A Utilitarian Approach of the Rawls's Difference Principle

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Highlights:

- Economic models are established based on utilitarianism and the Rawls's Difference Principle.
- An individual capability distribution is derived using the maximum entropy method.
- Work hours are long and the income inequality is large in a utilitarian government.
- Total utilities are more or less the same in both Rawlsian and utilitarian governments.
- Utilitarian government is good for the rich and Rawlsian government is good for the poor.

Abstract:
Rawlsian government (RG) maximizes the utility of the poorest group and utilitarian government (UG) maximizes total utility. Each government chooses tax parameters to achieve goals under balanced budget constraints. Individuals have different capabilities and maximize utilities through suitable work and leisure choices. The conclusion shows that Gini coefficients are lowered but work hours increase in both RG and UG even if people are allowed to work while they receive welfare payments.

JEL classification: D63, P52, D31

Keywords: Utilitarianism, Rawls's difference principle, Individual capabilities, Gini coefficient, Maximum entropy method

1. Introduction

Bentham (1789) and Mill (1861) introduced a utilitarian tradition to achieve the greatest good for the greatest number. They were interested in the aggregate sum of utilities, but not the distribution of utilities. Rawls (1999) initiated the concept of distributive justice. The difference

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principle requires that social and economic inequalities be organized so that they are of the greatest benefit to the least-advantaged members of society. Sen (2009) discussed formal equality of opportunities as necessary and sufficient conditions for distributive justice as having no barrier to education, position, or jobs. The Rawls′s difference principle provides intuitive guidelines for welfare distribution, but it is unclear on how to impose the principle in practical policy.

This paper introduces a neoclassical model. Cobb-Douglas functional forms are used for production and utility functions. Individual capability distribution is approximated using the maximum entropy method subject to the given Gini coefficient (US CPS data 1983). Ryu (1983, 2013) derives a probability density function using the maximum entropy method and Yitzhaki (2013) showed the equivalence of the first moment of income distribution with the Gini coefficient. Each government can achieve goals with proper choice of tax parameter. Individuals maximize their utilities with suitable work and leisure hour choices. Two welfare distribution systems are considered of when people receive unemployment benefits: not allowed to work (the first case) and allowed to work (the second case). The performances of RG and UG are compared.

The paradox of redistribution thesis suggests that a greater government targeting of benefits towards the poor results in a less likelihood that poverty and inequality will be reduced (Korpi and Palme 1998; Lindert 2004). This paper provides an opposite result. The RG produced higher utility for the poorest group and lower Gini coefficient compared to the UG.

2. Mathematical Model

There are 1000 persons in a society. Each person has different production capability as a function of position $z = [0,1]$. The least capable and poorest person is located at $z_1 = 0.001$ and the most capable and richest person at $z_{1000} = 1.0$. The distribution of production capability is assumed to have $a \exp(bz_i)$ because Ryu (2013) showed US CPS (1983) income shares can be well approximated with the above function with $b = 2.59,\ a = b / (\exp(b) - 1)$. Fig.1 compares approximated shares with the observed shares.

Let the production function of an individual depend on productivity and working hours,

$$y_i = a\ e^{bz_i} \sqrt{\bar{H} - l_i}\ . \qquad (1)$$

This shows decreasing marginal productivity of labor. One day is $\bar{H} = 24$ hours.

The after tax rate function is assumed to be

$$ATR = e^{-tz_i(\bar{H} - l_i)}\ . \qquad (2)$$

More capable person with higher z pays more tax and ATR decreased. If someone faces $t = 0.12$ and works 6 hours, then the after tax return rate becomes, ATR=$e^{-0.3z}$. The tax rate is close to zero for incapable persons but it increases to 52% for the most capable person.

Fig.1 Approximated and observed income shares

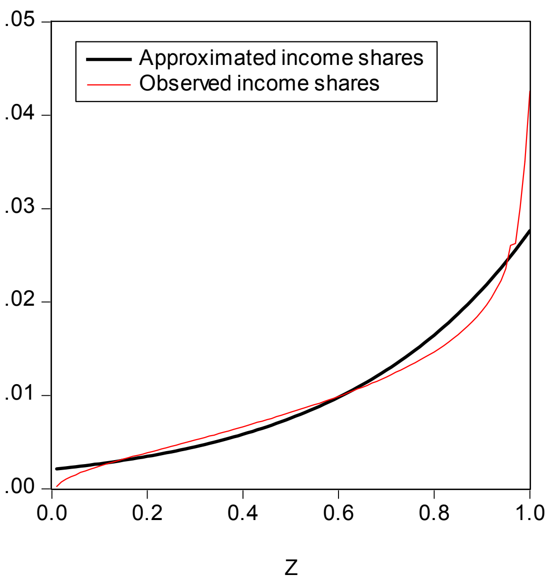
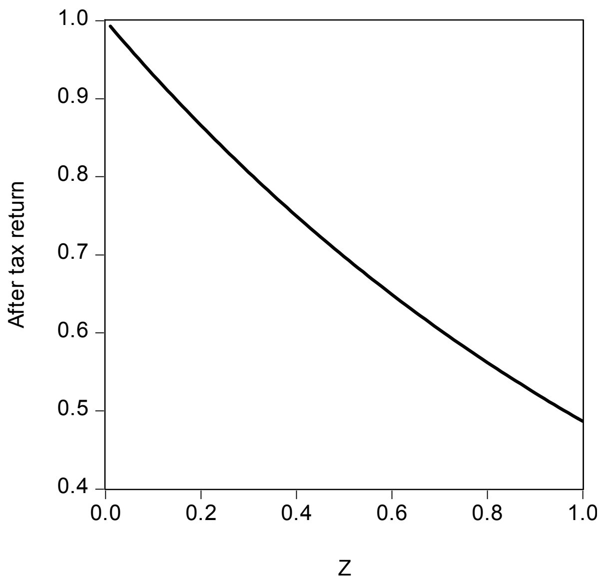


Fig.2 After tax return with respect to z



Consumption function for a person at z_i is

$$C_i = ATR_i \cdot y_i = e^{-tz_i(\bar{H} - l_i)} * a\ e^{bz_i} \sqrt{\bar{H} - l_i} \qquad (3)$$

The utility function is assumed to be $U_i = C_i^{1/3}\, l_i^{2/3}$ with leisure l_i. The model parameters $(1/3, 2/3)$ are chosen for convenience.[3]

$$\ln U_i = \left(\frac{1}{3}\right)\left[-tz_i(\bar{H} - l_i) + \ln a + bz_i + 0.5\ln(\bar{H} - l_i)\right] + \left(\frac{2}{3}\right)\ln l_i \qquad (4)$$

[3] Maximize utility, $a \ln C + (1-a)\ln l$, subject to $wage \cdot l + C = wage \cdot 24$. If optimal leisure is 16 hours, $a = 1/3$.

A person at position z_i will choose leisure hours l_i and work hours are $\bar{H} - l_i = 24 - l_i$ by maximizing utility.

$$\frac{\Delta \ln U_i}{\Delta l_i} = \left(\frac{1}{3}\right)\left[tz_i - \frac{0.5}{\bar{H} - l_i}\right] + \left(\frac{2}{3l_i}\right) = 0 \tag{5}$$

$$tz_i\, l_i^2 - (tz_i\, \bar{H} - 2.5)l_i - 2\bar{H} = 0$$

The optimal leisure hours are

$$l_i^*(z) = \frac{(tz_i\, \bar{H} - 2.5) + \sqrt{\left(tz_i\, \bar{H} - 2.5\right)^2 + 8tz_i\, \bar{H}}}{2tz_i} \tag{6}$$

The optimal working hours are $\bar{H} - l_i^*(z)$. Once the working hours are determined, the output level, tax amount, and utility level can be determined.

Output: $y_i = a\ e^{bz_i} \sqrt{\bar{H} - l_i^*}$ (7)

Consumption: $C_i = ATR_i \cdot y_i$ (8)

Tax Payment: $(1 - ATR_i)y_i = \left[1 - e^{-tz_i(\bar{H} - l_i)}\right]\ a\ e^{bz_i} \sqrt{\bar{H} - l_i^*}$ (9)

Utility: $\ln U_i = \left(\frac{1}{3}\right)\left[-tz_i\,(\bar{H} - l_i^*) + \ln a + bz_i + 0.5\ln(\bar{H} - l_i^*)\right] + \left(\frac{2}{3}\right)\ln l_i^*$ (10)

3. Welfare recipients not allowed to work

Suppose a person chooses the welfare group and decides not to work to receive welfare. There are N_{RG} $(i = 1, 2, \ldots, N_{RG})$ persons in the welfare group and $1000 - N_{RG}$ persons in the working group in RG. Similar notation goes for UG and there are N_{UG} welfare recipients. The boundary person has equal utility and they either belongs in the work group or in the welfare group. This boundary condition uniquely determines the number of welfare recipients (N_{RG} and N_{UG}).

Total tax collections for RG and UG are

$$TAX(N_{RG}) = \sum_{i=N_{RG}+1}^{1000} \left[\left(1 - e^{-t_{RG}\ z_i(\bar{H} - l_i^*)}\right) a\, e^{bz_i} \sqrt{\bar{H} - l_i^*}\right] \tag{11}$$

$$TAX(N_{UG}) = \sum_{i=N_{UG}+1}^{1000} \left[\left(1 - e^{-t_{UG}\; z_i(\bar{H}-l_i^*)}\right) a\, e^{bz_i} \sqrt{\bar{H} - l_i^*} \right] \tag{12}$$

Collected tax is evenly distributed to the welfare recipients in both RG and UG.

$$w_{RG} = \frac{TAX(N_{RG})}{N_{RG}} \qquad\qquad w_{UG} = \frac{TAX(N_{UG})}{N_{UG}} \tag{13}$$

Utility of welfare recipient: $U_{RG} = \left[w_{RG}\right]^{1/3} \left[\bar{H}\right]^{2/3}$ (14)

Utility of a worker after tax payment: $U_i = \left[e^{-tz_i(24-l_i)} * a\; e^{bz_i} \sqrt{24 - l_i} \;\right]^{1/3} \cdot\; l_i^{2/3}$ (15)

If a person at the boundary can enjoy $\bar{U}_i = \ln U_i$ from working, to make them to move to the welfare group, they need an equivalent consumption level C^* to keep them at the same utility level.

$$\bar{U}_i = \ln U_i = \ln U_{RG} = \frac{1}{3}\ln C^* + \frac{2}{3}\ln(24) \quad \rightarrow C^* = \frac{\exp(3\bar{U}_i)}{(24)^2} \tag{16}$$

The RG needs welfare spending of $N_{RG}\; C^*$ for N_{RG} recipients since everybody inside the welfare group receives the same amount. Match the welfare spending with the collected tax. This budget constraint uniquely determines the welfare recipient number N_{RG}. For the given tax rate, the welfare recipients utility is bigger than some worker's utility if N_{RG} is very small. This increases the welfare recipient number and the boundary condition will determine N_{RG} uniquely. Fewer persons will work and working hours will be short if tax parameter t_{RG} is very high. The aggregate output and total utility will decrease; however, the opposite happens if tax parameter t_{RG} is low. The total tax collection was

$$TAX(N_{RG}) = \sum_{i=N_{RG}+1}^{1000} \left[\left(1 - e^{-t_{RG}\; z_i(\bar{H}-l_i)}\right) a\, e^{bz_i} \sqrt{\bar{H} - l_i^*} \right] \tag{17}$$

Total utilities of RG are

$$\sum_{i=1}^{N_{RG}} \bar{U}(z_i) + \sum_{i=N_{RG}+1}^{1000} U(z_i) \tag{18}$$

The utility of the welfare recipient, society total utility, total output, total collected tax, the number of welfare recipients, and the Gini coefficients are plotted as a function of tax parameter (t).

Fig.3 Utility of welfare recipient

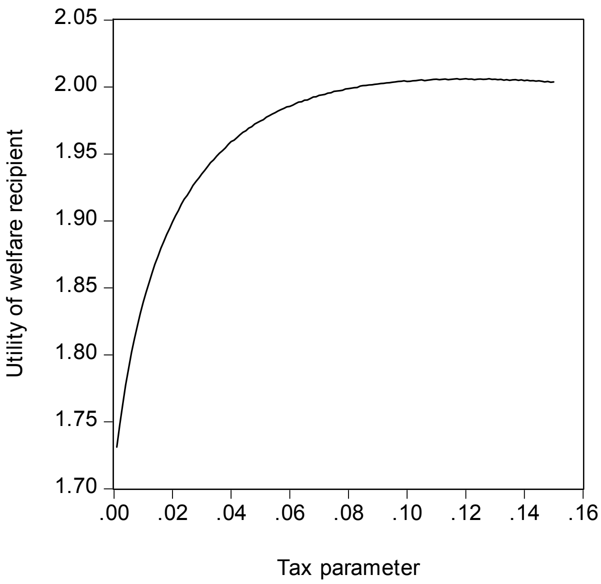
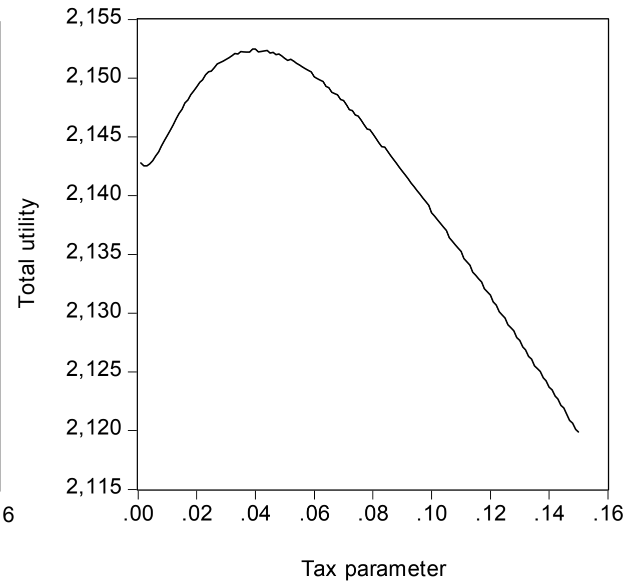


Fig.4 Total utility of the society



In Fig. 3, the utility of welfare recipient is maximized when $t = 0.12$ for RG. In Fig. 4, society total utility is maximized at $t = 0.033$. Fig. 5 and Fig. 6 show people will produce less and pay more tax as tax parameter increases. Fig. 7 shows the number of welfare recipient increases as the tax parameter increases. More people are discouraged from working as the tax parameter increases. Fig. 8 and Fig. 9 show that economic inequality is lowered and people will work less as tax parameters increase.

Fig. 5 Total output of society

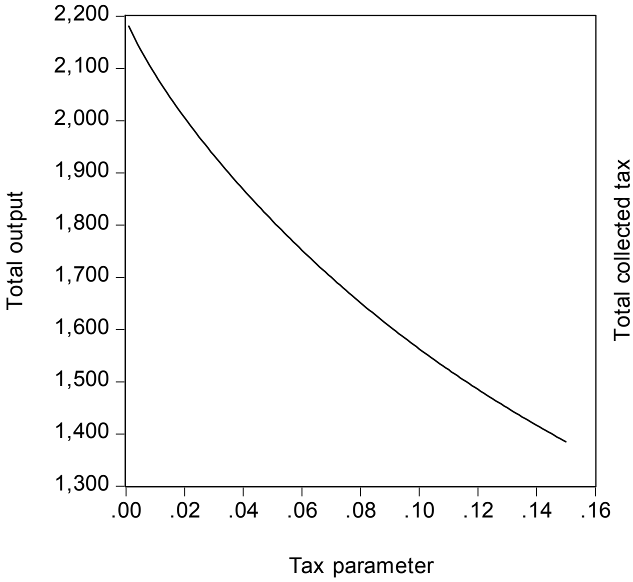


Fig.6 Total collected tax of society

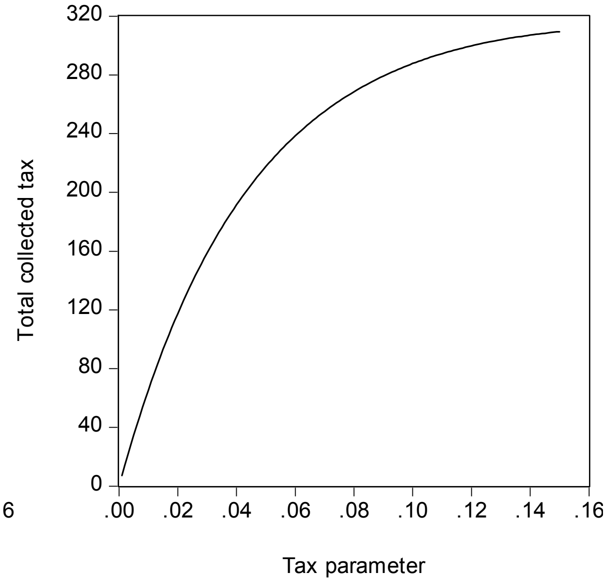


Fig.7 Number of welfare recipients

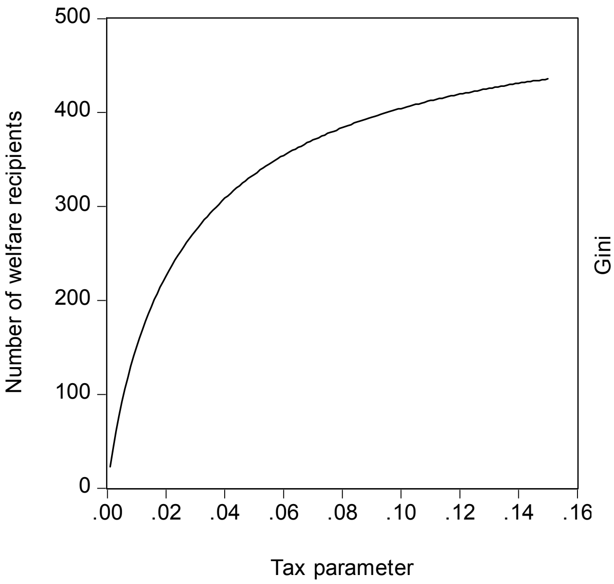


Fig.8 The Gini coefficients

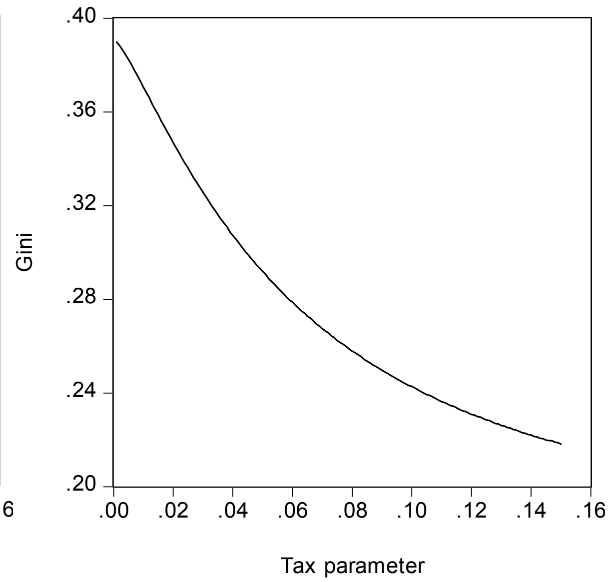


Fig.9 Total work hours

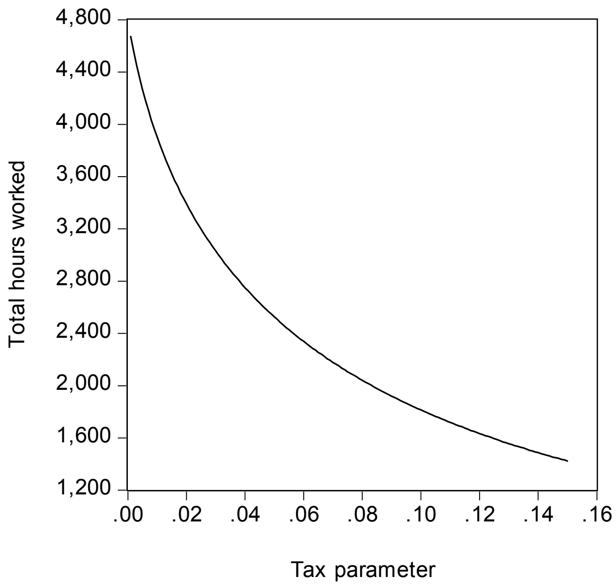


Table 1: Performance comparison of RG and UG

	RG	UG
Number of welfare recipients	420	286
Optimal tax parameter t	$t = 0.120$	$t = 0.033$
Log utility of welfare recipient	2.01	1.94
Total tax collected	300	169
Society total log utility	2132	2152
Gini coefficient	0.231	0.322
Total work hours	1632	2968
Total output	1486	1920

UG required long work hours and achieved larger output production; however, the aggregate utility was slightly higher than RG. Income transfer from the rich to the poor increased the utility of the poor. The society total utility is more or less the same in both RG and UG.

4. Welfare recipients allowed to work

Everybody participates in production to earn income, pay tax, and receive welfare. The welfare reception function is designed so that a more capable and rich person receives smaller welfare while a less capable and poor person receives larger welfare. The welfare function is assumed to be a monotonic decreasing function of z.

$$\text{Welfare} = \exp[r(1-z)] \tag{19}$$

The most capable and richest person at $z=1$ receives no welfare with welfare=1, but least capable and poorest person at $z=0$ receives 156% of his income if $r=0.94$. The consumption level for a person at position z_i is a multiplication of earned income and the welfare factor.

$$C_i^W = e^{-tz_i(\bar{H}-l_i^*)} * a\ e^{bz_i}\sqrt{\bar{H}-l_i^*}\ e^{r(1-z_i)}. \tag{20}$$

The corresponding welfare rate (r) is determined by the budget constraint for the given tax parameter t. Aggregate private consumption including welfare is equal to aggregate output.

$$\sum_{i=1}^{1000} C_i^W = \sum_{i=1}^{1000} y_i = \sum_{i=1}^{1000}\left[a\ e^{bz_i}\sqrt{\bar{H}-l_i^*}\right] \tag{21}$$

Suppose everyone knows their tax parameter (t) and welfare parameter (r). Each person maximizes the utility function with a proper choice of work hours,

$$\ln U_i = \left(\frac{1}{3}\right)\left[-tz_i(\bar{H}-l_i) + \ln a + bz_i + 0.5\ln(\bar{H}-l_i) + r(1-z_i)\right] + \left(\frac{2}{3}\right)\ln l_i \tag{22}$$

The optimal choice of working hours is the same as (6) because the welfare term come as an addition that does not affect the differentiation of utility with respect to labor choice. Total tax collection is a function of tax parameter (t).

$$TAX(t) = \sum_{i=1}^{1000}\left[\left(1-e^{-t\,z_i(\bar{H}-l_i^*)}\right)a\,e^{bz_i}\sqrt{\bar{H}-l_i^*}\right] \tag{23}$$

Different governments will achieve goals with different optimal tax parameters. UG maximizes aggregate utility and RG maximized the utility of the least advantaged group. The middle income group is defined as the range of 50% and 150% of median income. Following Rawls (1999), “all persons with less than half of the median income and wealth” (p.84) is considered the least advantaged group (poverty group). Table 2 summarizes the calculated results.

Table 2: Comparison of RG and UG when welfare recipients are allowed to work

	RG (max poverty group utility)	UG (max total utility)
Optimal tax parameter	0.607	0.072
Redistribution parameter	0.94	0.48
Total log utility	2082	2174
Total log utility of poorest group	431	416
Total collected tax	331	275
Gini coefficient	0.0862	0.268
Total work hours	1646	3774
Total output production	1042	1853

The RG can maximize the utility of the poverty group at $t = 0.607$. The UG maximizes total aggregate utility at $t = 0.072$. Redistribution is large in RG as it maximized the poverty group utility. Total utility and output were small in RG because it emphasized only the poor group and the tax burden is heavy. The capable and rich persons are discouraged from working long hours. The Gini coefficient is very low in RG and economic equality is well achieved. In Fig.10, RG is a good system for poor persons with relatively higher utilities, but the same system is bad for rich persons. In Fig. 11, produced output is larger in UG. In Fig. 12, poor persons work long hours but rich persons work less. Fig.13 shows that the poor pay more tax in RG than in UG, but their utility will increase with welfare reception (Fig. 10).

Fig.10 Individual log utility

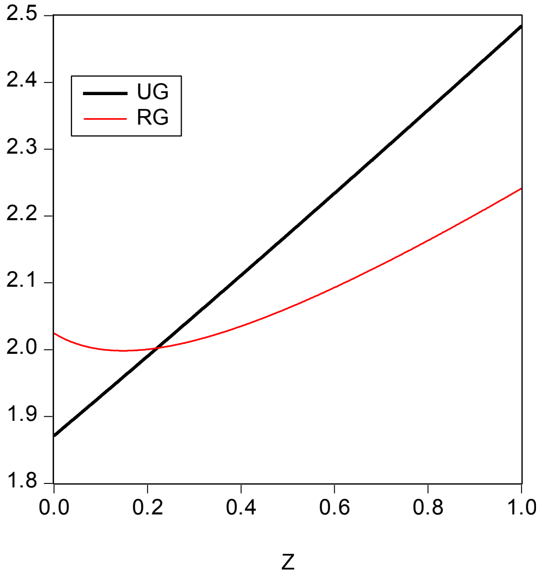


Fig. 11 Individual output

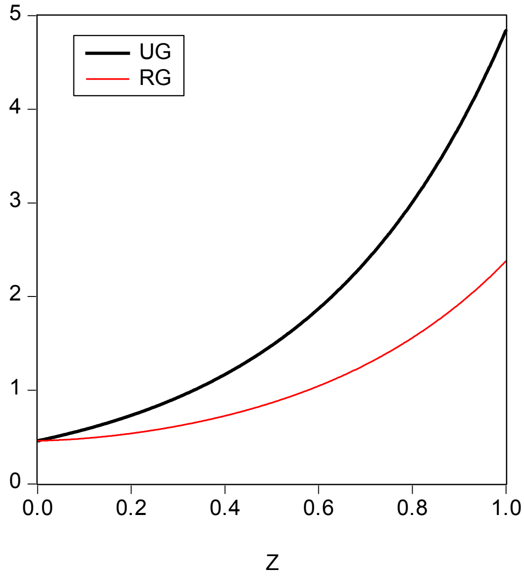


Fig.12 Individual work hours

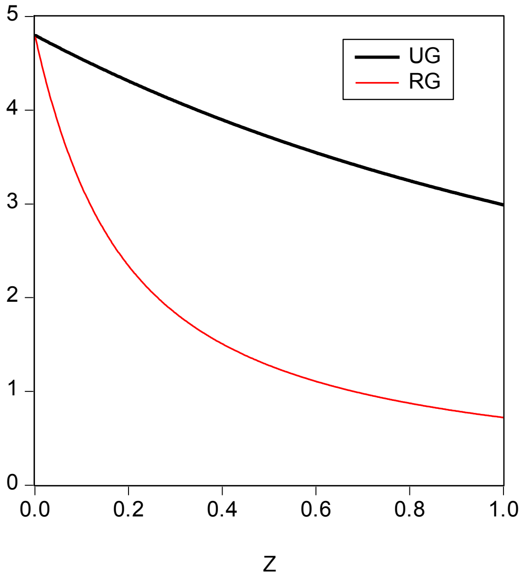
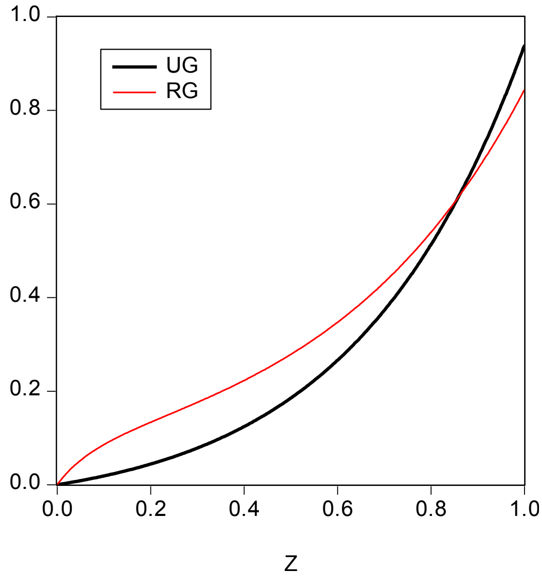


Fig. 13 Individual tax collected



5. Conclusion

UG maximizes total utility; therefore, government supports the rich at the cost of the poor as shown in Fig.10. The persons at z close to one are richer and enjoy a higher level of utilities in UG. RG maximizes poverty group utility; therefore, government supports the poor at the cost of the rich. At the national level, the difference in the government system made a significant difference in total output but a small difference in total utilities. UG with no work allowance for the welfare recipients is recommended for an output-oriented country because the tax burden is low and the produced output is the large. RG with work allowance is recommended for an equality-oriented country because the tax burden is heavy and the Gini coefficient is the smallest.

At the individual level, the produced output and utility depended on their capability level and choice of government form (RG vs UG). Individuals cannot change capability levels; however, they can participate in the selection of government because the poor are favored in RG and the rich are favored in UG.

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