*

Proof From the equilibrium conditions (B.1) to (B.3),

$$\sum_{j=0}^{2} \gamma_j p_j^{1-\epsilon} = 1$$
$$p_j = \left(\frac{\omega_j}{\alpha_{h,j}}\right)^{\alpha_{h,j}} \left(\frac{1}{\nu_e A_e \alpha_{e,j}}\right)^{\alpha_{e,j}} \left(\frac{1}{\nu_s A_s \alpha_{s,j}}\right)^{\alpha_{s,j}} N_e^{-\varphi_e \alpha_{e,j}} N_s^{-\varphi_s \alpha_{s,j}}, \text{ for } j = 0,1,2$$
$$w_{j-1} b(\hat{h}_j, j-1) = w_j b(\hat{h}_j, j), \text{ for } j = 1,2$$
$$\frac{\omega_{j-1} \int_{\hat{h}_{j-1}}^{\hat{h}_j} b(h, j-1)\mu(h)dh}{\omega_j \int_{\hat{h}_j}^{\hat{h}_{j+1}} b(h,j)\mu(h)dh} = \frac{\alpha_{h,j-1}\gamma_{j-1}}{\alpha_{h,j}\gamma_j} \left(\frac{p_{j-1}}{p_j}\right)^{1-\epsilon}, \text{ for } j = 1,2,$$

with $\sigma_s = \sigma_e = 1$.

Let $\Delta x = d\log(x)$. Then by differentiating above and using assumption 5,

$$\Delta p_j = \alpha_{h,j}\Delta\omega_j - \alpha_{e,j}\Delta A_e \tag{C.26}$$
$$\Delta\omega_{j-1} = \Delta\omega_j + \Delta b(\hat{h}_j, j) - \Delta b(\hat{h}_j, j-1) \tag{C.27}$$
$$\Delta\omega_{j-1} - \Delta\omega_j = (1-\epsilon)(\Delta p_{j-1} - \Delta p_j) \tag{C.28}$$
$$\sum_{j=0}^{2} \gamma_j p_j^{1-\epsilon} \Delta p_j = 0 \tag{C.29}$$

Eliminating ω_j's,

$$\left(\frac{1}{\alpha_{h,0}} - (1-\epsilon)\right)\Delta p_0 = \left(\frac{1}{\alpha_{h,1}} - (1-\epsilon)\right)\Delta p_1 + \left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,0}}{\alpha_{h,0}}\right)\Delta A_e \tag{C.30}$$
$$\left(\frac{1}{\alpha_{h,2}} - (1-\epsilon)\right)\Delta p_2 = \left(\frac{1}{\alpha_{h,1}} - (1-\epsilon)\right)\Delta p_1 + \left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,2}}{\alpha_{h,2}}\right)\Delta A_e \tag{C.31}$$

Since $1/\alpha_{h,j} > (1-\epsilon)$ for all j's and $\alpha_{e,1}/\alpha_{h,1} > \alpha_{e,j}/\alpha_{h,j}$ for $j = 0,2$, it is easy to check that $\Delta p_1 < 0$ by substituting equation (C.30) and (C.31) into equation (C.29).

Substituting equation (C.30) and (C.31) into equation (C.29), we also have

$$\left[\gamma_0 p_0^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}} - (1-\epsilon)}{\frac{1}{\alpha_{h,0}} - (1-\epsilon)}\right) + \gamma_2 p_2^{1-\epsilon} + \gamma_1 p_1^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}} - (1-\epsilon)}{\frac{1}{\alpha_{h,1}} - (1-\epsilon)}\right)\right]\Delta p_2$$
$$+\gamma_0 p_0^{1-\epsilon}\left[\frac{\left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,0}}{\alpha_{h,0}}\right) - \left(\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,2}}{\alpha_{h,2}}\right)}{\frac{1}{\alpha_{h,0}} - (1-\epsilon)}\right]\Delta A_e$$
$$-\gamma_1 p_1^{1-\epsilon}\frac{\frac{\alpha_{e,1}}{\alpha_{h,1}} - \frac{\alpha_{e,2}}{\alpha_{h,2}}}{\frac{1}{\alpha_{h,1}} - (1-\epsilon)}\Delta A_e = 0 \tag{C.32}$$

By assumption 6 and $\epsilon < 1$, we have

$$\left[\gamma_0 p_0^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}}-(1-\epsilon)}{\frac{1}{\alpha_{h,0}}-(1-\epsilon)}\right)+\gamma_2 p_2^{1-\epsilon}+\gamma_1 p_1^{1-\epsilon}\left(\frac{\frac{1}{\alpha_{h,2}}-(1-\epsilon)}{\frac{1}{\alpha_{h,1}}-(1-\epsilon)}\right)\right] > 0,$$

$$\gamma_0 p_0^{1-\epsilon}\left[\frac{\left(\frac{\alpha_{e,1}}{\alpha_{h,1}}-\frac{\alpha_{e,0}}{\alpha_{h,0}}\right)-\left(\frac{\alpha_{e,1}}{\alpha_{h,1}}-\frac{\alpha_{e,2}}{\alpha_{h,2}}\right)}{\frac{1}{\alpha_{h,0}}-(1-\epsilon)}\right] = 0,$$

$$\gamma_1 p_1^{1-\epsilon}\frac{\frac{\alpha_{e,1}}{\alpha_{h,1}}-\frac{\alpha_{e,2}}{\alpha_{h,2}}}{\frac{1}{\alpha_{h,1}}-(1-\epsilon)} > 0,$$

implying $\Delta p_2 > 0$ from equation (C.32). ∎

Now we show that lemma 6 implies a relative increase of software variety in the new steady state. Note that the profits from providing software and equipment variety are given by

$$\pi_s = \sum_j \frac{1-\nu}{\nu A_s} s_j \text{ and } \pi_e = \sum_j \frac{1-\nu}{\nu A_e} e_j.$$

From the FOC and using (C.4) ($p_e = 1/(\nu A_e)$ and $p_s = 1/(\nu A_s)$), demand for equipment and software for each task are $e_j = \nu_e A_e \alpha_{e,j} p_j T_j / N_e$ and $s_j = \nu_s A_s \alpha_{s,j} p_j T_j / N_s$.

From lemma 3, we know $\pi_e/\eta = \pi_s/\eta = \rho$ in any steady state equilibrium, and hence,

$$d\pi_e = (1-\nu)\Bigg[(1-\epsilon)(\alpha_{e,0}p_0^{-\epsilon}dp_0 + \alpha_{e,1}p_1^{-\epsilon}dp_1 + \alpha_{e,2}p_2^{-\epsilon}dp_2)Y$$
$$+(\sum_j \alpha_{e,j}p_j^{1-\epsilon})dY - \frac{1}{N_e}\sum_j \alpha_{e,j}p_j^{1-\epsilon}YdN_e\Bigg] = 0$$
$$d\pi_s = (1-\nu)\Bigg[(1-\epsilon)(\alpha_{s,0}p_0^{-\epsilon}dp_0 + \alpha_{s,1}p_1^{-\epsilon}dp_1 + \alpha_{s,2}p_2^{-\epsilon}dp_2)Y$$
$$+(\sum_j \alpha_{s,j}p_j^{1-\epsilon})dY - \frac{1}{N_s}\sum_j \alpha_{e,j}p_j^{1-\epsilon}YdN_s\Bigg] = 0$$

Combining,

$$(1-\epsilon)[(\alpha_{e,1}-\alpha_{s,1})p_1^{-\epsilon}dp_1 + (\alpha_{e,2}-\alpha_{s,2})p_2^{-\epsilon}dp_2]$$
$$= \sum_j \alpha_{e,j}p_j^{1-\epsilon}\left[\frac{dN_e}{N_e}-\frac{dY}{Y}\right] - \sum_j \alpha_{s,j}p_j^{1-\epsilon}\left[\frac{dN_s}{N_s}-\frac{dY}{Y}\right]$$
$$= \sum_j \alpha_{s,j}p_j^{1-\epsilon}\left[\frac{dN_e - dN_s}{N_s} - \left(1-\frac{N_e}{N_s}\right)\frac{dY}{Y}\right] < 0,$$

where the last equality is from no arbitrage condition (16) *(*$\frac{N_s}{N_e} = \frac{\sum_j \alpha_{s,j}\gamma_j p_j^{1-\epsilon}}{\sum_j \alpha_{e,j}\gamma_j p_j^{1-\epsilon}}$*), and the inequality is from lemma 6 and assumption 6.*

Hence, we have

$$dN_s > dN_e + (N_e - N_s)\frac{dY}{Y}.$$

Since decrease in the price of equipment raise the level of production, we have $dY/Y > 0$*. Hence, with the condition given in this proposition (*$N_e \geq N_s$*),* $(N_e - N_s)dY/Y \geq 0$ *and so* $dN_s > dN_e$*. Finally, since* $N_e \geq N_s$*, we have*

$$dN_s/N_s > dN_e/N_e,$$

which was to be shown. ∎

Proof of proposition 3 (skill demand reversal) *Suppose they cross at least once. It means that we have at least three points* $\tau_a < \tau_b < \tau_c$ *such that* $\hat{h}_1(\tau_a) = \hat{h}_2(\tau_a)$*,* $\hat{h}_1(\tau_b) = \hat{h}_2(\tau_b)$*, and* $\hat{h}_1(\tau_c) = \hat{h}_2(\tau_c)$*. Then, we have*

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)}\left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_a)-\alpha_h(\tau_b))(1-\epsilon)} = \frac{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)}{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)}\left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_b)-\alpha_s(\tau_a))} \tag{C.33}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)}\left(\frac{\omega_1(\tau_b)}{\omega_2(\tau_b)}\right)^{(\alpha_h(\tau_b)-\alpha_h(\tau_c))(1-\epsilon)} = \frac{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)}{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)}\left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_c)-\alpha_s(\tau_b))} \tag{C.34}$$

where $\varphi_s \equiv (1-\nu_s)/\nu_s$*.*

With small enough $\alpha_h'(\tau)$*, above equations can be approximated to*

$$\left(\frac{\omega_1(\tau_b)/\omega_1(\tau_a)}{\omega_2(\tau_b)/\omega_2(\tau_a)}\right)^{1-\alpha_h(\tau_a)(1-\epsilon)}\frac{\hat{h}_1'(\tau_b)/\hat{h}_1'(\tau_a)}{\hat{h}_2'(\tau_b)/\hat{h}_2'(\tau_a)} \approx \left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_b)-\alpha_s(\tau_a))} \tag{C.35}$$

$$\left(\frac{\omega_1(\tau_c)/\omega_1(\tau_b)}{\omega_2(\tau_c)/\omega_2(\tau_b)}\right)^{1-\alpha_h(\tau_c)(1-\epsilon)}\frac{\hat{h}_1'(\tau_c)/\hat{h}_1'(\tau_b)}{\hat{h}_2'(\tau_c)/\hat{h}_2'(\tau_b)} \approx \left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s(1-\epsilon)(\alpha_s(\tau_c)-\alpha_s(\tau_b))} \tag{C.36}$$

Again, since matching function is continuous and monotone, and $b(h,\tau)$ *is log supermodular, signs of log of LHS in both equation (C.35) and (C.36) should be different. However, since* $\alpha_s(\tau)$ *is strictly increasing, signs of log of RHS in equation (C.35) and (C.36) are same, which is contradiction.*

Finally, to show $\hat{h}_2(\tau) < \hat{h}_1(\tau)$ *for* $\tau \in (0,\bar{\tau})$*, recall that equilibrium condition (C.17)* *implies*

$$\left(\frac{\omega_1(\bar{\tau})/\omega_1(0)}{\omega_2(\bar{\tau})/\omega_2(0)}\right)^{1-\alpha_h(\bar{\tau})(1-\epsilon)} \frac{\hat{h}_1'(\bar{\tau})/\hat{h}_1'(0)}{\hat{h}_2'(\bar{\tau})/\hat{h}_2'(0)} = \left(\left(\frac{N_{s2}}{N_{s1}}\right)^{\varphi_s} \frac{\omega_2(0)}{\omega_1(0)}\right)^{(1-\epsilon)(\alpha_s(\bar{\tau})-\alpha_s(0))} \tag{C.37}$$

Since $(1-\epsilon)(\alpha_s(\bar{\tau})-\alpha_s(0)) > 0$ *and* $N_{s2} > N_{s1}$*, we have to have* $\omega_1(\bar{\tau})/\omega_1(0) > \omega_2(\bar{\tau})/\omega_2(0)$*, which implies* $\hat{h}_2(\tau) > \hat{h}_1(\tau)$*.* ∎

D Numerical Examples: Continuous Tasks

To illustrate the comparative statics, we provide some numerical examples. For this example, we set:

$$b(h,\tau) = h-\tau,\ \mathcal{M}(h) = \frac{1-h^{-a}}{1-\bar{h}^{-a}},\ \gamma(\tau) = 1,$$
$$\alpha_e(\tau) = -2.5(\tau-.5)^2+.6, \text{ and } \alpha_s(\tau) = .3\tau+.025.$$

For the parameter values, we use $\bar{\tau}=1$, $\bar{h}=4$, $d\tau=.005$, $a=2.5$, $\epsilon=0.7$, $\nu_s=0.65$, $\nu_s=.8$, $\eta_s=\eta_e=A_s=A_e=1$, $\theta=1$, and $\rho=.03$.

In the inner loop, we solve static equilibrium given N_s and N_e. The equilibrium assignment function is computed from equation (9) to (11). Specifically, we use $\hat{h}(0)=1$ and guess $\hat{h}'(0)$ and $\omega(0)$. With the guess, differential equation is solved using finite difference method. We iterate until $\hat{h}(1)=4$ and $\int p(\tau)^{1-\epsilon}d\tau = 1$ using Gauss-Newton method.

Then in the outer loop, we search for N_s and N_e that equate $\pi_s/\eta_s = \pi_e/\eta_e = \rho$, again using Gauss-Newton method.

Factor intensities and the equilibrium assignment function in this example are shown in figure A1. The equipment intensity $\alpha_e(\tau)$ is increasing on $\tau \in [0, 0.5]$ and decreasing on $\tau \in [0.5, 1]$, while the software intensity $\alpha_s(\tau)$ is increasing from 0 to 1. We can also see that the equilibrium assignment function $\hat{h}(\tau)$ is strictly increasing on τ.

Now we compare equilibrium with $A_e=1$ and $A_e=5$ in figure A2. The assignment function in the original equilibrium (with $A_e=1$), in the static equilibrium (with $A_e=5$) and the new steady state (with $A_e=5$) are depicted in figure A2(a). As expected from proposition 1 through proposition 3, we see that the assignment function in the static equilibrium (blue line) cross with the original assignment function (black line) at the middle of τ. The assignment function in the steady state (red line) is generally located above the assignment function in static equilibrium (blue line).

To see the changes in the employment structure more clearly, we also plot changes in the employment share by skill percentile in figure A2(b), similar to the graph shown in figure 2.2(a). To be specific, the horizontal axis shows the tasks ($\hat{\tau}(h)$) corresponding to each percentile in the skill distribution $\mathcal{M}$, and the vertical axis shows the changes in the employment share of those tasks from the original equilibrium to new static equilibrium (blue line) and from the new static equilibrium to new steady state (red line). For example, the first two points on the horizontal axis is two task $\hat{\tau}_1(h_1)$ and $\hat{\tau}_1(h_2)$ where $\hat{\tau}_1$ represents the original (inverse) assignment function and $h_1 = 1$ and $h_2 = \mathcal{M}^{-1}(1)$. Then the first point on the blue line is difference between $\mathcal{M}(\hat{h}_2(\hat{\tau}_1(h_1))) - \mathcal{M}(\hat{h}_2(\hat{\tau}_1(h_2))$ and $\mathcal{M}(h_2) - \mathcal{M}(h_1)$, where $\hat{h}_2$ is the assignment function in the static equilibrium corresponding to $A_e = 5$.

For the relative size of software variety to equipment variety, it was initially .74 in the original equilibrium, and increases to .77 in the new steady state, which is about 6% increase.

CES Task Production To characterize the analytical results, we assume unitary elasticity between labor and capital. However, the crucial characteristic is that the elasticity of substitution between labor and capital is greater than the elasticity of substitution between tasks (ϵ). Furthermore, we expect that task production need not be Cobb-Douglas in generating responses consistent with propositions 1 to 3, at least nu-

Fig. A1: Factor intensities and assignment function

(a) Intensities $\alpha(\tau)$ (b) Assignment function $\hat{h}(\tau)$

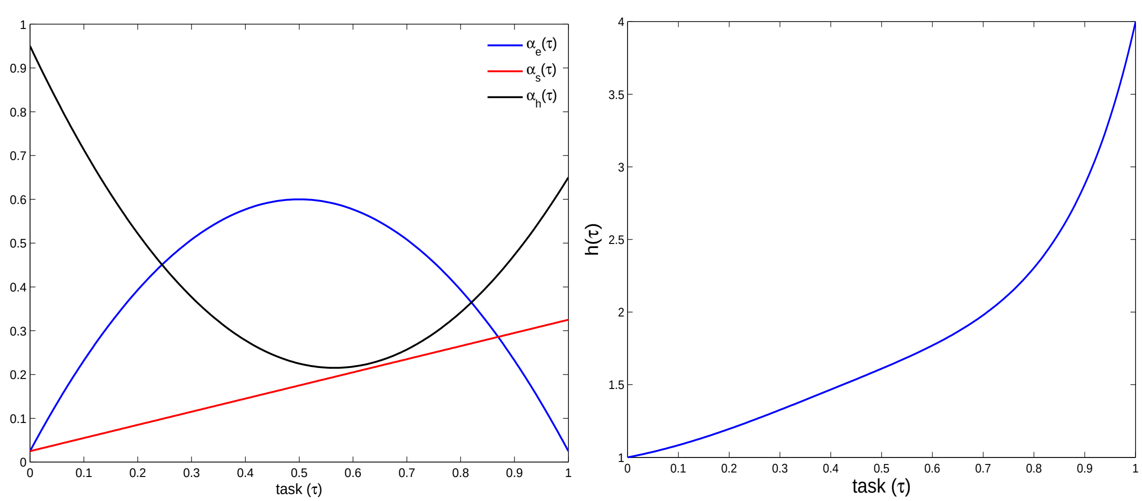
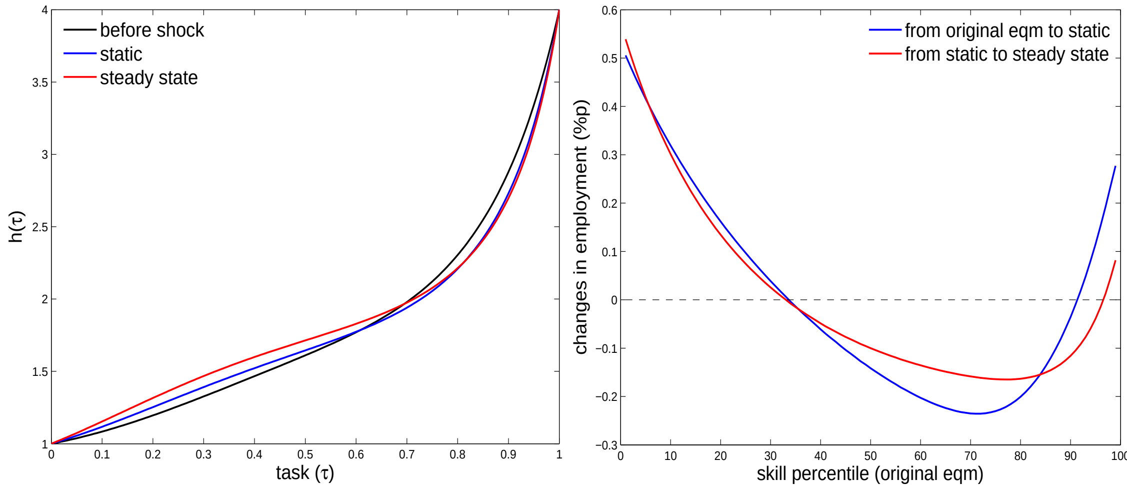


Fig. A2: Equilibrium comparison with $A_e = 1$ and $A_e = 5$

(a) Assignment function $\hat{h}(\tau)$ (b) Changes in employment by skill percentile



merically.

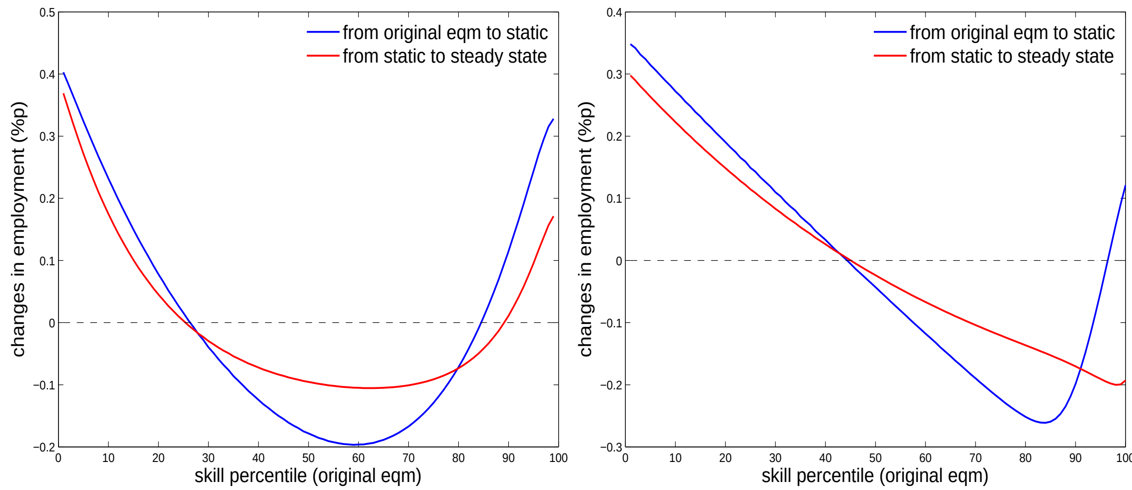
What could we expect if the elasticity of substitution between labor and capital is different from one? We predict that the larger the elasticity of substitution between labor and equipment becomes, the higher the polarization effect would appear. We also expect that the skill demand reversal effect (decreasing high-skill demand) and the rise of software would be enhanced as the elasticity of substitution between labor and software increases.

Intuitively, when the elasticity of substitution between equipment and labor is greater than one, a decrease in the price of equipment lowers the demand for middle-skill tasks not only through the adjustment in the assignment but also through the adjustment between labor and equipment within a task. Additionally, when the elasticity of substitution between software and labor is greater than one, corresponding increases in software would substitute high-skill labor more than before.

To confirm the intuition, we provide several numerical illustrations in figure A3 (details can be found in appendix D). As expected, the magnitude of the decreasing middle increases as σ_e increases, and the decrease in high-skill demand is enhanced as σ_s increases.

Fig. A3: Equilibrium comparison with $A_e = 1$ and $A_e = 5$: CES task production

(a) Changes in employment with $\sigma_e = 1.2$ (b) Changes in employment with $\sigma_s = 1.2$



E Data Construction for Section 5

For relative employment by industry, we use a ratio of employment of routine occupations and employment of cognitive occupations. Routine occupations include machine operators, office and sales, mechanics, construction and production, and transportation occupations. Cognitive occupations are management, professionals, and technicians. The level of employment is obtained from Census 1980, 1990, and 2000, and American Community Survey (ACS) 2010, received from IPUMS. We made a concordance between consistent industry code `ind1990` and `indnaics` using employment in Census 2000. Then employment by `indnaics` is merged into 61 BEA industry code based on a concordance between BEA industry code and NAICS.

The price of equipment and software by industry is from Section 2 of Fixed Asset Table from BEA. The price index is constructed by dividing nominal investment by real investment. We use private non-residential equipment investment by industry for the benchmark, although other series (e.g. industrial equipment) also give similar results.

For growth of software innovation, we use log difference of own account software investment by industry, which captures software investment made in-house by firms. We believe this as a good proxy for software innovation, as in-house software investment is made to develop new software for firm's production process.

It is not straightforward to measure R&D for equipment related innovation from industry level data, as BEA records R&D expenditures only by sources of funds. We think that R&D expenditures funded by equipment producing industries are likely to be used for equipment related innovation, but they should be only a subset of total equipment related innovations. It is also likely that most of these expenditures are used by equipment producing industries, not others, which makes it difficult to capture industry variation. Therefore, we use total R&D expenditures other than software as a benchmark series for N_e, and examine robustness using many different combinations of R&D data. All combinations, including a case with own-account software only, show similar positive relation against relative price.

F Calibration Procedure

This section describes the detailed calibration procedure. We normalize exogenous variables M_j's, A_e, and A_s to one in 1980.

1. We start from $\hat{h}_j$'s that correspond to employment share of occupation j in 1980 and fix ϵ, σ_s and σ_e arbitrarily.
2. By indifference between tasks at the threshold level of skills, we have

   $$\frac{w_j}{w_{j-1}} = \frac{\hat{h}_j - \chi_{j-1}}{\hat{h}_j - \chi_j},$$

   and so $w_j = w_0 \prod_{k=1}^{j} (\hat{h}_k - \chi_{k-1}) / (\hat{h}_k - \chi_k)$. Therefore, payroll share of occupation j is given by

   $$\frac{\prod_{k=1}^{j} (\hat{h}_k - \chi_{k-1}) / (\hat{h}_k - \chi_k) \int_{\hat{h}_{j-1}}^{\hat{h}_j} (h - \chi_j) h^{-a-1} dh}{\sum_j \prod_{k=1}^{j} (\hat{h}_k - \chi_{k-1}) / (\hat{h}_k - \chi_k) \int_{\hat{h}_{j-1}}^{\hat{h}_j} (h - \chi_j) h^{-a-1} dh}.$$

   We set 8 parameters χ_j's and 1 parameter a to minimize distance between payroll share in data and the model for 9 occupations.
3. Guess $\alpha_{j,e}$ and $\alpha_{j,s}$. We find γ_j's that match with $\hat{h}_j$'s in equilibrium.
4. We iterate over $\alpha_{j,e}$ and $\alpha_{j,s}$ untill aggregate labor share, E_j and S_j in the model match with aggregate labor share, equipment and software investment by occupation in data.
5. We solve for M_j's for routine occupations ($j = 2, 3, 4, 5, 7, 8$) to match employment share of routine occupations in data. Note that we already have different values of A_e and A_s for each period obtained from data.

6. Iterate over σ_s and σ_e so that labor share with and without software match with trend implied level in 2010.

7. Iterate over ϵ so as to minimize an average distance between changes in payroll share by occupation in the model and data.

The procedure gives all the parameters needed to be calibrated. For ν_e and ν_s, we use estimated value as described in the section 6.

G Software Embodied in Equipment

We highlight the different use of software and equipment by occupations. However, the software has to be integrated into the equipment to be utilized by a human. We conceptually differentiate additional software made to support works by human and software embodied in equipment in the production of equipment. The software investment highlighted in the model is the former, not including latter. One may wonder whether the software integrated with equipment from the production of equipment has a similar pattern with software investment or equipment investment.

Regarding data, the former appears as software investment in NIPA, and the latter appears as intermediate use of software by equipment producing industries in the Input-Output table. To investigate the pattern of two, we construct software-embodied-equipment series from IO table.

To be specific, we firstly get intermediate consumption of software commodities, 511200 (software publishers) and 541511 (custom computer programming services) from a detailed Input-Output table. This information is only available for the year 1997, 2002, and 2007. Hence, we linearly inter- and extrapolate the ratio of commodities 511200 to 511 and 541511 to 5415 to the periods 1980 to 2014. Then by multiplying the estimated ratio to intermediate use of industry 511 and 5415, we get estimates for software intermediate for 1980 and 2014. Figure A1 depicts software intermediates divided by equipment investment, showing no clear trend in equipment-embodied software. This implies that the rise of software is phenomenon refined to the use of software by human workers, not machine.

Fig. A1: Estimated series for intermediate use of software / equipment investment