Pyramidal Business Groups and Asymmetric Financial Frictions

Duksang Cho*

Korea Development Institute

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Abstract

Given capital market imperfections, an entrepreneur can alleviate financial frictions by creating a pyramidal business group in which a parent firm offers its subsidiary firm internal finance. This endogenous creation of pyramidal business groups can beget asymmetric financial frictions between business-group and stand-alone firms. I build a model to show that these asymmetric financial frictions can have sizable effects on resource allocation. On one hand, the financial advantage of pyramidal business groups can foster productive firms by incorporating them as subsidiaries. On the other hand, the asymmetrically large amount of external capital controlled by pyramidal business groups can be used up by unproductive business-group firms and push up the price of capital in an equilibrium. The model suggests that with fine investor protection or low financial frictions, the benefits of pyramidal business groups can be dominated by their costs because the probability of fostering productive subsidiaries diminishes as the efficiency of external capital markets improves, while the prevalence of pyramidal business groups does not attenuate due to their asymmetric financial advantage continuing to exist.

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1 Introduction

A pyramidal business group is a collection of legally independent corporations controlled by a coterie of shareholders. It is a common ownership structure for a country's largest firms, with exceptions of some countries such as the United States or the United Kingdom.[1] The economy-wide repercussions of pyramidal business groups, however, have been unclear although they are salient economic institutions too sizable to be ignored. For instance, pyramidal business groups in South Korea not only have been acclaimed as engines of growth for the country's fast development but also have been the subjects of controversy for their economic concentration.[2]

In this paper, I build a model of pyramidal business groups in a general equilibrium framework and aim to answer a question: Can pyramidal business groups affect the efficiency of an economy? I focus on a pyramidal ownership structure, which arises due to capital market imperfections and gives rise to asymmetric financial frictions between business-group and stand-alone firms.[3]

Built on the span of control model developed by Lucas [1978], two assumptions are introduced. First, I assume that the investor protection of an economy is imperfect so that a firm's ability to raise external capital is constrained.[4] I assume a limited commitment problem such that an entrepreneur controlling his or her firms can divert τ fraction of the firms' cash flow before outside investors are reimbursed. In the model, this realized diversion keeps the expected rate of return on external equity finance identical to the risk-free interest rate. Thus, an entrepreneur can earn positive profits as the private benefits of control and have an incentive to create a business group with flotation costs. Note that a common implementation of financial frictions in the literature hinges on an out-of-equilibrium path and that the diversion does not occur in an equilibrium.

Second, I allow for a business group as a private means that can alleviate financial frictions.

[1] La Porta, Lopez-de Silanes, and Shleifer [1999] examine 27 wealthy countries and show that most of the largest corporations in a country are business groups controlled by families or the state through pyramidal ownership schemes. La Porta, Lopez-de Silanes, Shleifer, and Vishny [2000] argue that the degree of investor protection is closely related to the corporate governance structure and that business groups are common in countries with poor investor protection. Masulis, Pham, and Zein [2011] examine 28,635 listed firms in 45 countries, including developing economies, and reaffirm that pyramidal business groups are a common ownership structure around the world. They show that the prevalence of business groups is negatively associated with the capital availability of an economy, but insignificantly associated with the degree of investor protection. They argue that business groups emerge in order to alleviate financial frictions.

[2] As of 2004 in South Korea, business groups controlled by a few families hold 56% of market capitalization in the country according to Masulis, Pham, and Zein [2011].

[3] Given the fact that a business group is a dominant ownership structure of the largest corporations in a country, this study revisits a question raised by many others: if the size distribution of firms in a country affects its economic efficiency.

[4] e.g. Buera, Kaboski, and Shin [2011]

A business group is defined as a collection of two firms connected through a pyramidal ownership structure such that a business-group entrepreneur controls a parent firm that controls a subsidiary firm. There is no limited commitment problem between the parent and the subsidiary because both firms are controlled by the common entrepreneur. Thus, the parent can offer internal finance as much as possible to the subsidiary without financial frictions. Specifically, the financial advantage of a pyramidal business group in the model is twofold. Not only does the subsidiary use its internal equity finance offered by the parent as leverage to raise external capital, but also the parent uses its equity shares of the subsidiary as leverage to raise external capital. Thus, it is the financial advantage of a business group that makes it possible for an entrepreneur to build up a business group as a competitive ownership structure in an equilibrium.

The model shows that business groups can have non-monotonic impact on resource allocation given the degree of financial frictions. In an economy with poor investor protection, the internal capital markets of business groups substitute for underdeveloped external capital markets and foster financially constrained but productive firms. A numerical example of the model shows that the rich become business-group entrepreneurs by hiring the poor but talented as business-group managers. It also shows that an economy with business groups accumulates larger capital stock than an economy without business groups because the rich save more in order to create business groups featuring internal capital markets. This implies that business groups can be efficient private institutions at the early stage of economic development where financial frictions are rampant.

In an economy with fine investor protection, however, the asymmetric financial frictions between business-group and stand-alone firms become a source of resource misallocation. The rich but unproductive choose to create business groups despite flotation costs because they can earn ex-ante positive profits by incorporating productive subsidiaries, while their ex-post profits can be negative because the probability of launching productive subsidiaries declines with the rising managerial compensation as investor protection improves. Moreover, business-group entrepreneurs use their financial advantage to consume more and save less by raising a larger amount of external capital without increasing net capital in production. Thus, the larger demand and the lower supply of capital push up the price of capital in an equilibrium and force stand-alone entrepreneurs, most of whom are financially constrained, to raise less external capital, produce less, and consume less. The numerical example shows that the stand-alone entrepreneurs' wealth drops significantly and that an economy dominated by business groups features decreasing capital stock and stagnating aggregate consumption as the fraction of diversion τ goes to zero.

An interesting lesson we can learn from the model is that the relative number of business-group firms does not decrease endogenously with the improvement of investor protection. This result is consistent with Masulis, Pham, and Zein [2011] who report an insignificant association between the prevalence of family business groups and the degree of investor protection. Given the observation that the direction of effects business groups have on an economy is reversed as investor protection improves, the unvarying number of business-group firms implies that mitigating capital market imperfections may barely reduce factor misallocation or even worsen it without due consideration of pyramidal business groups, which are prevalent in many economies including developed countries.

Although I simplify the problem of business groups by focusing on the financial advantage of their internal capital markets, there is a larger pool of questions about business groups that should be examined such as monopoly, political economy, risk sharing, or intangible assets of business groups. For example, Khanna and Yafeh [2007] review the issues of business groups and conclude that their origins and effects are largely unknown. Note that the objective of this paper is to narrow down the problem and understand a trait of business groups, their internal capital markets, in a general equilibrium framework.

In the literature, the pyramidal ownership structure of a business group has been understood with two different viewpoints. First, a traditional view is that it is an expropriation device. The main argument of this view is that the pyramidal ownership structure creates discrepancy between ownership and control. Although the controlling shareholder of a business group, typically a family, owns a small portion of shares of business-group affiliates, its pyramidal scheme allows the family to take control over the business group and to earn the private benefits of control at the expense of other shareholders. This separation of ownership from control can generate agency problems, resource misallocation, and economic entrenchment. See Morck, Wolfenzon, and Yeung [2005] for the review of this perspective.

Second, more recent studies examine pyramidal business groups as start-up breeders. They focus on the role of business groups that offer internal finance to start-up firms and help them grow larger by supplementing the inefficiency of external capital markets. Almeida and Wolfenzon [2006b] offer a theory of business groups based on the financial advantage of pyramidal business groups. In their model, the controlling shareholder of a parent firm uses the firm's retained earnings to launch a subsidiary firm that provides cash flow to the controlling shareholder. Despite the discrepancy between ownership and control, business groups can be economically beneficial because subsidiary firms would be dismissed without the help of internal capital markets due to setup costs that cannot be raised from external capital markets given financial frictions. Bena and Ortiz-Molina [2013] use data from 38 European countries and show that business groups do play a significant role in creating new firms.

These two perspectives on pyramidal business groups are not mutually exclusive. They are rather opposite sides of the same coin, in that the first can cause the second. The opportunity to earn additional cash flow from a subsidiary firm is an incentive for the controlling shareholder of a parent firm, which offers internal finance and helps to launch its subsidiary firm.

A natural question arises. Between these two viewpoints, which aspect of business groups is dominant? Simply put, are business groups good or bad for an economy? In spite of its relevance, the answer has been unclear. This is because most researchers have focused on the internal efficiency of an individual business group. Few researchers have developed models of business groups in a general equilibrium framework.

Among them, Almeida and Wolfenzon [2006a] show that the financial advantage of business groups can cause asymmetric financial frictions between business-group firms and stand-alone firms, which results in factor misallocation in an equilibrium. Despite its novel insight, their model is stylized so that it is hard to capture dynamic aspects of an economy allowing for forward-looking behaviors of individuals such as savings or self-financing. This can be a problem if we want to examine the economic impact of the asymmetric financial frictions because the wealth distribution of an economy is endogenously determined by the agents' dynamic optimization, which might undo factor misallocation steming from financial frictions (e.g. Moll [2014]).

Ševčík [2015] examines the economic impact of business groups using a heterogeneous agent model with financial frictions, in which the wealth distribution of an economy is endogenously determined. He studies to what extent internal capital markets of business groups can alleviate financial frictions and concludes that aggregate output in Canada would be reduced by 3% if its business groups were shut down. Business groups in his model, however, are partnerships rather than pyramids. This can be a problem if we want to examine the economic repercussions of pyramidal business groups that feature the separation of ownership from control. Specifically, in his model the degree of financial frictions captured by the ratio of capital to wealth is a given constant identical to all firms, while in my model the ratio is endogenously determined and business-group entrepreneurs leverage their wealth into control over capital worth vastly more through a pyramidal ownership structure.

In order to deal with these limitations, I introduce the following feature in my model. First, each individual chooses his or her consumption, savings, and occupation every period. Thus, the joint distribution of individuals' wealth and occupation is endogenously determined. Second, an individual who chooses to be an entrepreneur also chooses his or her firms' ownership structure. I connect corporate capital structures with corporate ownership structures given capital market imperfections. A pyramidal business group is introduced as a private means of an entrepreneur alleviating financial frictions. Thus, asymmetric financial frictions among firms arise from the endogenous choice of firms' ownership structure.

The rest of this paper proceeds as follows. In Section 2, I introduce an individual's problem of occupational choices given the heterogeneity in managerial talent and wealth throughout the population. Every period, each individual chooses his or her occupation among a worker, a stand-alone entrepreneur, a business-group entrepreneur, and a manager who can be hired by a business-group entrepreneur.

In Section 3, financial frictions and three types of capital markets such as external debt, external equity, and internal equity markets are specified. These three types of capital markets are used to build up three types of firms: a private company, a publicly held corporation, and a pyramidal business group. This variety of firms' ownership structures captures private institutions stemming from agents' endogenous reactions against capital market imperfections, which generates asymmetric financial frictions among firms in the model.

In Section 4, a stationary equilibrium is defined by introducing a matching rule between a business-group entrepreneur and a manager. In Section 5, I remark on the model. The costs and benefits of pyramidal business groups are discussed. In Section 6, a numerical example of the model is constructed and the results of the model are presented. Lastly in Section 7, I discuss the limitation of the model and propose future research.

2 A Heterogeneous Agent Model with Occupational Choices

2.1 Economic Environment

An economy consists of infinitely lived individuals. Every period, each individual is endowed with an indivisible labor force and characterized by his or her own managerial talent z that changes over periods following a Markov chain.[5] Let's assume that an individual consumes out of his or her own wealth a such that $c \in [0, a]$ and that a utility function $u(c)$ satisfies standard conditions such that $u'(c) > 0$, $u''(c) < 0$, and $\lim_{c \to 0} u'(c) = \infty$.

Given (z, a), an individual chooses his or her next period occupation $o(z, a)$ among a worker (W), a stand-alone entrepreneur (SA), or a business-group entrepreneur (BG). At the beginning of the next period, a worker sells his or her indivisible labor force and earns wage w, and an entrepreneur runs a firm and earns from the firm's stochastic cash flow π.

An entrepreneur raises her firm's capital k given (z, a). At the beginning of the next period, the entrepreneur observes a shock to the managerial talent z' and hires labor ℓ given k.[6] Then, the firm produces cash flow π that is defined as the optimized gross output net of labor costs $w\ell$ and capital depreciation $\delta' k$ such that

$$\pi\left(z', \delta' | z, k\right) = \max_{\ell} z' k^{\alpha} \ell^{\theta} - w\ell + (1 - \delta')k, \quad \alpha, \theta > 0, \quad \alpha + \theta < 1 \tag{1}$$

where $\alpha + \theta < 1$ is a span of control shaping the production function into decreasing returns to scale. Suppose that the capital depreciation rate $\delta' \in (0, 1)$ is a random variable independent of z'.

A stand-alone entrepreneur can run either a private company or a publicly held corporation. A private company is a firm fully owned by its stand-alone entrepreneur, which raises capital from external debt markets. A publicly held corporation can be incorporated by its stand-alone entrepreneur who pays flotation costs k^F. It can raise capital from external equity markets as well as external debt markets.

A business group is defined as a collection of two corporations: a parent that offers internal

[5] An exogenous process of managerial talents can be understood as a parsimonious way of capturing the impact of financial frictions on factor allocations by abstracting away from the endogenous nature of managerial talents. In Section 6, I will specify a state space and a transition probability of managerial talent z.

[6] We can think of this timing structure, raising k given z and then producing cash flow π after observing z', as an entrepreneur's investment decision taking risks.

equity finance and a subsidiary that receives internal equity finance. An individual of (z_1, a_1)[7], who chooses to be a business-group entrepreneur $o(z_1, a_1) = BG$, runs the parent with z_1 and hires a manager of (z_2, a_2) who runs the subsidiary with z_2. The business-group entrepreneur can choose z_2, while a_2 is randomly drawn with probability $P^{BG}(z_2, a_2)$. The business-group entrepreneur earns from both firms' cash flow at the beginning of the next period.

An individual of (z, a), who chooses to be a worker or a stand-alone entrepreneur $o(z, a) \in \{W, SA\}$, can be matched to a business-group entrepreneur with probability $P^M(z, a)$. If the matching is realized, the individual becomes a manager and earns managerial compensation $w^M(z, a)$ at the beginning of the next period. Note that the managerial compensation w^M is a function of (z, a) that are pinned down when the matching is realized, even though the subsidiary firm's production will be realized with z' next period.

Figure 1 summarizes the timing of an individual's problem within a period. Given (z, a), firstly an individual chooses his or her occupation, secondly the matching between business-group entrepreneurs and the others are realized, and lastly output is produced with realized shocks to managerial talents z' at the beginning of the next period.

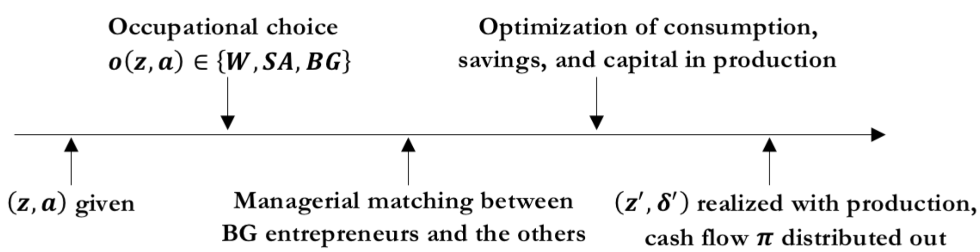


Figure 1: Time-line of an Individual's Problem Within a Period

2.2 An Individual's Problem

Every period, each individual solves the following problem given his or her managerial talent z and wealth a such that

$$V(z, a) = \max_{o \in \{W, SA, BG\}} \left\{ V^W(z, a), V^{SA}(z, a), V^{BG}(z, a) \right\} \tag{2}$$

[7] I use (z_1, a_1) instead of (z, a) because (z_1, a_1) is convenient for comparing a parent's managerial talent z_1 indexed by 1 to a subsidiary's managerial talent z_2 indexed by 2.

given $\left\{r, w, w^M(z,a), P^M(z,a), P^{BG}(z_2,a_2)\right\}$, which respectively stand for the rate of return on capital, wage for a worker, managerial compensation, the probability of being matched with a business-group entrepreneur, and the probability of being matched with a manager featuring (z_2, a_2).

$V^W(z,a)$ is the value if an individual chooses to be a worker such that

$$\begin{aligned} V^W(z,a) &= \left(1 - P^M(z,a)\right) \cdot V_0^W(z,a) + P^M(z,a) \cdot \max\left\{V_0^W(z,a), V^M(z,a)\right\}, \\ V_0^W(z,a) &= \max_{s\in[0,a]} \; u(a-s) + \beta \mathbb{E}_{z'}\left[V\left(z', w + (1+r)s\right) \middle| \, z\right] \end{aligned} \tag{3}$$

where s is the risk-free asset matured in the next-period with interest rate r.

$V^M(z,a)$ is the value if an individual becomes a manager given $w^M(z,a)$ such that

$$V^M(z,a) = \max_{s\in[0,a]} \; u(a-s) + \beta \mathbb{E}_{z'}\left[V\left(z', w^M(z,a) + (1+r)s\right) \middle| \, z\right]. \tag{4}$$

Note that both the next-period wealth for a worker,$w + (1+r)s$, and that for a manger,$w^M(z,a) + (1+r)s$, are realized without uncertainty.

$V^{SA}(z,a)$ is the value if an individual chooses to be a stand-alone entrepreneur who runs a private company or a publicly held corporation such that

$$\begin{aligned} V^{SA}(z,a) &= \left(1 - P^M(z,a)\right) \cdot V_0^{SA}(z,a) + P^M(z,a) \cdot \max\left\{V_0^{SA}(z,a), V^M(z,a)\right\}, \\ V_0^{SA}(z,a) &= \max_{k^C,k^D,k^E} u\left(a - k^C\right) + \beta \mathbb{E}_{z',\delta'}\left[V\left(z', a'\right) \middle| \, z, k\left(k^C, k^D, k^E\right)\right] \end{aligned} \tag{5}$$

where the firm's capital in production k is a function of private finance k^C, external debt finance k^D, and external equity finance k^E. The entrepreneur's next-period wealth a' is a function of shocks to managerial talent z' and capital depreciation rate δ' given $\left\{k^C, k^D, k^E\right\}$.

Lastly, $V^{BG}(z_1, a_1)$ is the value if an individual of (z_1, a_1) chooses to be a business-group entrepreneur who controls a business group consisting of two corporations, a parent with (z_1, k_1) and a subsidiary with (z_2, k_2). The business-group entrepreneur determines both firms' capital k_1 and k_2 by choosing $\left\{k_i^C, k_i^D, k_i^E\right\}_{i\in\{1,2\}}$ given $\left\{z_2, w^M(z_2, a_2)\right\}$. k_1^C is the private finance that the business-group entrepreneur offers to the parent, and k_2^C is the internal equity finance that the parent offers to the subsidiary. I will specify how the business-group entrepreneur optimizes k_1 and k_2 in the following section. For now, let's focus on that the business-group entrepreneur chooses

z_2, the optimal managerial talent for the subsidiary, given $w^M(z_2, a_2)$ and $P^{BG}(z_2, a_2)$ such that

$$V^{BG}(z_1, a_1) = \max_{z_2} \left[\left(1 - \sum_{a_2} P^{BG}(z_2, a_2)\right) \cdot V_0^{SA}(z_1, a_1) + \sum_{a_2} P^{BG}(z_2, a_2) \cdot \max\left\{ V_0^{SA}(z_1, a_1), V_0^{BG}(z_1, a_1 | \, z_2, a_2) \right\} \right]$$

$$V_0^{BG}(z_1, a_1 | \, z_2, a_2) = \max_{\left\{k_i^C, k_i^D, k_i^E\right\}_{i \in \{1,2\}}} u\left(a_1 - k_1^C\right) + \beta \mathbb{E}_{z_1', z_2', \delta_1', \delta_2'}\left[V\left(z_1', a_1'\right) | \, z_1, z_2, k_1, k_2 \right]. \tag{6}$$

The business-group entrepreneur's next-period wealth a_1' is a function of $(z_1', \delta_1', z_2', \delta_2')$ given the firms' capital structure $\left\{k_i^C, k_i^D, k_i^E\right\}_{i \in \{1,2\}}$. Note that the probability of matching with a manager $P^{BG}(z_2, a_2)$ is endogenously determined in an equilibrium and that its sum can be less than one such that $\sum_{a_2} P^{BG}(z_2, a_2) \leq 1$. If the demand of z_2 is higher than the supply of z_2, some business-group entrepreneurs would fail to be matched with their targeted managers featuring z_2.

Figure 2 is an expository diagram of an individual's occupational choice given his or her managerial talent z and wealth a.[8] First, it shows that the poor and untalented are likely to become workers because they are not productive enough to run firms and because they do not have enough wealth to hire managers. Secondly, it shows that the more talented, the more likely to become entrepreneurs. A declining line separating SA from W captures financial frictions with which would-be entrepreneurs could become workers if they have not enough wealth. Lastly, it shows that the rich tend to become business-group entrepreneurs because they can pay managerial compensation and hire talented individuals as business-group managers running subsidiary firms.

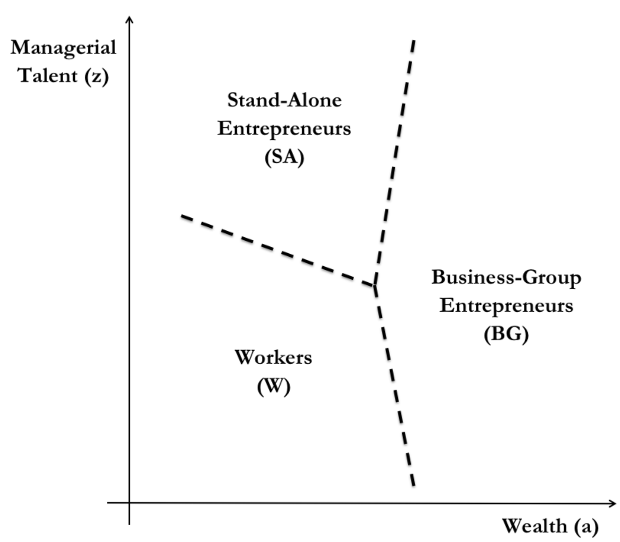


Figure 2: Occupational Choice Given Managerial Talent z and Wealth a

[8]Figure 2 is not the equilibrium output of the model. It is an example constructed for clarifying the idea of an individual's occupational choice problem.

3 Financial Frictions and Three Types of Firms

Suppose that an entrepreneur who controls her firm can divert τ fraction of the firm's cash flow. The tunneling ratio τ captures the degree of financial frictions in an economy. Accordingly, $(1 - \tau)$ captures the degree of investor protection in an economy because $(1 - \tau)$ is the residual cash flow investors can enforce on a firm if the firm does not make reimbursement.

Given financial frictions, an entrepreneur can choose her firms' ownership structure: a private company, a publicly held corporation, or a pyramidal business group. Specifically, an entrepreneur can run her private company that is only allowed to access external debt markets with the help of the entrepreneur's wealth as collateral. I assume that the external debt finance is bounded above by the firm's lowest cash flow in order to guarantee its repayment.

For raising more external finance, an entrepreneur can pay flotation costs and incorporate a publicly held corporation that can tap into external equity markets. I assume that an entrepreneur owns all shares of her firm at the onset of its incorporation, which can be sold to outside shareholders to raise external equity finance. The extent of external equity finance her firm can raise is assumed to be proportional to the firm's expected cash flow and the fraction of shares sold to outside shareholders.

Lastly, an entrepreneur can hire a manager with managerial compensation and build up a business group that consists of two corporations, a parent run by the entrepreneur and a subsidiary run by the manager. The business-group entrepreneur uses a pyramidal ownership structure to control both firms and makes the parent offer internal equity finance to the subsidiary without financial frictions. Similar to stand-alone corporations, both the parent and the subsidiary can sell their shares to outside shareholders and raise external equity finance.

3.1 A Private Company

Given her managerial talent and wealth, (z, a), an entrepreneur can run a private company that is a firm fully owned by her. Due to the lack of external equity finance, a private company relies on external debt finance. The firm's capital in production k is determined as follows. First, the entrepreneur of a private company is obliged for the company's liability so that her wealth net of consumption $a - c$ becomes the firm's collateral k^C such that

$$k^C = a - c \geq 0. \tag{7}$$

Second, given the collateral k^C and the opportunity of tunneling $\tau\pi$, the firm's capital in production k is bounded above as follows.[9]

$$\underbrace{(1+r)k}_{\text{Debt Repayment}} \leq \underbrace{(1+r)k^C}_{\text{Collateral}} + \underbrace{(1-\tau)\inf_{z',\delta'} \pi\left(z', \delta' | z, k\right)}_{\text{Secured Cash Flow to Debtholders}} \tag{8}$$

Lastly, the entrepreneur of a private company can choose k and decide how much external debt finance will be raised. I assume that the firm, or the entrepreneur, can invest in a risk-free asset by taking $k < k^C$. Thus, the entrepreneur can earn the risk-free residual cash flow from the firm such that

$$(1+r)(k^C - k) + \inf_{z',\delta'}[\pi\left(z', \delta' | z, k\right)]. \tag{9}$$

To summarize, a stand-alone entrepreneur running a private company solves

$$V_0^{SA}(z, a) = \max_{k^C \in [0,a], k} u\left(a - k^C\right) + \beta \mathbb{E}_{z',\delta'}\left[V(z', a')|z\right] \tag{10}$$

subject to

$$\begin{aligned} a' &= \pi(z', \delta' | z, k) + (1+r)(k^C - k) \\ k &\leq k^C + \frac{1-\tau}{1+r} \inf_{z',\delta'}[\pi\left(z', \delta' | z, k\right)]. \end{aligned} \tag{11}$$

3.2 A Publicly Held Corporation

An entrepreneur of (z, a) can choose to incorporate her firm into a publicly held corporation with flotation costs $k^F > 0$. After its incorporation, a publicly held corporation can tap into external equity markets. The corporation's capital in production k is determined by the sum of private finance k^C, external debt finance k^D, and external equity finance k^E net of flotation costs k^F such that

$$k = k^C + k^D + k^E - k^F. \tag{12}$$

[9]Unlike publicly held corporations or business groups, expropriation does not occur in a private company that are fully owned by its entrepreneur.

Each type of capital is determined as follows. First, the entrepreneur can transfer a fraction of her wealth k^C to her corporation. k^C is determined by the entrepreneur's wealth a net of her consumption c and private risk-free asset s. I assume that the flotation costs k^F should be paid by the entrepreneur with k^C before the firm's incorporation such that[10]

$$k^C = a - c - s \geq k^F. \tag{13}$$

In contrast to a private company, the entrepreneur's wealth cannot be used as collateral for her corporation because a publicly held corporation is a legal entity that is separated from its entrepreneur. By construction, however, the wealth transfer from its entrepreneur to the publicly held corporation works as collateral, and this is why I abuse the notation of k^C.

Second, a publicly held corporation can use external debt finance k^D. Given the assumption that an entrepreneur controlling her firm can divert τ fraction of the firm's cash flow π, the external debt finance k^D is constrained in order to guarantee its repayment as follows. Note that a publicly held corporation can make an investment in a risk-free asset by taking $k^D < 0$.

$$(1+r)k^D \leq (1-\tau) \inf_{z',\delta'} [\pi(z', \delta' | z, k)] \tag{14}$$

Third, a publicly held corporation can tap into external equity markets. The corporation can raise external equity $k^E = k^E(\sigma)$ by selling its $\sigma \in [0, \bar{\sigma}_{SA}]$ fraction of shares. Suppose that $(1 - \bar{\sigma}_{SA}) > 0$ fraction of the firm's shares is required for an entrepreneur to take control of his or her stand-alone corporation. I assume that external capital markets are competitive and well diversified so that the publicly held corporation can raise external equity with the risk-free interest rate r.

$$\underbrace{(1+r)k^E}_{\substack{\text{Expected Payoff} \\ \text{to Outside Shareholders}}} = \sigma \cdot \mathbb{E}_{z',\delta'} \left[\underbrace{(1-\tau)\pi(z', \delta' | z, k)}_{\text{Cash Flow after Tunelling}} - \underbrace{(1+r)k^D}_{\text{Debt Reimbursement}} \right], \quad \sigma \in [0, \bar{\sigma}_{SA}], \quad \bar{\sigma}_{SA} < 1 \tag{15}$$

As can be seen in the above equation, the firm's cash flow π is sequentially distributed to the

[10] k^F captures expenses such as underwriting fees, legal fees, or registration fees of issuing shares. Although in the real world flotation costs consist of fixed costs as wells costs proportional to the extent of shares issued, only the fixed costs are employed in the model with k^F. I exclude the proportional costs that can be paid with external financing after issuing shares because the efficiency of these back loaded costs is hardly distinguished from the degree of financial frictions τ. Moreover, in the model k^F is paid every periods if an entrepreneur runs a publicly held corporation successively.

entrepreneur with tunneling $\tau\pi$, to creditors with debt reimbursement $(1+r)k^D$, and to shareholders with residual claims.

To summarize, a stand-alone entrepreneur running a publicly held corporation solves

$$V_0^{SA}(z,a) = \max_{s\geq 0,\, k^C, k^D, \sigma\in[0,\bar{\sigma}_{SA}]} u\left(a - s - k^C\right) + \beta \mathbb{E}_{z',\delta'}\left[V(z',a')|z\right] \tag{16}$$

subject to

$$\begin{aligned}
a' &= (1+r)s + \tau\pi(z',\delta'|z,k) + (1-\sigma)\left\{(1-\tau)\pi(z',\delta'|z,k) - (1+r)k^D\right\} \\
k &= k^C + k^D + k^E - k^F \\
k^C &\in \left[k^F, a - s\right] \\
k^D &\leq \frac{1-\tau}{1+r}\inf_{z',\delta'}\left[\pi\left(z',\delta'|z,k\right)\right] \\
k^E &= \frac{\sigma}{1+r}\mathop{\mathbb{E}}_{z',\delta'}\left[(1-\tau)\pi\left(z',\delta'|z,k\right) - (1+r)k^D\right].
\end{aligned} \tag{17}$$

Condition 1. *The value function $V(z,a)$ satisfies the following condition.*

$$\mathbb{E}_{z',\delta'}\left[V_a(z',a')\cdot\left\{\mathbb{E}_{z',\delta'}\pi(z',\delta'|z,k) - \pi(z',\delta'|z,k)\right\}\middle|\, z,k\right] > 0 \tag{18}$$

Condition 1 describes that the entrepreneur running a firm is risk-averse. Although the utility function of an individual is concave by construction, Condition 1 is not guaranteed in general because of the non-convexity of the individual's choice set. The individual's value function $V(z,a)$ might be locally convex. We need an additional structure to hold Condition 1. From now on, let's assume that for all (z,k), a minimum cash flow $\inf_{z',\delta'}\pi(z',\delta'|z,k)$ is low enough to satisfy Condition 1. Note that the marginal utility of consumption goes to infinity as consumption goes to zero by construction. Thus, a low enough minimum cash flow can make the marginal value of wealth $V_a(z',a')$ large enough to hold Condition 1.

Proposition 1. *Given the risk-free investment opportunity for a corporation, $k^D < 0$, a stand-alone entrepreneur weakly prefers not to hold private asset such that*

$$s = 0.$$

Given Condition 1 and the risk-free investment opportunity, a stand-alone entrepreneur of a publicly held corporation strictly prefers a full external equity finance such that

$$\sigma = \bar{\sigma}_{SA}.$$

Proof. See Appendix F. □

Corollary 1. *From Proposition 1, the stand-alone entrepreneur's choice variables degenerate into* $\left\{k^C, k^D, \sigma\right\}$. *Thus, we can simplify the problem of a private company and that of a publicly held corporation into the common problem of a stand-alone entrepreneur such that*

$$V_0^{SA}(z,a) = \max_{k^C, k^D, \sigma\in\{0,\bar{\sigma}_{SA}\}} u\left(a - k^C\right) + \beta \mathbb{E}_{z',\delta'}\left[V(z',a')|z\right] \tag{19}$$

subject to

$$\begin{aligned}
a' &= \tau\pi(z',\delta'|z,k) + (1-\sigma)\left\{(1-\tau)\pi(z',\delta'|z,k) - (1+r)k^D\right\} \\
k &= k^C + k^D + k^E - k^F \cdot \mathbb{1}_{\sigma=\bar{\sigma}_{SA}} \\
k^C &\in \left[k^F \cdot \mathbb{1}_{\sigma=\bar{\sigma}_{SA}},\, a\right] \\
k^D &\le \frac{1-\tau}{1+r} \inf_{z',\delta'}\left[\pi\left(z',\delta'|z,k\right)\right] \\
k^E &= \frac{\bar{\sigma}_{SA}\cdot \mathbb{1}_{\sigma=\bar{\sigma}_{SA}}}{1+r} \mathbb{E}_{z',\delta'}\left[(1-\tau)\pi\left(z',\delta'|z,k\right) - (1+r)k^D\right].
\end{aligned} \tag{20}$$

3.3 A Business Group

A business group is defined as a collection of two publicly held corporations, Firm 1 and Firm 2, which are controlled by a business-group entrepreneur. Let z_1 be the productivity of Firm 1 that inherits from the business-group entrepreneur and let z_2 be the productivity of Firm 2 that inherits from the manager.

Assume that a business group is connected through a pyramidal ownership structure such that Firm 2 is owned and controlled by Firm 1 that is owned and controlled by a business-group entrepreneur. More specifically, the business-group entrepreneur incorporates Firm 1 with private finance k_1^C, keeps at least $(1-\bar{\sigma}_{BG})$ shares of Firm 1, and controls Firm 1. Similarly, Firm 1 incorporates Firm 2 with internal equity finance k_2^C, keeps at least $(1-\bar{\sigma}_{BG})$ shares of Firm 2, and controls Firm 2. I assume that the manager of Firm 2 takes managerial compensation $w^M(z_2, a_2)$, relinquishes her control rights and cash flow rights over Firm 2, and hands them over to Firm 1. As a result, the entrepreneur of a business group can control both firms and divert cash flow from both firms.

Two things are worth noting. First, the pair of managerial talent z_2 and its corresponding managerial compensation $w^M(z_2, a_2)$ can be understood as a contract between an entrepreneur who buys z_2 and a manager who sells z_2 with the price of $w^M(z_2, a_2)$. Thus, how to pin down $w^M(z_2, a_2)$ can be critical in the model. Given the lack of managerial talent markets, I assume that $w^M(z_2, a_2)$ is a certainty equivalent for an individual, who can run a stand-alone firm or become a worker as outside options. It will be formally specified in the following section.

Second, I assume that $(1 - \bar{\sigma}_{BG})$ fraction of shares is required to acquire control rights over a business group. $\bar{\sigma}_{BG}$ can be different from that of a stand-alone firm, $\bar{\sigma}_{SA}$, because $(1-\bar{\sigma}_{BG})$ needs to capture large enough block shares in order to ensure exclusive control rights over business-group firms, while $(1-\bar{\sigma}_{SA})$ only captures stand-alone entrepreneur's payoff structure proportional to the firm's cash flow. Thus, I assume that $\bar{\sigma}_{BG} \leq \bar{\sigma}_{SA}$ although the model lacks the micro foundation about how to pin down $\bar{\sigma}_{SA}$ and $\bar{\sigma}_{BG}$.

3.3.1 Capital Structure of Firm 2

For now, suppose that Firm 2 is run by a manager who has z_2 and a_2. I assume that the flotation costs k^F and the managerial compensation $w^M = w^M(z_2, a_2)$ should be paid by Firm 1 through internal equity finance k_2^C such that

$$k_2^C \geq k^F + w^M. \tag{21}$$

This implies that Firm 2 should be incorporated before tapping into external capital markets. Firm 2 raises external debt finance k_2^D under the following constraint given the assumption that the business-group entrepreneur, who controls Firm 1 that controls Firm 2, can expropriate cash flow from Firm 2.

$$k_2^D \leq \frac{1-\tau}{1+r} \inf_{z_2', \delta_2'} \pi\left(z_2', \delta_2' | z_2, k_2\right) \tag{22}$$

Firm 2 raises external equity finance k_2^E by selling its σ_2 fraction of shares.

$$\underbrace{(1+r)k_2^E}_{\substack{\text{Expected Payoff}\\ \text{to Outside Shareholders}}} = \sigma_2 \cdot \mathop{\mathbb{E}}_{z_2', \delta_2'} \left[\underbrace{(1-\tau)\pi\left(z_2', \delta_2' | z_2, k_2\right)}_{\text{Cash Flow after Tunelling}} - \underbrace{(1+r)k_2^D}_{\text{Debt Reimbursement}} \right], \quad \sigma_2 \leq \bar{\sigma}_{BG} \tag{23}$$

From the above equations, the capital in production of Firm 2, k_2, is determined by the sum of internal equity finance k_2^C, external debt finance k_2^D, and external equity finance k_2^E net of flotation costs k^F and managerial compensation w^M such that

$$k_2 = k_2^C + k_2^D + k_2^E - k^F - w^M. \tag{24}$$

3.3.2 Capital Structure of Firm 1

A business-group entrepreneur of (z_1, a_1) can transfer her wealth k_1^C to Firm 1. I assume that both firms' flotation costs and Firm 2's managerial compensation should be paid by the entrepreneur with k_1^C such that

$$k_1^C = a_1 - c - s \geq \underbrace{k^F}_{\substack{\text{Flotation Costs}\\ \text{of Firm 1}}} + \underbrace{k^F + w^M}_{\substack{\text{Gross Flotation Costs}\\ \text{of Firm 2}}}. \tag{25}$$

This is not only because the timing of incorporating both Firm 1 and Firm 2 is simultaneous in the model but also because the contract between the entrepreneur and the manager should be set up before incorporating Firm 2.

Given the capital structure of Firm 2, $\left\{k_2^C, k_2^D, k_2^E\right\}$, and its cash flow, $\pi(z_2', \delta_2' | z_2, k_2)$, Firm 1 raises external debt finance k_1^D under the following constraint.

$$(1+r)k_1^D \leq (1-\tau)\,\pi_1 \quad \forall\left(z_1', z_2', \delta_1', \delta_2'\right)$$

where π_1 is the gross cash flow from Firm 1 defined by

$$\pi_1 = \underbrace{\pi\left(z_1', \delta_1' | z_1, k_1^* = k_1 - k_2^C\right)}_{\substack{\text{Gross Output Net of Labor Costs and}\\ \text{Capital Depreciation from Firm 1}}} + \underbrace{(1-\sigma_2)\left\{(1-\tau)\,\pi_2 - (1+r)\,k_2^D\right\}}_{\text{Residual Claims of Firm 1 from Firm 2}},$$

$$\pi_2 = \underbrace{\pi\left(z_2', \delta_2' | z_2, k_2\right)}_{\substack{\text{Gross Output Net of Labor Costs and}\\ \text{Capital Depreciation from Firm 2}}}.$$

We can rewrite the above inequality such that

$$k_1^D \leq \frac{1-\tau}{1+r}\left[\inf_{z_1',\delta_1'}\left[\pi(z_1',\delta_1'|z_1,k_1^*)\right]+(1-\sigma_2)\left\{(1-\tau)\inf_{z_2',\delta_2'}\left[\pi(z_2',\delta_2'|z_2,k_2)\right]-(1+r)k_2^D\right\}\right]. \quad (26)$$

Conceptually, the internal equity finance k_2^C used by Firm 2 should be raised from Firm 1's retained earnings (e.g. Almeida and Wolfenzon [2006b]). Given the limitation that firms are created and liquidated every period, however, I use Firm 1's capital k_1 as the proxy for the Firm 1's retained earnings. Thus, the internal equity finance k_2^C is raised out of k_1, and Firm 1's capital in production becomes $k_1^* = k_1 - k_2^C > 0$.

Lastly, Firm 1 raises external equity finance k_1^E by selling its σ_1 fraction of shares to outside shareholders such that

$$\underbrace{(1+r)k_1^E}_{\substack{\text{Expected Payoff}\\ \text{to Outside Shareholders}}} = \sigma_1 \cdot \mathop{\mathbb{E}}_{z_1',z_2',\delta_1',\delta_2'}\left[(1-\tau)\pi_1-(1+r)k_1^D\right], \quad \sigma_1 \leq \bar{\sigma}_{BG}.$$

It can be rewritten as follows.

$$k_1^E = \frac{\sigma_1}{1+r}\left[(1-\tau)\mathop{\mathbb{E}}_{z_1',\delta_1'}\left[\pi(z_1',\delta_1'|z_1',k_1^*)\right]+(1-\tau)(1-\sigma_2)\left\{(1-\tau)\mathop{\mathbb{E}}_{z_2',\delta_2'}\left[\pi(z_2',\delta_2'|z_2,k_2)\right]-(1+r)k_2^D\right\}-(1+r)k_1^D\right] \quad (27)$$

From the above equations, the capital in production of Firm 1, k_1^*, is determined by the sum of private finance k_1^C, external debt finance k_1^D, and external equity finance k_1^E net of flotation costs k^F and internal equity finance k_2^C such that

$$\begin{aligned} k_1^* &= k_1 - k_2^C \\ &= k_1^C + k_1^D + k_1^E - k^F - k_2^C. \end{aligned} \quad (28)$$

3.3.3 A Business-Group Entrepreneur's Problem

Given (z_2, a_2) and $w^M = w^M(z_2, a_2)$, a business-group entrepreneur of (z_1, a_1) solves

$$V_0^{BG}\left(z_1, a_1|\, z_2, w^M\right) = \max_{\substack{s\geq 0\\ \left\{k_i^C,k_i^D,k_i^E\right\}_{i\in\{1,2\}}}} u\left(a_1 - s - k_1^C\right) + \beta\mathbb{E}_{z_1',z_2',\delta_1',\delta_2'}\left[V(z_1',a_1')|z_1,z_2\right] \quad (29)$$

subject to

$$
\begin{aligned}
a_1' &= (1+r)s + \tau\pi(z_1', \delta_1'|z_1, k_1^*) + \tau\pi(z_2', \delta_2'|z_2, k_2) \\
&\quad + (1-\sigma_1)\left\{(1-\tau)\pi(z_1', \delta_1'|z_1, k_1^*) - (1+r)k_1^D\right\} \\
&\quad + (1-\sigma_1+\sigma_1\tau)(1-\sigma_2)\left\{(1-\tau)\pi(z_2', \delta_2'|z_2, k_2) - (1+r)k_2^D\right\}
\end{aligned} \tag{30}
$$

Equation (21) - (28)

Condition 2. *The value function $V(z_1, a_1)$ satisfies the following conditions:*

$$
\begin{aligned}
&\mathbb{E}_{(z_i', \delta_i')_{i\in\{1,2\}}}\left[V_a(z_1', a_1') \cdot \left\{\mathbb{E}_{z_1', \delta_1'}\pi(z_1', \delta_1'|z_1, k_1^*) - \pi(z_1', \delta_1'|z_1, k_1^*)\right\}\middle| z_1, z_2, k_1^*, k_2\right] > 0, \\
&\mathbb{E}_{(z_i', \delta_i')_{i\in\{1,2\}}}\left[V_a(z_1', a_1') \cdot \left\{\mathbb{E}_{z_2', \delta_2'}\pi(z_2', \delta_2'|z_2, k_2) - \pi(z_2', \delta_2'|z_2, k_2\right\}\middle| z_1, z_2, k_1^*, k_2\right] > 0.
\end{aligned} \tag{31}
$$

Proposition 2. *Given the non-negative financial frictions, $\tau > 0$, and the risk-free investment opportunity of Firm 2 such that $k_2^D < 0$, a business-group entrepreneur weakly prefers no private risk-free asset and a full external debt finance of Firm 1 such that*

$$
\begin{aligned}
s &= 0, \\
k_1^D &= \frac{1-\tau}{1+r}\left[\inf_{z_1', \delta_1'}\left[\pi(z_1', \delta_1'|z_1, k_1^*)\right] + (1-\sigma_2)\left\{(1-\tau)\inf_{z_2', \delta_2'}\left[\pi(z_2', \delta_2'|z_2, k_2)\right] - (1+r)k_2^D\right\}\right].
\end{aligned}
$$

Given Condition 2 and the risk-free investment opportunity of Firm 2, a business-group entrepreneur strictly prefers a full external equity finance of both firms such that

$$\sigma_1 = \sigma_2 = \bar{\sigma}_{BG}.$$

Proof. See Appendix E. □

Corollary 2. *From Proposition 2, the business-group entrepreneur's choice variables degenerate into $\left\{k_1^C, k_2^C, k_2^D\right\}$. Thus, we can rewrite the business-group entrepreneur's problem as follows.*

$$
V_0^{BG}\left(z_1, a_1 \middle| z_2, w^M\right) = \max_{k_1^C, k_2^C, k_2^D} u\left(a_1 - k_1^C\right) + \beta\mathbb{E}_{z_1', z_2', \delta_1', \delta_2'}\left[V(z_1', a_1')|z_1, z_2\right] \tag{32}
$$

subject to

$$
\begin{aligned}
&k_1^C \in \left[2k_F + w^M, a\right], \quad k_2^C \in \left[k^F + w^M, k_1\right], \quad k_2^D \leq \frac{1-\tau}{1+r} \inf_{z_2',\delta_2'} \left[\pi_2\left(z_2', \delta_2' | z_2, k_2\right)\right] \\
&k_1^* = k_1^C - k^F - k_2^C + \frac{1-\tau}{1+r}\left\{\bar{\sigma}_{BG}\mathbb{E}_{z_1',\delta_1'}\left[\pi(z_1',\delta_1'|z_1,k_1^*)\right] + (1-\bar{\sigma}_{BG})\inf_{z_1',\delta_1'}\left[\pi(z_1',\delta_1'|z_1,k_1^*)\right] - (1-\bar{\sigma}_{BG})(1+r)k_2^D\right\} \\
&\quad + \frac{(1-\tau)^2(1-\bar{\sigma}_{BG})}{1+r}\left[\bar{\sigma}_{BG}\mathbb{E}_{z_2',\delta_2'}\left[\pi(z_2',\delta_2'|z_2,k_2)\right] + (1-\bar{\sigma}_{BG})\inf_{z_2',\delta_2'}\left[\pi(z_2',\delta_2'|z_2,k_2)\right]\right] \\
&k_2 = k_2^C - k^F - w^M + (1-\bar{\sigma}_{BG})k_2^D + \frac{1-\tau}{1+r}\bar{\sigma}_{BG}\mathbb{E}_{z_2',\delta_2'}\left[\pi(z_2',\delta_2'|z_2,k_2)\right] \\
&a_1' = \tau\pi(z_1',\delta_1'|z_1,k_1^*) + \tau\pi(z_2',\delta_2'|z_2,k_2) \\
&\quad + (1-\bar{\sigma}_{BG})(1-\tau)\left[\pi(z_1',\delta_1'|z_1,k_1^*) - \inf_{z_1',\delta_1'}\left[\pi(z_1',\delta_1'|z_1,k_1^*)\right] - (1-\bar{\sigma}_{BG})\left\{(1-\tau)\inf_{z_2',\delta_2'}\left[\pi(z_2',\delta_2'|z_2,k_2)\right] - (1+r)k_2^D\right\}\right] \\
&\quad + (1-\bar{\sigma}_{BG}+\bar{\sigma}_{BG}\tau)(1-\bar{\sigma}_{BG})\left\{(1-\tau)\pi(z_2',\delta_2'|z_2,k_2) - (1+r)k_2^D\right\}.
\end{aligned}
\tag{33}
$$

Note that in Corollary 2, Firm 1's capital in production k_1^* decreases with k_2^C but increases with the cash flow of Firm 2, $\pi(z_2',\delta_2'|z_2,k_2)$ in the right hand side of k_1^*. Given that $\pi(z_2',\delta_2'|z_2,k_2)$ increases with k_2 and that k_2 increases with k_2^C, we can see that the financial advantage of a business group derives not only from no limited commitment problem such that $k_2^C < k_1$, but also from an increase in the cash flow from Firm 2 to Firm 1.

4 A Matching Rule and a Stationary Equilibrium

4.1 A Matching Rule Between Business-Group Entrepreneurs and the Others

To complete the model, let's consider an ad-hoc matching rule. It is designed for mitigating the gap between the model and the real world. Although the model assumes one-period matching between a business-group entrepreneur and a manager by construction, in the real world the matching between a business-group entrepreneur of (z,a) and a subsidiary Firm 2 of z_2 is stable over time.

First, let's assume that the managerial compensation $w^M(z_2,a_2)$ is equal to the certainty equivalent for a manager who has outside options such that

$$
V^M(z_2,a_2|w^M(z_2,a_2)) = \max\left\{V_0^W(z_2,a_2), V_0^{SA}(z_2,a_2)\right\}. \tag{34}
$$

This assumption implies that a business-group entrepreneur acquires all of gains from building a business group and that the manager of Firm 2 will have less wealth in the next period than the expected wealth a stand-alone entrepreneur would have because of the risk-averse preference.

Second, suppose that the business-group entrepreneur can choose z_2 but cannot choose a_2. A business-group entrepreneur and its manager of Firm 2 who has a_2 are randomly matched given z_2. As a result, while an individual always accepts the offer of being a manger given the managerial compensation as a certainty equivalent, a business-group entrepreneur of (z, a) can turn down the opportunity of launching a subsidiary Firm 2 if the matched manager has too high a_2 that induces $w_M(z_2, a_2) > \bar{w}^M(z, a|z_2)$, where $\bar{w}^M(z, a|z_2)$ is the largest managerial compensation a business-group entrepreneur of (z, a) can be better off such that

$$\bar{w}^M(z, a|z_2) = \sup\left\{w^M > 0 : V_0^{BG}(z, a|z_2, w^M) \geq V_0^{SA}\left(z, a|\, \sigma \leq \bar{\sigma}_{BG}\right)\right\}. \tag{35}$$

Lastly, assume that a business-group entrepreneur, who screens out $w^M(z_2, a_2) > \bar{w}^M(z, a|z_2)$ and gives up the opportunity of launching a subsidiary Firm 2, should keep at least $(1 - \bar{\sigma}_{BG})$ shares of Firm 1. This assumption begets a business group without Firm 2, which sells only $\bar{\sigma}_{BG}$ fraction of shares, not $\bar{\sigma}_{SA}$. Although the capital structures of a business group without Firm 2 is ex-post suboptimal, it is ex-ante optimal for a business-group entrepreneur who wants to launch Firm 2 with the possibility of being matched with $w^M(z_2, a_2) \leq \bar{w}^M(z, a|z_2)$. The possibility of no subsidiary Firm 2 can be understood as an opportunity cost for a business-group entrepreneur. Given the limitation of the model defining a business group as a collection of two corporations, a business group without Firm 2 can be understood as a business group with less pyramidal layer.

4.2 A Stationary Equilibrium

Given the matching rule, a stationary equilibrium consists of a stationary joint distribution of managerial talent and wealth $F(z, a)$; the probability of being hired as a manager $P^M(z, a)$ and the probability of being matched with a manager $P^{BG}(z_2, a_2)$; prices $\left\{r, w, w^M(z_2, a_2)\right\}$; and individual policy functions such as (i) occupation $o(z, a)$ for an individual, (ii) private risk-free asset $s(z, a)$ for a worker or a manager, (iii) private finance $k^C(z, a)$, external debt finance $k^D(z, a)$, and external equity finance $k^E(z, a)$ for a stand-alone entrepreneur, (iv) the optimal managerial talent for a subsidiary firm $z_2(z, a)$, private finance $k_1^C(z, a|z_2, a_2)$, internal equity finance $k_2^C(z, a|z_2, a_2)$, and external debt finance $k_2^D(z, a|z_2, a_2)$ for a business-group entrepreneur matched with $w^M(z_2, a_2) \leq \bar{w}^M(z, a|z_2)$, and (v) private finance $k^C(z, a)$ and external debt finance $k^D(z, a)$ for a business-group entrepreneur matched with $w^M(z_2, a_2) > \bar{w}^M(z, a|z_2)$ such that

1. Given the stationary joint distribution of managerial talent and wealth $F(z, a)$, the probability of being hired as a manager $P^M(z, a)$, the probability of being matched with a manager $P^{BG}(z_2, a_2)$, and prices $\left\{r, w, w^M(z_2, a_2)\right\}$, the individual policy functions solve the individual's problem in Section 2.2;

2. The joint distribution of managerial talent and wealth $F(z, a)$ is stationary such that

$$F(z, a) = \int_{\{(\tilde{z},\tilde{a})|\, z'(\tilde{z}) \leq z,\, a'(z',\delta'|\tilde{z},\tilde{a}) \leq a\}} dF(\tilde{z}, \tilde{a}); \tag{36}$$

3. The probability of a worker or a stand-alone entrepreneur being hired as a manager, $P^M(z_2, a_2)$, and the probability of a business-group entrepreneur being matched with a manager, $P^{BG}(z_2, a_2)$, satisfy the following condition

$$\int_{o(z_2,a_2)\in\{W,SA\}} P^M(z_2, a_2) \cdot F(z_2, da_2) = \int_{\substack{o(z,a)=BG \\ z_2(z,a)=z_2}} \int_{\substack{o(z_2,a_2)\in\{W,SA\} \\ w^M(z_2,a_2)\leq \bar{w}^M(z,a|z_2)}} P^{BG}(z_2, a_2) da_2 dF(z, a) \quad \forall z_2; \tag{37}$$

4. Capital market and labor market clear. See Appendix A for the full description.

5 Remarks on the Model

5.1 Financial Advantage of Business Groups

In order to gauge how well internal capital markets can alleviate exogenous financial frictions in the model, let's consider how much private wealth of an entrepreneur is required to raise a fixed amount of capital in production given the ownership structure of firms.

Suppose that a business group consists of two firms that replicate a stand-alone firm's capital structure with identical managerial talents such that

$$k = k_1^* = k_2, \quad z = z_1 = z_2, \quad \sigma = \bar{\sigma}_{SA} = \bar{\sigma}_{BG}.$$

Let's compare the required level of private finance for a stand-alone firm k^C to that for a business group k_1^C in order to raise $k = k_1^* = k_2$. For a stand-alone firm, the feasible capital in production

k is determined by the following equation.

$$k = k^C - k^F + \frac{1-\tau}{1+r}\left\{\bar{\sigma}\mathbb{E}\pi(z', \delta'|z,k) + (1-\bar{\sigma})\inf \pi(z', \delta'|z,k)\right\} \tag{38}$$

Similarly, the set of feasible capital in production for a business group, i.e. k_1^* for Firm 1 and k_2 for Firm 2, is determined by the following equations.

$$\begin{aligned} k_1^* &= k_1^C - k^F - k_2^C + \frac{1-\tau}{1+r}\left\{\bar{\sigma}\mathbb{E}\pi(z_1', \delta_1'|z,k_1^*) + (1-\bar{\sigma})\inf \pi(z_1', \delta_1'|z,k_1^*)\right\} \\ &\quad + \frac{(1-\tau)^2(1-\bar{\sigma})\bar{\sigma}}{1+r}\left\{\mathbb{E}\pi\left(z_2', \delta_2'|z,k_2\right) - \inf \pi\left(z_2', \delta_2'|z,k_2\right)\right\}, \\ k_2 &= k_2^C - k^F - w^M + \frac{1-\tau}{1+r}\left\{\bar{\sigma}\mathbb{E}\pi(z_2', \delta_2'|z,k_2) + (1-\bar{\sigma})\inf \pi(z_2', \delta_2'|z,k_2)\right\} \end{aligned} \tag{39}$$

By solving for the above equations with $k_1^* = k_2 = k$,

$$\begin{aligned} k_1^C &= 2k^C + w^M - \frac{(1-\tau)^2(1-\bar{\sigma})\bar{\sigma}}{1+r}\left\{\mathbb{E}\pi\left(z_2', \delta_2'|z,k\right) - \inf \pi\left(z_2', \delta_2'|z,k\right)\right\} \\ &= 2k^C + w^M - (1-\tau)(1-\bar{\sigma})k^E \end{aligned} \tag{40}$$

where k^E is the feasible external equity finance that a stand-alone firm with managerial talent z can raise given k^C.

Now, we can compare the effective degree of financial frictions between business-group firms and stand-alone firms. By fixing capital in production $k = k_1^* = k_2$, let's observe the ratio of capital in production to private finance for a stand-alone entrepreneur (SA) and for a business-group entrepreneur (BG) such that

$$\lambda_{SA} = \frac{k}{k^C}, \quad \lambda_{BG} = \frac{k_1^* + k_2}{k_1^C}. \tag{41}$$

Then, the financial advantage of a business group can be measured by the following ratio.

$$\left.\frac{\lambda_{BG}}{\lambda_{SA}}\right|_{k=k_1^*=k_2} = \frac{1}{1+\frac{1}{2}\left\{\frac{w^M}{k^C} - (1-\tau)(1-\bar{\sigma})\frac{k^E}{k^C}\right\}} \tag{42}$$

The ratio depends both on the cost of building up a subsidiary firm, w^M, and the efficiency of external capital markets, $(1-\tau)(1-\bar{\sigma})k^E$. If the latter outweighs the former, the ratio becomes greater than 1. This implies that a business group raises more external finance than a stand-alone firm does given the same amount of private finance. For instance, suppose that $\frac{w^M}{k^C} = 0.4$ and

$\frac{k^E}{k^C} = 20$ given $\tau = 0.3$ and $\bar{\sigma} = 0.9$. Then, the ratio becomes 2 and it means that a business group raises twice larger capital than a stand-alone firm does given the same amount of private finance.

The asymmetric financial advantage of business groups can be lessened if business groups are subject to a lower fraction of equity shares sold to outside shareholders such that $\bar{\sigma}_{BG} < \bar{\sigma}_{SA}$. With this conditions, the above ratio can be rewritten as follows.

$$\frac{1}{1 + \frac{1}{2}\left[\frac{w^M}{k^C} + \left\{2\left(1 - \frac{\bar{\sigma}_{BG}}{\bar{\sigma}_{SA}}\right) - (1-\tau)(1-\bar{\sigma}_{BG})\frac{\bar{\sigma}_{BG}}{\bar{\sigma}_{SA}}\right\}\frac{k^E}{k^C}\right]} \tag{43}$$

Given the same specification with the above but $\bar{\sigma}_{BG} = 0.87$ and $\bar{\sigma}_{SA} = 0.9$, we can observe that the ratio becomes 1.01 and the asymmetric financial advantage of business groups is almost nullified. It teaches us that the minimum equity shares $(1 - \bar{\sigma}_{BG})$, which the controlling shareholder of a business group should keep to control over the business group, can have sizable effects on mitigating the asymmetric financial advantage of business groups. However, note that this example is made up for a stark comparison and business-group entrepreneurs can choose z_2 and optimize their external financing. Thus, we can guess that $\bar{\sigma}_{BG}$ should be much lower in order to lessen the asymmetric financial advantage of business groups in an equilibrium.

5.2 Asymmetric Financial Frictions

Given the finite amount of capital stock in an economy, the asymmetric financial advantage of business groups is in other words the asymmetric financial frictions between business-group firms and stand-alone firms, which can result in factor misallocation in a general equilibrium. Note that managerial compensation w^M is a certainty equivalent proportional to the firm's expected cash flow net of risk premium while external equity finance k^E is proportional to the firm's expected cash flow. This implies that as τ decreases, k^E can grow faster than w^M and that $(1-\tau)(1-\bar{\sigma})k^E$ can grow much faster than w^M. Thus, improvement of investor protection captured by declining τ can increase the gap of external finance raised by business-group firms and stand-alone firms.

The asymmetric financial frictions are of concern because they can be another source of factor misallocation. In an equilibrium, alleviated financial frictions for business groups can increase the demand of external capital and push up the price of capital. For stand-alone firms, however, the higher price of capital r acts like the higher degree of financial frictions τ in that financial constraints of external finance always come with $\frac{1}{1+r}$ as well as $(1-\tau)$. Thus, given the lack of internal capital markets with the higher price of capital, stand-alone firms cannot raise as much capital as they could do in an economy without business groups. As a result, an economy with business groups can give rise to the higher price of capital and lower aggregate output due to factor misallocation. Moreover, since the asymmetric financial frictions between business-group firms and stand-alone firms are intensified as the degree of financial frictions are mitigated, we can guess that costs of business groups are more likely to dominate their benefits in an equilibrium as financial frictions decrease. Last but not the least, the financial advantage of business groups increasing with investor protection $(1-\tau)$ implies that the prevalence of business groups needs not attenuate as investor protection improves.

5.3 External Finance Substituting for Private Finance

As the degree of financial frictions τ decreases, the model shows that both the volume of external equity finance k^E and corporate savings, or corporate lending $-k^D$, can expand without increasing capital in production k. Suppose that firms are financially unconstrained and that the degree of financial frictions is lessened such that

$$d\tau < 0, \quad dk = dk_1^* = dk_2 = 0. \tag{44}$$

From Corollary 2, we can see that a business-group entrepreneur can be better off by increasing consumption $dc > 0$ and decreasing both private finance $dk_1^C < 0$ and external debt finance $dk_2^D < 0$ without altering the next-period wealth $da' = 0$ such that

$$\begin{aligned} dc &= -dk_1^C, \\ da'|_{dk_1^*=dk_2=0} &= (+)d\tau - \tau(1-\bar{\sigma}_{BG})(1+r)dk_2^D = 0, \\ dk_1^* + dk_2 &= (-)d\tau + dk_1^C + \tau(1-\bar{\sigma}_{BG})dk_2^D = 0. \end{aligned} \tag{45}$$

Note that a decrease in private finance $dk_1^C < 0$ without changing capital in production $dk_1^* = dk_2 = 0$ means larger net external finance such that

$$d(k_1^D + k_1^E) > 0, \quad d(k_2^D + k_2^E) > 0. \tag{46}$$

Moreover, from Corollary 2 with $dk_2 = 0$, we can observe that internal equity finance k_2^C increases with corporate savings $-k_2^D$ such that

$$dk_2^C = -(1 - \bar{\sigma}_{BG})dk_2^D > 0. \tag{47}$$

Similarly, from Corollary 1, a stand-alone entrepreneur can be better off by increasing consumption $dc > 0$ and decreasing both private finance $dk^C < 0$ and external debt finance $dk^D < 0$ without altering the next-period wealth $da' = 0$ such that

$$\begin{aligned} dc &= -dk^C, \\ da'\big|_{dk=0} &= (+)d\tau - (1-\sigma)(1+r)dk^D = 0, \\ dk &= (-)d\tau + dk^C + (1 - \bar{\sigma}_{SA})dk^D = 0, \end{aligned} \tag{48}$$

A decrease in private finance $dk^C < 0$ without changing capital in production $dk = 0$ means larger net external finance such that

$$d(k^D + k^E) > 0. \tag{49}$$

The above results show that the excessive amount of external equity finance can be reinvested through corporate savings for risk sharing. In case of business groups, a parent firm's excessive external finance flows into its internal equity finance that is used by the subsidiary firm's investment for risk sharing. Moreover, by raising more external finance, an entrepreneur can reduce wealth transferred to her firm, consume more, and save less. The declining savings ratio of the rich, most of whom are business-group entrepreneurs financially unconstrained, can result in declining capital stock of an economy. Thus, in the model, a strictly positive correlation between the price of capital and aggregate capital in production of an economy can be broken as financial frictions decrease.

6 A Numerical Example of the Model

6.1 Setup

I construct a numerical example of the model and use it to compare two economies: an economy with business groups in which an entrepreneur can choose to create a business group and an economy without business groups in which building a business group is not an option for an entrepreneur.

Description	Parameter
Time discounting factor	$\beta = 0.85$
Relative risk aversion	$\gamma = 1.2$
Span of control	$\alpha + \theta = 0.8$
Capital share	$\alpha = {}^{0.8}\!/_{3}$
Labor share	$\theta = {}^{0.8*2}\!/_{3}$
Average capital depreciation rate	$\mathbb{E}\delta' = 0.059$
Flotation costs	$k^F = 20$
Stand-alone firm's equity share sold out	$\bar{\sigma}_{SA} = 0.9$
Business-group firm's equity share sold out	$\bar{\sigma}_{BG} = 0.7$

Table 1: Parameters

Table 1 summarizes parameters used in the numerical example. A CRRA utility function is employed such that $u(c) = \frac{c^{1-\gamma}-1}{1-\gamma}$. I choose parameters that are conventional in the literature with one exception, a time discounting factor β, which is intentionally chosen very low for the fast convergence of numerical calculation. Model specific parameters such as flotation costs and maximum equity shares sold to outside shareholders are based on the rule of thumb.[11]

The wealth space is discretized into 20 exponentially increasing grids from $a(1) = 1.0 \times 10^{-4}$ to $a(20) = 1.0 \times 10^{6}$. The managerial talent space is discretized into 20 exponentially increasing girds from $z(1) = 1$ to $z(20) = 4$. The transition probability of the managerial talent from $z = z(i)$ to $z' = z(j)$ is defined such that [12]

$$\forall i \in \{1, 2, ..., 19, 20\}\,,\, j = \max\{1, \min\{20, i + k\}\} \text{ with probability } p_k,\ k \in \{-9, -8, ..., 8, 9\},$$

p_{-9}	p_{-8}	p_{-7}	p_{-6}	p_{-5}	p_{-4}	p_{-3}	p_{-2}	p_{-1}	p_0	p_{+1}	p_{+2}	p_{+3}	p_{+4}	p_{+5}	p_{+6}	p_{+7}	p_{+8}	p_{+9}
0.005	0.005	0.01	0.01	0.01	0.01	0.1	0.2	0.3	0.2	0.1	0.01	0.01	0.01	0.01	0.0025	0.0025	0.0025	0.0025

Table 2: The Transition Probability of Managerial Talent

[11] For example, I choose $\bar{\sigma}_{BG} = 0.7$ because it is one of the criteria Fair Trade Commission in South Korea uses to identify if a corporation is a business-group subsidiary.

[12] Note that given the exponentially increasing managerial talent space, the transition probability defined in Table 2 mimics a scale-free growth process bounded below $z' = z(1)$ with negative drift, which can approximate a stationary Pareto distribution (e.g. Gabaix [1999]).

Lastly, I assume that the capital depreciation rate δ' is a simple random variable, which is independent of the shocks to managerial talent such that

$$\delta' = \begin{cases} \bar{\delta} = 0.8 & \text{with probability } 0.05 \\ \underline{\delta} = 0.02 & \text{with probability } 0.95 \end{cases} . \tag{50}$$

6.2 Observations

Observation 1 (Occupational Choice)**.** *The rich choose to become business-group entrepreneurs. The poor but talented are hired as business-group managers with positive probabilities. The north-west region of* (z, a)*, where individuals with the positive probabilities of being hired as managers reside, becomes smaller as investor protection improves. The poor, untalented become workers.*

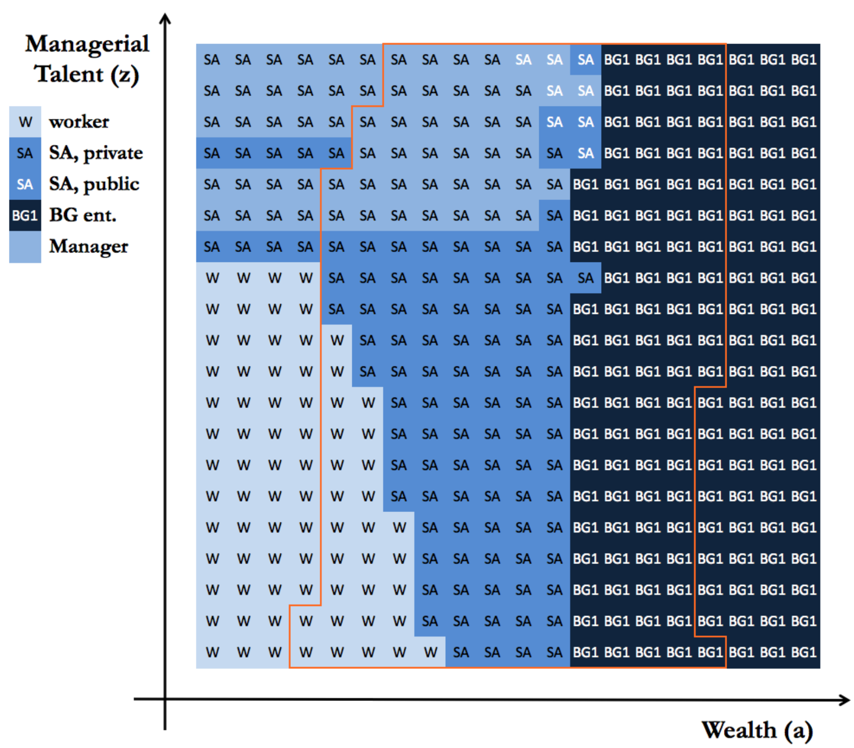


Figure 3: Occupational Map in an Economy With Business Groups Given $\tau = 0.5$. No population exists outside the border of orange line.

Figure 3 shows occupational choices of individuals given a moderate degree of financial frictions, $\tau = 0.5$. We can see that the east where the rich reside is filled with business-group entrepreneurs and that the northwest where the poor but talented reside is filled with stand-alone entrepreneurs who can be hired as business-group managers. This occupational policy function shows that pyramidal business groups work as start-up breeders that can foster productive firms given capital market imperfections. In the southwest, a declining line separating a SA region from a W region captures that wealth is required for an individual to become a stand-alone entrepreneurs given financial frictions.

As the fraction of diversion decreases to $\tau = 0.1$, two changes are observed in the following Figure 4, which depicts occupational choices of individuals in an economy with business groups given $\tau = 0.1$. First, the rich but untalented still become business-group entrepreneurs because they expect to earn ex-ante positive profits by hiring the talented as managers. We will see that these unproductive business-group entrepreneurs can be a source of resource misallocation. If we shut down the possibility of creating pyramidal business groups, the southeast region in Figure 6 shows that the rich but untalented business-group entrepreneurs would become workers in an economy without business groups given $\tau = 0.1$.

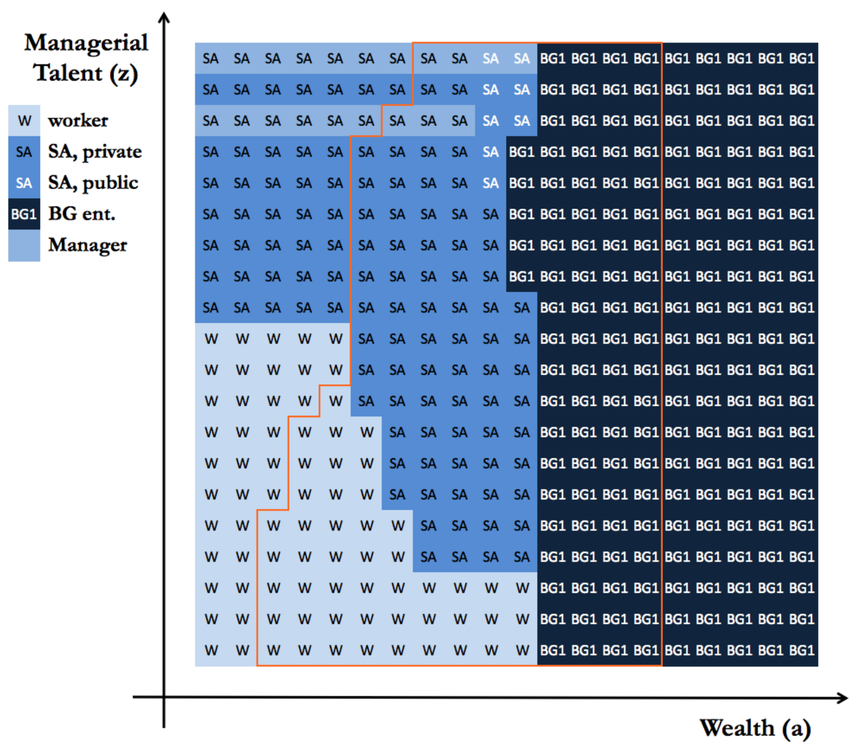


Figure 4: Occupational Map in an Economy With Business Groups Given $\tau = 0.1$. No population exists outside the border of orange line.

Second, Figure 4 shows that fewer individuals are hired as business-group managers. Note that the managerial compensation $w^M(z_2, a_2)$ is likely to be increasing as financial frictions decrease because an outside option of running a stand-alone firm should be a better option with lower financial frictions. Thus, business-group entrepreneurs have to hire the more talented but still financially constrained in order to earn positive profits. The contracted upper northwest region in Figure 4 captures this rising cut-off value of managerial talents, which can give business-group entrepreneurs positive profits with high enough managerial talent but small enough managerial compensation.

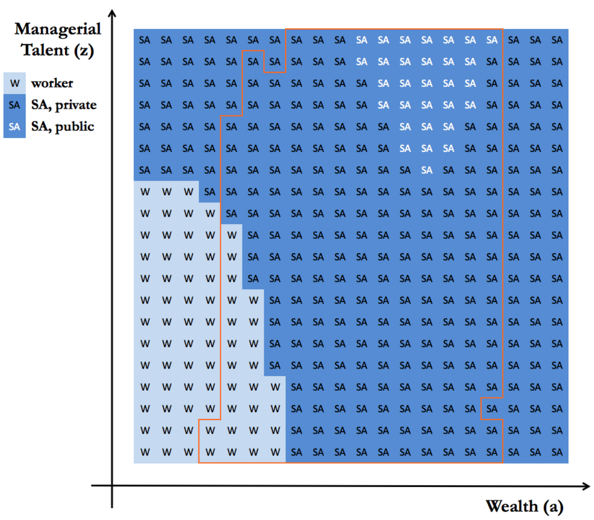


Figure 5: Occupational Map in an Economy Without Business Groups Given $\tau = 0.5$

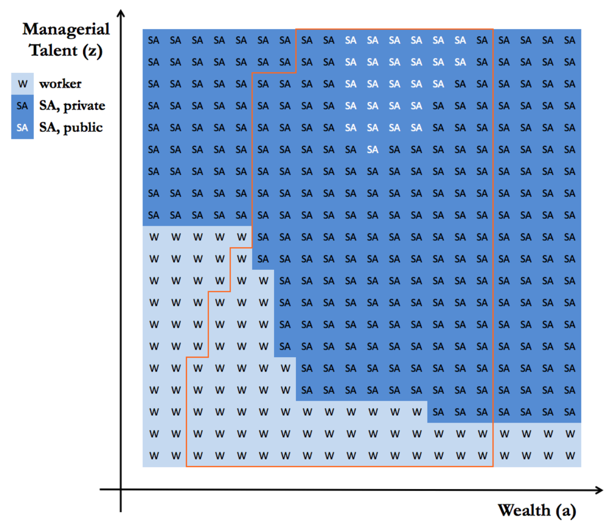


Figure 6: Occupational Map in an Economy Without Business Groups Given $\tau = 0.1$

Observation 2 (The Relative Number of Business-Group Firms). *The prevalence of business groups shows insignificant correlation with the strength of investor protection measured by* $(1 - \tau)$*. Specifically, the relative number of business-group firms out of all corporations does not decrease with* $(1 - \tau)$*.*

Observation 2 can be understood as a corollary of Observation 1, which states that the rich become business-group entrepreneurs regardless of the degree of financial frictions. The following Figure 7 shows us two interesting features about the prevalence of business groups. First, business-group firms cannot thrive under too severe financial frictions such as $\tau \geq 0.7$. This is because too severe financial frictions undermine the financial advantage of a pyramidal ownership structure that leverages on external capital markets.

Second, although the total number of business-group firms is unvarying, the number of subsidiary firms decreases as financial frictions decrease. Observation 1 already shows that the number of individuals who have the positive probability of being hired as managers decreases as financial frictions decrease. We will see in the following observations that subsidiaries are more productive than parents and that this decreasing ratio of subsidiary firms can weaken the benefits of pyramidal business groups as start-up breeders.

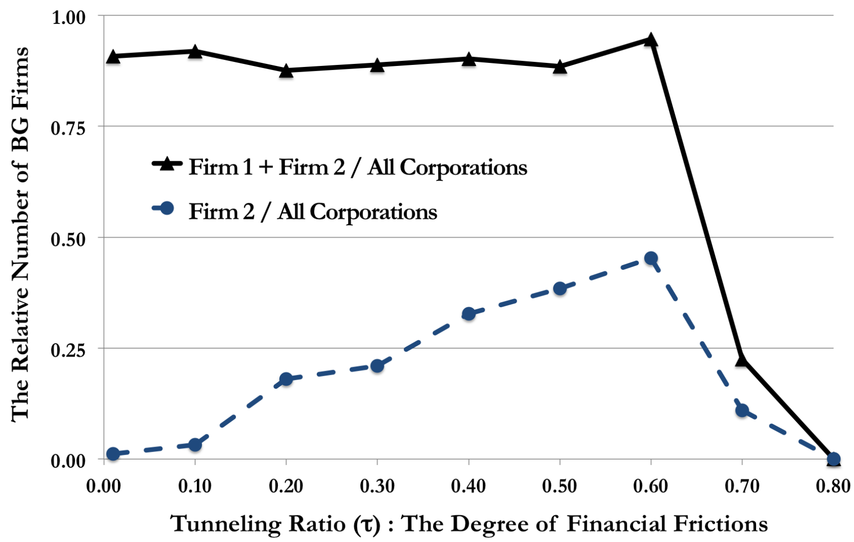
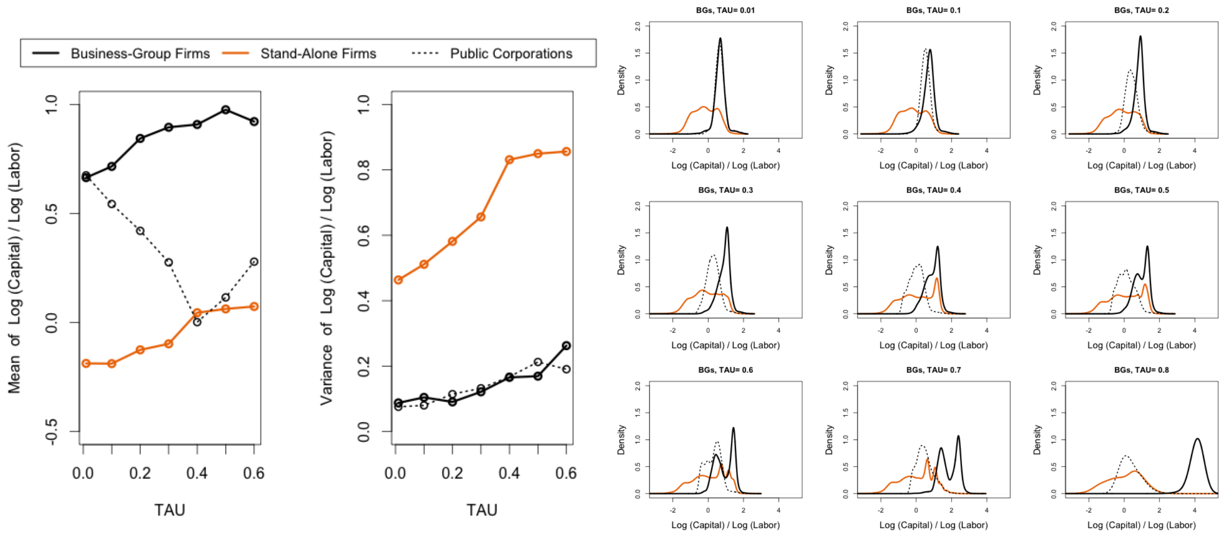


Figure 7: The Prevalence of Business Groups Measured by the Relative Number of Firms

Observation 3 (Asymmetric Financial Frictions Between Business-Group and Stand-Alone Firms)**.** *Business-group firms have a larger ratio of capital to labor than stand-alone firms. The variance of capital to labor ratios is smaller within business-group firms than within stand-alone firms.*

Given the Cobb-Douglas production function, the ratio of capital to labor would be identical to all types of firms if an economy had no financial frictions and no shocks to managerial talents. Thus, business-group firms' higher mean and smaller variance of capital-to-labor ratios suggest that business-group firms have better financial conditions than stand-alone firms. Figure 8 shows that these asymmetric financial frictions persist and hardly vary even though investor protection improves.

Figure 8 also shows that public corporations achieve almost identical capital-to-labor ratios to business groups as τ goes to zero. This implies that firms would be financially unconstrained if they could use external equity finance with fine investor protection. However, the asymmetric financial frictions between business-group and stand-alone firms do not wane because most stand-alone entrepreneur don't pay flotation costs k^F and turn down the option of tapping into external equity markets. As can be seen in Figure 7, most corporations are business-group firms, and the relative number of public corporations using external equity finance decreases as τ decreases.



(a) (b)

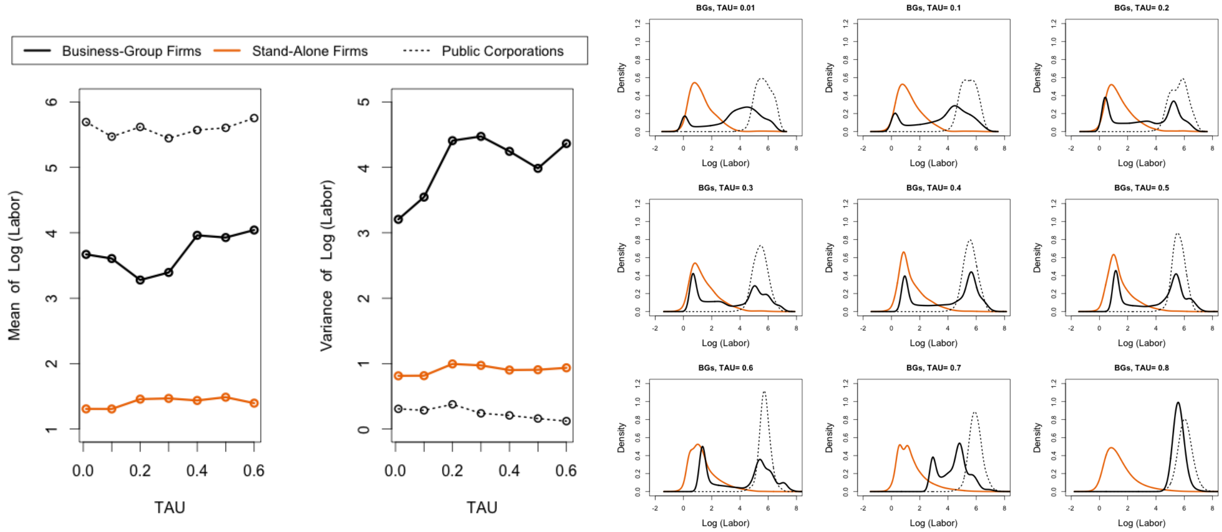
Figure 8: Distributions of Capital-to-Labor Ratio

Then, the question is if these asymmetric financial frictions have sizable effects on resource allocation. The following Observation 4 gives an answer to the question.

Observation 4 (Firm Size Distributions)**.** *Business-group firms have the larger mean and variance of employment and also have the larger mean and variance of TFP than stand-alone firms.*

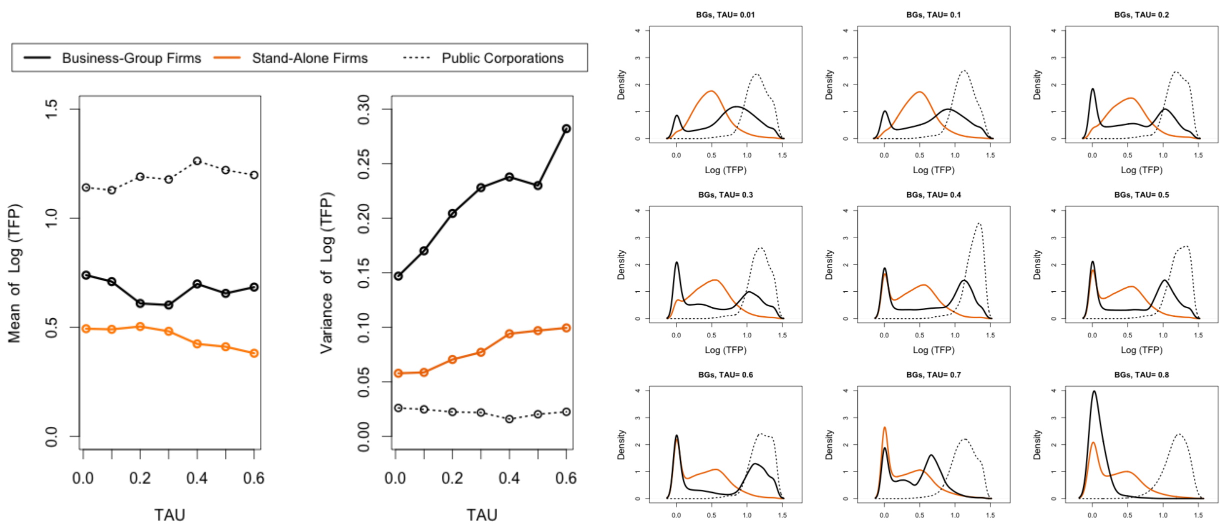
The following Figure 9 shows that business-group firms are larger than stand-alone firms on average. This is because business-group firms not only have better financial conditions (Figure 8) but also have better managerial talents on average (Figure 10).

Business-group firms, however, also have larger variances of employment and managerial talents. Given the persistence of asymmetric financial frictions, the large number of unproductive business-group firms can distort resource allocation in an equilibrium. Note that the distributions of business-group firms are bimodal. Small, unproductive business-group firms coexist with large, productive business-group firms. This observation complies with the occupational choice that the rich but unproductive choose to become business-group entrepreneurs regardless of the degree of financial frictions (Figure 4).



(a) (b)

Figure 9: Firm Size Distributions Measured by Employment



(a) (b)

Figure 10: Distributions of Managerial Talent (TFP)

Given that pyramidal business groups have financial advantage but also have more dispersed productivities, the effects of pyramidal business groups on resource allocation are ambiguous. Their financial advantage makes business-group firms not only to raise more capital but also to allocates more capital to low productive business-group firms. The following Observation 5 shows that the net effects of pyramidal business groups depend on the level of financial frictions, τ.

Observation 5 (Factor Prices and Aggregate Inputs)**.** *As the strength of investor protection* $(1-\tau)$ *improves in an economy with business groups, both the rate of return on capital and wage increase monotonically, while both the capital stock and labor force increase first and then decrease.*

Figure 11 captures correlations between factor prices and aggregate inputs in the degree of financial frictions. It shows that positive correlations between factor prices and aggregate inputs are broken under the prevalence of business groups. The existence of business groups helps an economy achieve a large amount of aggregate inputs under the moderate level of financial frictions such as $\tau \in [0.3, 0.7]$. However, a further decrease in financial frictions from $\tau = 0.2$ only pushes up factor prices and results in the smaller aggregate inputs of an economy. This non-monotonicity contrasts with strictly positive correlations between factor prices and aggregate inputs in an economy without business groups.

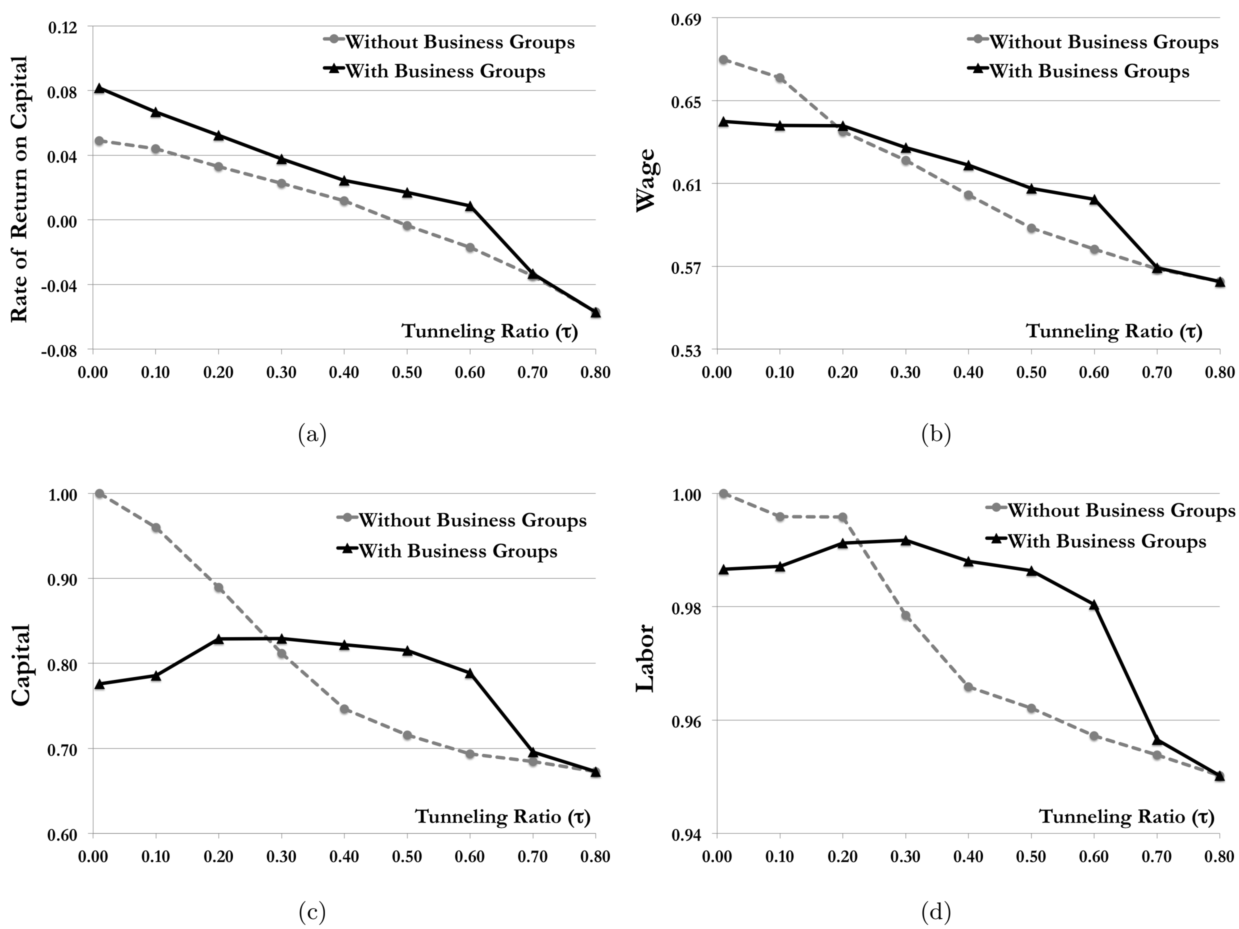


Figure 11: Factor Prices and Aggregate Inputs

This negative correlation observed in Figure 11 derives from a decrease in savings of the rich.

The following Table 3 captures savings of the rich[13] whose wealth is top 0.14% in an economy with business groups. It shows that the rich who choose to create business groups save less as financial frictions decrease. The level of their savings decreases from 0.88 to 0.53, and the share of their savings decreases from 52% to 33%. This decrease in savings can be supported by the financial advantage of business-group entrepreneurs, which allows them to substitute external finance for private finance. With the same amount of wealth, business-group entrepreneurs can consume more and save less by raising more external capital as financial frictions decrease.

It is interesting that the savings of the rich would be monotonically increasing with investor protection if we shut down the possibility of creating business groups. In an economy without business groups, savings of the rich increase from 35% to 47% as financial frictions decrease from $\tau = 0.5$ to $\tau = 0.1$. Note that the population of the rich increases,[14] which implies that the lower financial frictions help the talented accumulate wealth in an economy without business groups.

The Degree of Financial Frictions	$\tau = 0.1$	$\tau = 0.5$
An Economy with Business Groups		
Savings of the Rich (Share of Capital Stock)	0.53(33%)	0.88(52%)
Population of the Rich	0.14%	0.15%
An Economy without Business Groups		
Savings of the Rich (Share of Capital Stock)	0.93(47%)	0.52(35%)
Population of the Rich	0.21%	0.11%

Table 3: Savings of the Rich

The Degree of Financial Frictions	$\tau = 0.1$	$\tau = 0.5$
Wealth of SA Entrepreneurs (Share of Total Wealth)	0.46(18%)	1.04(40%)
Population of SA Entrepreneurs	5.24%	5.78%

Table 4: Wealth of Stand-Alone Entrepreneurs in an Economy with Business Groups

In an economy dominated by business groups, however, its stagnating population of the rich

[13] I choose $a(13) = 398$ as the criteria for the rich because the population of individuals whose wealth is greater than or equal to 398 hardly changes as financial frictions decrease: the population changes from 0.147% with $\tau = 0.5$ to 0.136% with $\tau = 0.1$%.

[14] Given the criteria of the rich, $a \geq a(13) = 398$, the population of the rich in an economy without business groups increases from 0.11% with $\tau = 0.5$ to 0.21% with $\tau = 0.1$.

suggests that the poor but talented suffer from the asymmetric financial frictions and have a difficulty to accumulate their wealth. The above Table 4 shows this possibility. The absolute level of stand-alone entrepreneurs' wealth decreases from 1.04 to 0.46 as financial frictions decrease from $\tau = 0.5$ to $\tau = 0.1$, and the share of their wealth also decreases from 40% to 18%. Note that the population of stand-alone entrepreneurs is barely changed as financial frictions decrease. This implies that the decrease in stand-alone entrepreneurs' wealth derives from a decrease in their wealth on average, not from a decrease in the number of their population.[15]

Observation 6 (Aggregate Flotation Costs)**.** *An economy with business groups consumes larger flotations costs than an economy without business groups.*

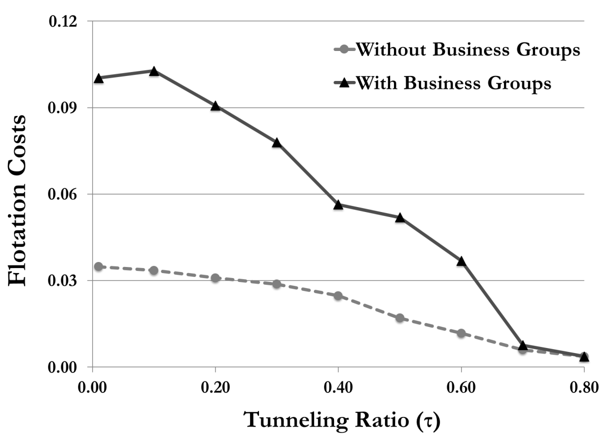


Figure 12: Flotation Costs

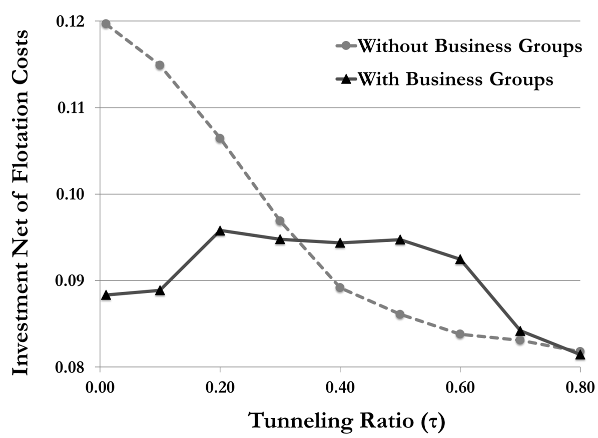


Figure 13: Investment Net of Flotation Costs

Observation 6 teaches us creating a business group can be an efficient choice for an individual, but not for an economy. As can be seen in Figure 12, flotation costs of an economy with business groups increase faster than an economy without business groups. Remember that the rich but untalented create business groups by paying flotation costs in order to launch productive subsidiaries. Thus, incorporating pyramidal business groups requires larger fixed costs. It is the problem that even though the more parent firms are incorporated, the fewer subsidiary firms are launched as financial frictions decrease.

Figure 13 shows the aggregate flotation costs in an economy with business groups are sizable. The aggregate investment net of flotation costs decreases as τ goes to zero. This complies with the observation that as τ goes to zero, the capital stock of an economy with business groups declines.

[15]Appendix B shows that pyramidal business groups can lower the wealth mobility from the bottom to the top.

One might ask why the net investment declines even though financial frictions decrease and the rate of return on capital keeps rising. The following Figure 14 gives an explanation. It shows that the investment rate of an economy with business groups is not only larger than an economy without business groups but also increases monotonically. Thus, a decrease in financial frictions indeed increases the investment rate of an economy. The excessive flotation costs used up by business groups, however, overwhelm the increase in investment and result in the decrease in net investment used for replenishing capital depreciation in a stationary state equilibrium as τ goes to zero.

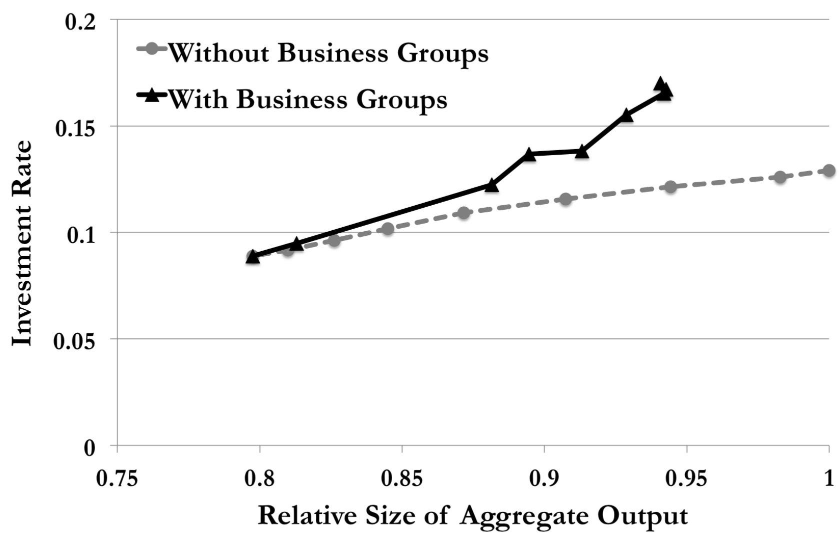


Figure 14: Investment Rate

Observation 7 (Aggregate Output)**.** *Let's define aggregate output of an economy as the sum of aggregate consumption and aggregate investment. Then, the aggregate output of an economy with business groups does not monotonically increase with investor protection. When the level of investor protection is strong enough such as* $(1-\tau) \geq 0.8$*, an increase in investor protection does not increase the aggregate output of an economy under the prevalence of business groups.*

Pyramidal business groups make the aggregate output of an economy regress toward a moderate level over the degree of financial frictions. Figure 15 shows that business groups can partially nullify the impact of financial frictions on aggregate output. At the early stage of its development where financial frictions are rampant, business groups help an economy produce larger aggregate output.[16]

[16] Figure 15 shows that a little development of investor protection is required for business groups to help an economy produce more aggregate output. This is because the internal equity finance of business groups works as leverage for

When the tunneling ratio τ goes to zero, however, Figure 15 shows that the aggregate output of an economy with business groups is stagnating.

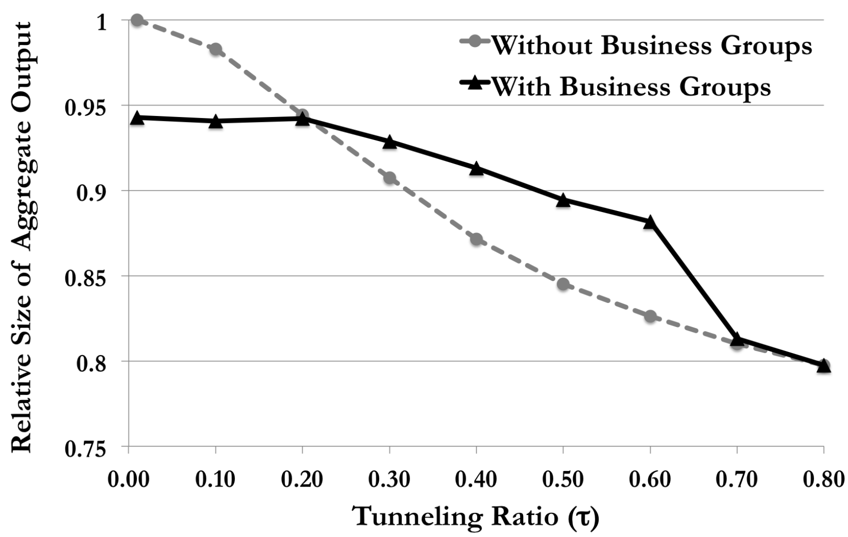


Figure 15: Aggregate Output

Observation 7 rebuts an argument that the economic impact of business groups would spontaneously vanish if investor protection improves. The stagnating aggregate output rather suggests that achieving good investor protection is not enough to lessen the effects of business groups on an economy and that aggregate output may not grow without restraining the prevalence of business groups. As argued in the previous remarks, business groups can be asymmetrically benefited by the improvement of investor protection in the model. The stagnating aggregate output of an economy with business groups in Figure 15 suggests that the asymmetric financial frictions become sizable and the benefits of business groups can be dominated by their costs when the degree of financial frictions is low enough such that $\tau \leq 0.2$.

The following Observation 8 shows how sizable the asymmetric financial frictions between business-group and stand-alone firms are and why dealing with pyramidal ownership structure is necessary for the development of external capital markets.

Observation 8 (External Capital Markets)**.** *Let's define the size of external capital markets as*

raising capital from external markets. Too large financial frictions can weaken the efficiency of the financial advantage of pyramidal business groups.

the sum of external debt finance and external equity finance used by all firms such that

$$
\begin{aligned}
\textit{External Capital Markets} = & \int_{o(z,a)=SA} \left\{1 - P^M(z,a)\right\} \cdot \left\{k^D(z,a) + k^E(z,a)\right\} dF(z,a) \\
& + \int_{o(z,a)=BG} \sum_{i \in \{1,2\}} \mathbb{E}_{a_2} \left[k_i^D(z,a|z_2(z,a),a_2) + k_i^E(z,a|z_2(z,a),a_2)\right] dF(z,a).
\end{aligned} \tag{51}
$$

Controlling for aggregate output, the external capital markets of an economy with business groups are smaller than those of an economy without business groups.

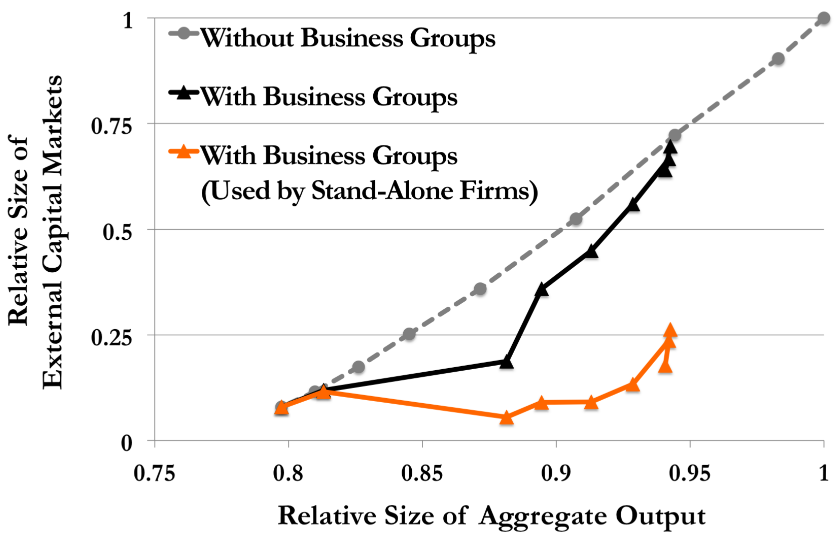


Figure 16: External Capital Markets

Figure 16 shows that the underdevelopment of external capital markets can be associated with the prevalence of business groups in an equilibrium. However, it does not mean that shutting down business groups increases the size of external capital markets. External capital markets of an economy with business groups are larger than those of an economy without business groups given the moderate degree of financial frictions such that $\tau \in [0.3, 0.6]$, while they are smaller given the low degree of financial frictions such that $\tau \leq 0.2$.[17] It is a more precise interpretation of the result that business groups decrease the size of external capital used by stand-alone firms in an equilibrium. Figure 16 shows that more than a half of external capital is used by business groups and that external capital used by stand-alone firms is smaller than its counterparts in an economy without business groups.

[17]Note that in Figure 16, each point on a line is connected with two adjacent points of tunneling ratio.

This underdevelopment of external capital markets in an economy with business groups arises due to the asymmetric financial frictions between business-group firms and stand-alone firms in the model. Note that given the same degree of financial frictions, the price of capital is always higher in an economy with business groups than that without business groups. The higher price of capital impairs stand-alone firms' external financing. Thus, stand-alone firms, which lack internal capital markets, should suffer from the tighter financial constraints and cannot but raise less external capital in an economy dominated by business groups.

7 Concluding Remarks and Future Research

Financial frictions can cause resource misallocation. They are understood as one of the major hindrances to economic development. Although many researchers have shown why and to what extent financial frictions affect an economy, few macroeconomic models have investigated private institutions that can arise as endogenous reactions against financial frictions. In this paper, I study the endogenous creation of pyramidal business groups and focus on the repercussions of their financial advantage given capital market imperfections.

There are three main implications of the model. First, pyramidal business groups can be efficient private institutions if external capital markets are underdeveloped due to severe financial frictions. Second, the asymmetric financial frictions between business-group and stand-alone firms can create inefficiencies that impair stand-alone firms' external financing in an equilibrium. Third, the prevalence of business groups does not spontaneously shrink as investor protection improves.

The last implication can be viewed as a limitation of this paper, in that the unvarying number of business-group firms in the model cannot explain why the prevalence of business groups differs across developed countries. Thus, finding a rationale for the cross-country difference can be an interesting topic for future research. For instance, Kandel, Kosenko, Morck, and Yafeh [2015] argue that the U.S. pyramidal business groups have almost disappeared because the U.S. government pursued specific policy measures to regulate business groups such as the Public Utility Holding Company Act (1935) and rising inter-corporate dividend taxation (after 1935). We can use the model developed in this paper to do a counter-factual analysis that examines how effectively the regulations adopted in the U.S. can reduce the prevalence of business groups and undo factor misallocation spawned by business groups.

Another follow-up research agenda can be the effects of pyramidal business groups on wealth inequality and socioeconomic mobility. The model developed in this paper suggests that the rich can entrench their wealth by building up pyramidal business groups, which results in a decrease in the probability of the poor accumulating wealth. Given the assumption that the inequality of entrepreneurial productivity stems from luck, business groups could be an institution that allows the rich to insure their wealth against their bad luck. This entrenchment of the rich implies that in an equilibrium, the prevalence of business groups can prevent the poor from exploiting their good luck. Thus, we can use the model to study how pyramidal business groups can change the patterns of wealth inequality and socioeconomic mobility. Preliminary results are presented in Appendix B.

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Appendix

A Market Clearing Conditions

Capital market clears such that

$$
\begin{aligned}
&\int \{a - c(z,a)\}\, dF(z,a) \\
&= \int_{o(z,a)=SA} \left\{k(z,a) + \mathbb{1}_{\sigma(z,a)>0} \cdot k^F\right\} \cdot \left\{1 - P^M(z,a)\right\}\, dF(z,a) \\
&+ \int_{o(z,a)=BG} \int_{\substack{o(z_2,a_2)=\{W,SA\} \\ w^M(z_2,a_2) \le \bar{w}^M(z,a|z_2)}} \left\{2k^F + w^M(z_2,a_2) + k_1^*(z,a|z_2,a_2) + k_2(z,a|z_2,a_2)\right\} \\
&\quad \cdot P^{BG}(z_2(z,a),a_2)\, da_2 dF(z,a) \\
&+ \int_{o(z,a)=BG} \int_{\substack{o(z_2,a_2)=\{W,SA\} \\ w^M(z_2,a_2) > \bar{w}^M(z,a|z_2)}} \left\{k^F + k_1(z,a|k_2^C = 0)\right\} \cdot P^{BG}(z_2(z,a),a_2)\, da_2 dF(z,a) \\
&+ \int_{o(z,a)=BG} \int_{o(z_2,a_2)=\{W,SA\}} \left\{k^F + k_1(z,a|k_2^C = 0)\right\} \cdot \left\{1 - P^{BG}(z_2(z,a),a_2)\right\}\, da_2 dF(z,a);
\end{aligned} \tag{52}
$$

and labor market clears such that

$$
\begin{aligned}
&\int_{o(z,a)=W} \left\{1 - P^M(z,a)\right\} dF(z,a) \\
&= \int_{o(z,a)=SA} \int_{z'} \ell(z', k(z,a))\, dG(z'|z) \cdot \left\{1 - P^M(z,a)\right\}\, dF(z,a) \\
&+ \int_{o(z,a)=BG} \int_{\substack{o(z_2,a_2)=\{W,SA\} \\ w^M(z_2,a_2) \le \bar{w}^M(z,a|z_2)}} \left\{ \int_{z'} \ell(z', k_1^*(z,a|z_2,a_2))\, dG(z'|z) + \int_{z_2'} \ell(z_2', k_2(z,a|z_2,a_2))\, dG(z_2'|z_2) \right\} \\
&\cdot P^{BG}(z_2(z,a),a_2)\, da_2 dF(z,a) \\
&+ \int_{o(z,a)=BG} \int_{\substack{o(z_2,a_2)=\{W,SA\} \\ w^M(z_2,a_2) > \bar{w}^M(z,a|z_2)}} \int_{z'} \ell(z', k_1(z,a|k_2^C = 0))\, dG(z'|z) \cdot P^{BG}(z_2(z,a),a_2)\, da_2 dF(z,a) \\
&+ \int_{o(z,a)=BG} \int_{o(z_2,a_2)=\{W,SA\}} \int_{z'} \ell(z', k_1(z,a|k_2^C = 0))\, dG(z'|z) \cdot \left\{1 - P^{BG}(z_2(z,a),a_2)\right\}\, da_2 dF(z,a)
\end{aligned} \tag{53}
$$

where $G(z'|z)$ is a conditional cdf derived from the transition probability of managerial talents.

B Pyramidal Business Groups and Wealth Inequality

The wealth Gini coefficient of an economy with business groups rises first and then declines as the strength of investor protection $(1 - \tau)$ improves. See the following Figure 17.

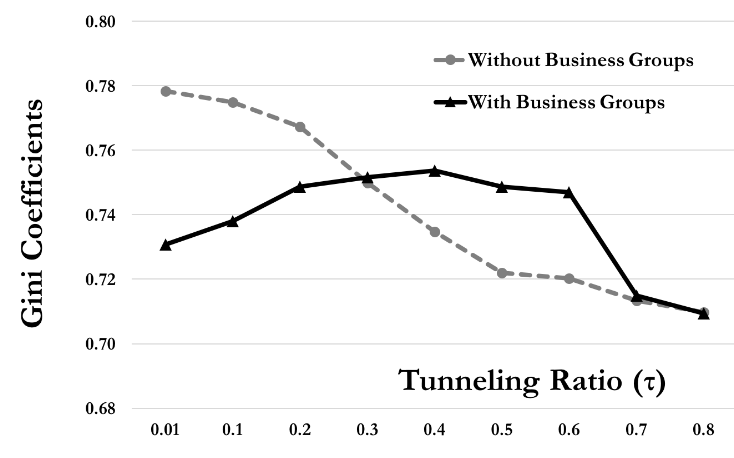

Figure 17: Wealth Gini Coefficients

This non-monotonic change in wealth inequality under the prevalence of business groups is not a trivial result because by construction, the model intends to feature a rising wealth inequality with investor protection. Given capital market imperfections, the less financial frictions allows entrepreneurs to use the more capital in production and to accumulate the more wealth. Wealth inequality in the model would follow the underlying managerial-talent inequality generated by a Markov process if no financial frictions existed. Thus, the hump-shaped Gini coefficients observed in Figure 17 suggests that business groups hinder the talented from accumulating wealth as τ goes to zero.

Figure 18 shows that business groups can lower a upward wealth mobility. It captures the probability of individuals moving from $a = a(6)$ to $a' \geq a(13)$ after ten periods. An individual of $a(6)$ living in an economy with business groups has a lower probability of being rich than that in an economy without business groups.

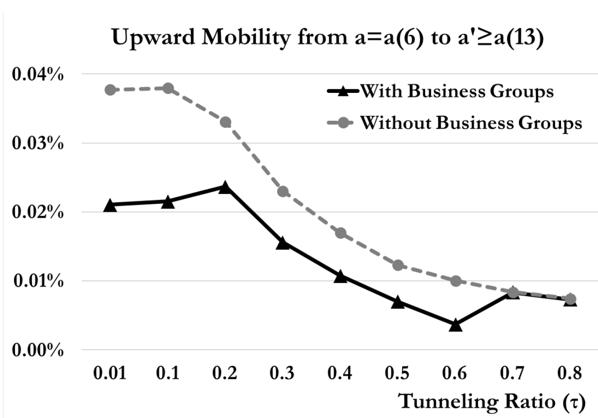

Figure 18: Upward Wealth Mobility

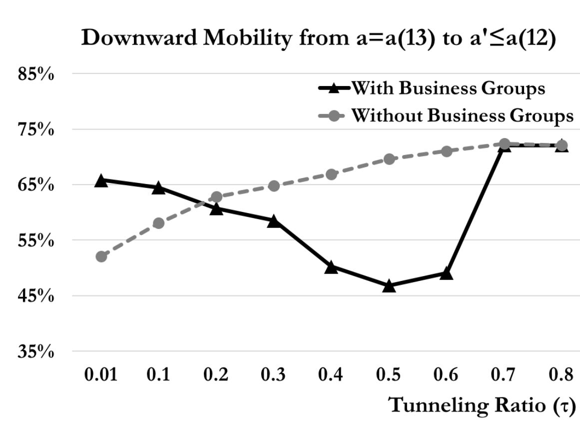

Figure 19: Downward Wealth Mobility

In contrast, Figure 19 shows that pyramidal business groups can lower a downward wealth mobility. However, this effect of lowering the downward wealth mobility decreases as investor protection improves. Figure 19 depicts the probability of individuals moving from $a = a(6)$ to $a' \leq a(12)$ after ten periods. Given the observation that the prevalence of business groups increases abruptly at $\tau = 0.6$ and persists as τ decreases, this rising downward mobility from $\tau = 0.5$ teaches us that creating a business group as an insurance for the rich becomes less efficient as τ decreases. Remember that the probability of launching a subsidiary firm is decreasing as τ decreases, which can be seen in Figure 7.

With the low upward wealth mobility and the U-shaped downward wealth mobility, business groups result in the non-monotonic population of the rich with respect to τ. Figure 20 captures the population of the rich increases first and then decreases as investor protection $(1-\tau)$ improves. Note that the population of the rich follows the similar pattern with the capital stock (Figure 11) and with the net aggregate investment (Figure 13) in an economy with business groups. This implies that a lower upward wealth mobility can be a symptom of inefficiencies pyramidal business groups induce in an equilibrium.

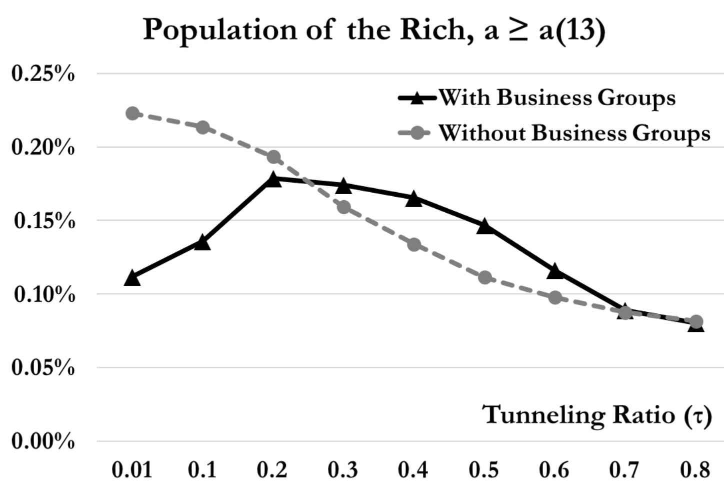

Figure 20: Population of the Rich

C Auxiliary Results of the Model

Observation 9 (Internal Capital Markets)**.** *Let's define the size of internal capital markets as the sum of internal equity finance used by all business groups such that*

$$\textit{Internal Capital Markets} = \int_{o(z,a)=BG} \mathbb{E}_{a_2}\left[k_2^C(z,a|z_2(z,a),a_2)\right] dF(z,a). \tag{54}$$

Then, internal capital markets of business groups are larger than external capital markets if the degree of financial frictions is moderate ($\tau \in [0.4, 0.6]$) *or low enough* ($\tau = 0.01$)*.*

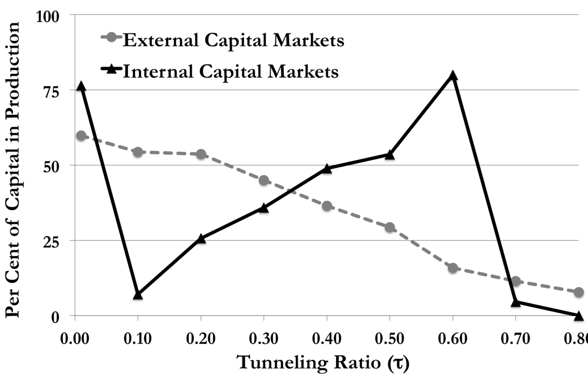

Figure 21: Internal Capital Markets

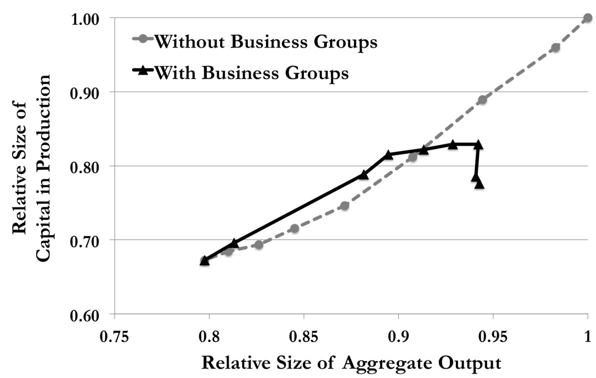

Figure 22: Capital Stock

An economy with business groups features sizable internal capital markets of business groups growing up early, lasting long. Figure 21 shows that the internal capital markets are even bigger and matured earlier than external capital markets given the moderate degree of financial frictions. As can be seen in Figure 22, controlling for aggregate output, the early development of internal capital markets makes the capital in production of an economy with business groups larger than that of an economy without business groups given the high degree of financial frictions such that $\tau \geq 0.5$.[18] The faster growth of internal capital markets suggests that business groups can be good substitutes for underdeveloped external capital markets in the early stage of an economy where financial frictions are rampant.

Another salient observation is the longevity of internal capital markets of business groups. Figure 21 shows that internal capital markets do not monotonically wane as financial frictions decrease. The size of internal capital markets decreases when the degree of financial frictions

[18]Each line is connected with two adjacent tunneling ratios such that $\tau \in \{0.01, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8\}$.

decreases from $\tau = 0.6$ to $\tau = 0.1$. However, it increases again when the degree of financial frictions decreases from $\tau = 0.1$ to $\tau = 0.01$. With low enough financial frictions, business groups are likely to be financially unconstrained and accumulate corporate savings. The rebounding internal capital markets in Figure 21 captures these excessive corporate savings of business groups.[19]

Observation 10 (Corporate Savings)**.** *Let's define corporate savings as the sum of assets that are not used in production but re-invested in external capital markets such that*

$$\begin{aligned} \textit{Corporate Savings} = & \int_{o(z,a)=SA} \left\{1 - P^M(z,a)\right\} \cdot \left\{-k^D(z,a) \cdot \mathbb{1}_{k^E(z,a)>0}\right\} dF(z,a) \\ & + \int_{o(z,a)=BG} \sum_{i \in \{1,2\}} \mathbb{E}_{a_2}\left[-k_i^D(z,a|z_2,a_2)\middle| P^{BG}(z_2(z,a),a_2)\right] dF(z,a). \end{aligned} \tag{55}$$

Then, given the level of aggregate output, the corporate savings of an economy with business groups are larger than those of an economy without business groups, and the gap of corporate savings between these two economies is enlarged as the degree of financial frictions decreases.

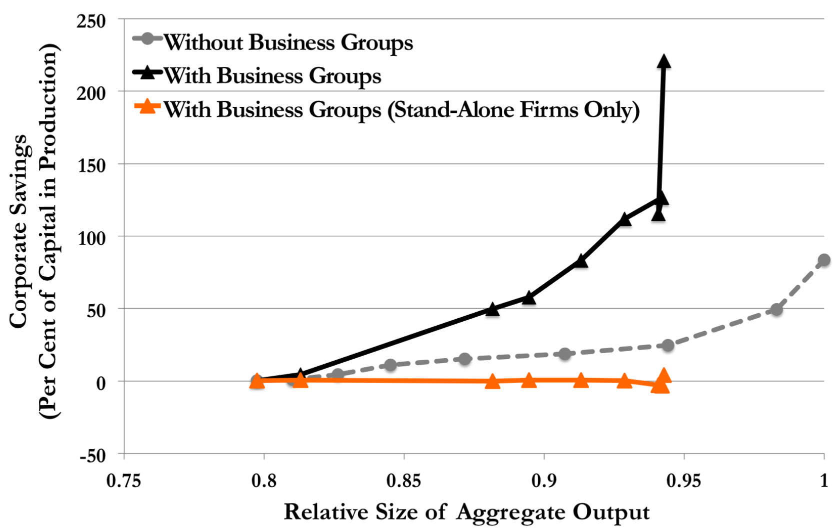

Figure 23: Corporate Savings

Given the asymmetric financial advantage of business groups, an economy with business groups features excessive corporate savings. Figure 23 shows that corporate savings grow much faster in an economy with business groups than those in an economy without business groups do. It also shows that most corporate savings are piled up by business groups.

[19]See Appendix C for the more detailed description of corporate savings. I define corporate savings as the assets corporations hold, which are not used in production but re-invested in external capital markets for risk sharing.

Corporate savings per se is good for an entrepreneur because the firm's minimum cash flow increases with corporate savings in the model. However, the asymmetrically large corporate savings of business groups can induce factor misallocation in an equilibrium. With smaller financial frictions ($d\tau < 0$), a business-group entrepreneur can raise more external equity finance ($dk_i^E > 0$), which is used not only to reduce private finance ($dk_1^C < 0$) for more consumption ($dc > 0$) but also to increase corporate savings ($d(-k_2^D) > 0$) for risk sharing, without increasing capital in production ($dk_i = 0$). Thus, the excessive corporate savings of business groups imply that business-group entrepreneurs can save less ($dk_1^C < 0$) by taking control of larger external capital ($d(k_i^D + k_i^E) > 0$). Note that in an economy with business groups, capital stock (Figure 22) is shrinking while corporate savings of business groups (Figure 23) are soring up as the tunneling ratio τ goes to zero.

Observation 11 (Aggregate Consumption and Investment Rate)**.** *Controlling for aggregate output, aggregate consumption in an economy with business groups is smaller than that in an economy without business groups, and accordingly the investment rate of an economy with business groups is higher than that of an economy without business groups.*

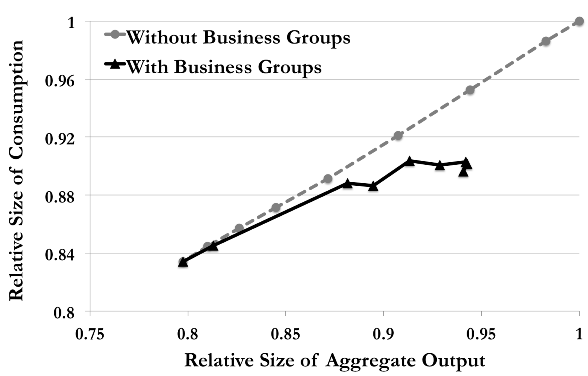

Figure 24: Consumption Level

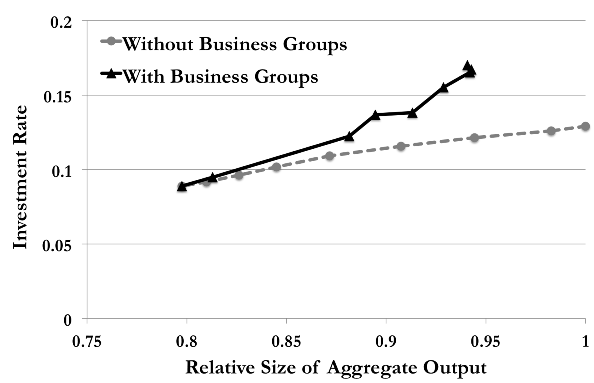

Figure 25: Investment Rate

Figure 24 shows that business groups can lower the aggregate consumption and make it stagnate even though the degree of financial frictions decreases.

The smaller aggregate consumption in an economy with business groups derives from a decrease in consumption of stand-alone entrepreneurs. Although the stand-alone entrepreneurs' population barely changes, their consumption decreases from 0.22 to 0.11 as financial frictions decrease from $\tau = 0.5$ to $\tau = 0.1$. As the share of aggregate consumption, stand-alone entrepreneurs' consumption accounts for 24% given $\tau = 0.5$ but only for 12% given $\tau = 0.1$. This contrasts with the rising consumption share of business-group entrepreneurs: from 13% to 17%.

Figure 25 shows that the investment rate of an economy with business groups is significantly higher than that of an economy without business groups with $\tau < 0.4$. Since a stationary equilibrium is employed in the model, the lower consumption level makes a pair with the higher investment rate. Note that both the lower consumption level and higher investment rate of an economy with business groups become more salient as the degree of financial frictions decreases. The declining capital stock of an economy with business groups in Figure 22 implies that the rising investment rate depicted in Figure 25 does not increase capital stock but is used as flotation costs for propping up pyramidal ownership structures which can be seen in Figure 12.

D Comparing the Model with Literature and Data

D.1 Replicating Masulis, Pham, and Zein [2011]

Masulis, Pham, and Zein [2011] report that the prevalence of family business groups in a country has strong negative association with the availability of external capital. They also report a positive but insignificant association between the prevalence of family business groups and the degree of investor protection. Their observations are replicated by the numerical example of the model. Let's define the prevalence of business groups as the ratio of the number of business-group firms to the number of firms incorporated, which is the definition Masulis, Pham, and Zein [2011] use in their paper.

First, Figure 26 shows that business groups initially thrive when severe financial frictions are mitigated and that they are partially disciplined but do not vanish as moderate financial frictions attenuate. Thus, the degree of investor protection, which is the inverse of financial frictions, can have a positive but insignificant association with the prevalence of business groups.

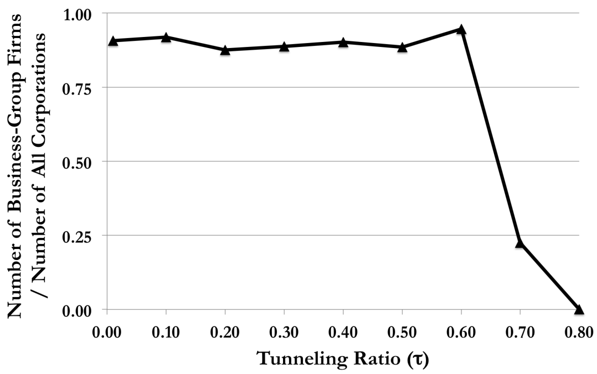

Figure 26: Prevalence of Business Groups

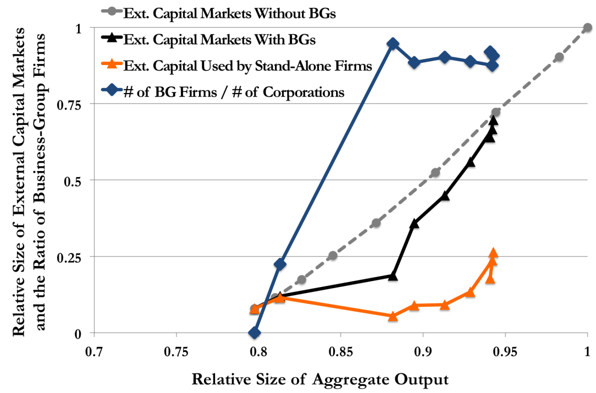

Figure 27: Underdeveloped Ext. Cap. Markets

Second, Figure 27 shows that the prevalence of business groups moves in opposite direction with the difference in the size of external capital markets between two economies with and without business groups. This negative correlation implies that the prevalence of business groups is significantly associated with the availability of capital in an economy. Admittedly, the size of external capital markets in the model is not the exact counterpart of capital availability for stand-alone firms Masulis, Pham, and Zein [2011] use in their paper because the former captures an equilibrium output but the latter captures the supply of capital.

This negative correlation between business groups and external capital markets can be explained by two competing channels. First, business groups can deter the development of external capital markets. Second, business groups can arise because of the underdevelopment of external capital markets.

The model developed in this paper suggests that the second channel is dominant with severe financial frictions and that the first channel is dominant with low enough financial frictions. Given severe financial frictions, an economy with business groups has larger capital stock with larger external capital markets than an economy without business groups. In contrast, given the low degree of financial frictions, an economy with business groups has smaller capital stock and smaller external capital markets than an economy without business groups. Since the prevalence of pyramidal business groups does not change in the degree of financial frictions, these reversals in capital markets imply that the direction of causality for the negative correlation can be reversed as the degree of financial frictions decreases.

D.2 Faster Growth of Corporate Savings, Lower Household Consumption, and Higher Consumption of Fixed Capital

I examine three characteristics of an economy with business groups that the model predicts: faster growth of corporate savings, lower household consumption, and higher consumption of fixed capital.

First, the model shows that corporate savings in an economy with business groups grow faster than those in an economy without business groups do when financial frictions attenuate. Figure 28 and Figure 29 depict annual trends of changes in corporate savings for 23 countries from 2004 to 2014. The Annual trend of corporate savings for a country is estimated with changes in corporate-savings ratios, which are non-financial corporate gross savings divided by gross national disposable income collected from OECD.

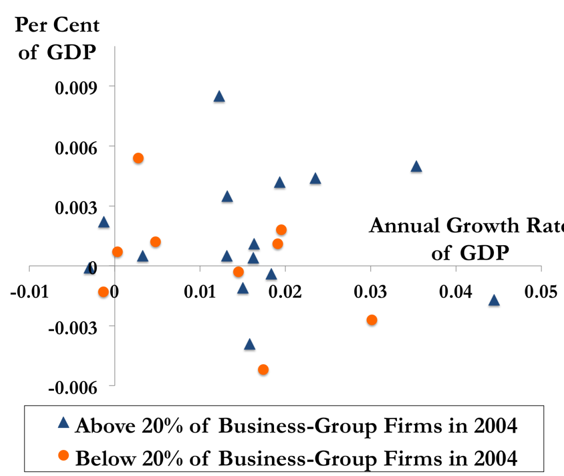

Figure 28: Annual Trend of Changes in Corporate Savings for Each Country Since 2004 (Data is Collected from OECD, PWT8.1, and Masulis et al. [2011])

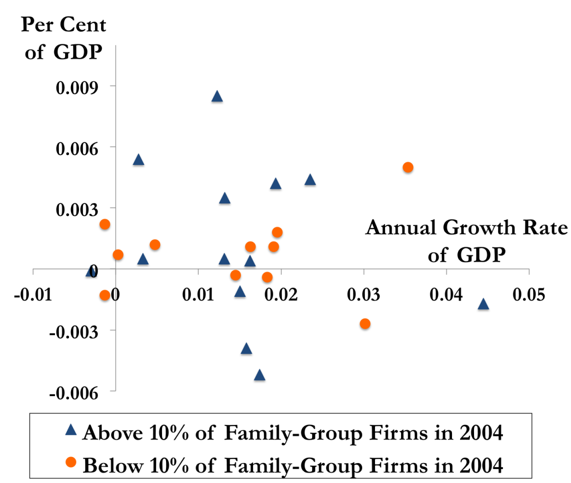

Figure 29: Annual Trend of Changes in Corporate Savings for Each Country Since 2004 (Data is Collected from OECD, PWT8.1, and Masulis et al. [2011])

I assume that the degree of financial frictions has been decreased since 2004 as the global capital markets have been expanded. I also assume that each country has its own prevalence of business groups because of country specific environment such as government regulations. Given the assumptions, the model predicts that corporate savings in a country with the higher prevalence of business groups grow faster than those in a country with the lower prevalence of business groups do.

Figure 28, in which the prevalence of business groups is measured by the relative number of family and non-family business-group firms, shows that countries with more than 20% of the prevalence tend to have corporate savings growing faster. The average growth rate of corporate savings in countries with above 20% of business-group firms is 0.15%, and that in countries with below 20% of business-group firms is almost 0%. Figure 29, in which the prevalence of business groups is measured by the relative number of family-group firms only, shows that there is no strict association between the prevalence of family groups and the growth rates of corporate savings. The average growth rate of corporate savings in countries with above 10% of family-group firms is 0.12%, and that in countries with below 10% of family-group firms is 0.076%.

Second, the model shows that the consumption level of an economy with business groups is significantly lower than that of an economy without business groups unless financial frictions are too severe. Figure 30 and Figure 31 show that the share of household consumption has negative association with the prevalence of business-group firms within the group of countries above $30,000 real GDP per capita. Note that the negative correlation disappears if real GDP per capita is less than $30,000.

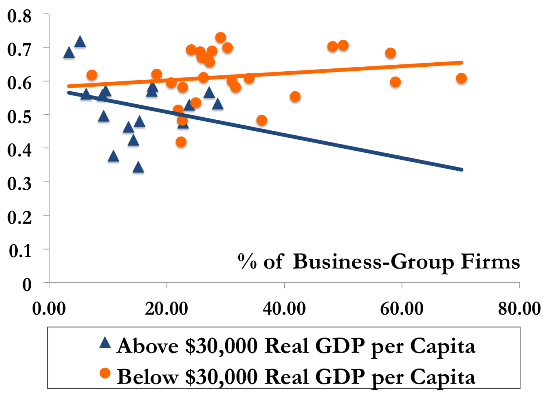

Figure 30: Share of Household Consumption (2004, Current PPPs, Collected from PWT8.1 and Masulis et al. [2011])

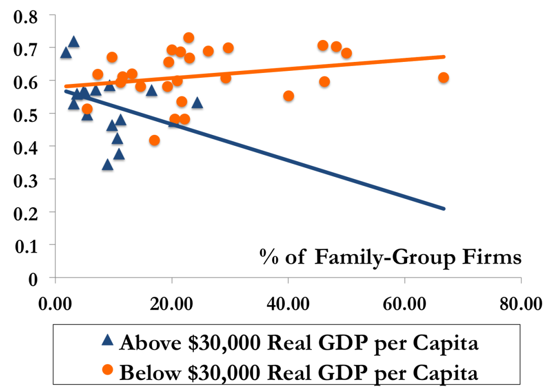

Figure 31: Share of Household Consumption (2004, Current PPPs, Collected from PWT8.1 and Masulis et al. [2011])

Because the model predicts significant lower aggregate consumption if an economy dominated by business groups produces higher aggregate output, I divide countries into two groups, one with real GDP per capita greater than or equal to $30,000 and the other with real GDP per capita less than $30,000.[20] The data of real GDP per capita and the share of household consumption is collected from Penn World Table 8.1 for 44 countries. I use the year of 2004 data points because Masulis, Pham, and Zein [2011] collect the prevalence of business groups as of 2004.

[20] Out of 44 countries in the sample, the number of countries with above $30,000 real GDP per capital is 17, and the number of countries with below $30,000 real GDP per capital is 27.

Lastly, the model predicts that the consumption of fixed capital is significantly higher in an economy with business groups than that in an economy without business groups unless financial frictions are too severe. Figure 32 and Figure 33 show that the consumption of fixed capital is positively associated with the prevalence of business groups and that the association is stronger within the group of countries above \$30,000 real GDP per capita.

Note that the model employs a stationary equilibrium and so that investment in the model is equivalent to the consumption of fixed capital or capital depreciation. Given that the model predicts significantly higher investment rates of an economy with business groups if it produces higher aggregate output, countries are divided into two groups, one with real GDP per capita greater than or equal to \$30,000 and the other with real GDP per capita less than \$30,000.[21] Consumption of fixed capital as a share of GDP for 23 countries in 2004 is collected from OECD and real GDP per capita in 2004 is collected from Penn World Table 8.1.

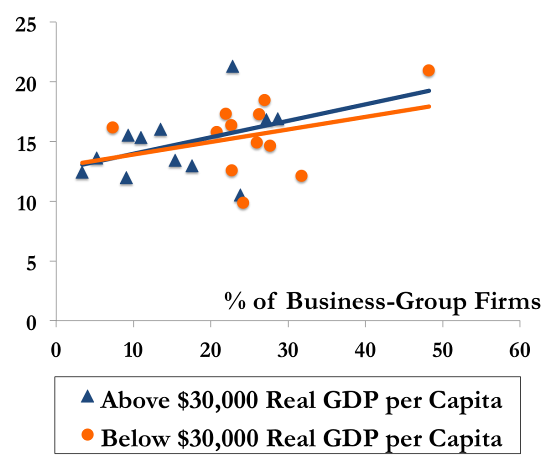

Figure 32: Consumption of Fixed Capital (2004, Per Cent of GDP, Collected from OECD and Masulis et al. [2011])

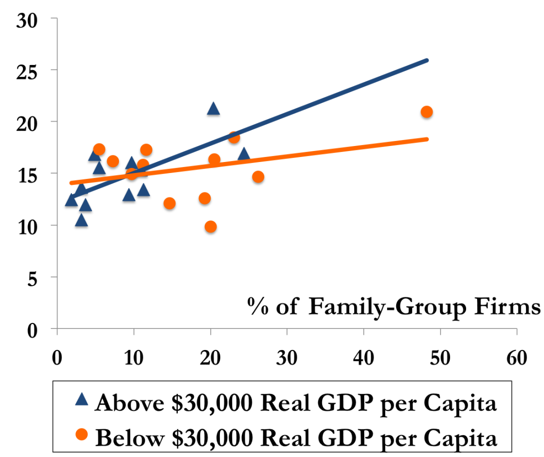

Figure 33: Consumption of Fixed Capital (2004, Per Cent of GDP, Collected from OECD and Masulis et al. [2011])

E Proof of Proposition 1

Let's define $\phi \in \left[\underline{\phi}, 1\right]$ and $\nu \leq 1$ such that

$$k^C = \phi a, \quad k^D = \frac{1-\tau}{1+r}\nu \inf_{z',\delta'} \left[\pi(z', \delta'|z, k)\right]. \tag{56}$$

[21] \$30,000 real GDP per capita is the median of the sample.

Then, a stand-alone entrepreneur running a publicly held corporation solves the following problem.

$$\mathcal{L}(z,a) = u\left((1-\phi)a - s\right) + \beta\mathbb{E}_{z',\delta'}\left[V(z',a')|z\right] + \lambda_s s + \lambda_\phi(\phi - \underline{\phi}) + \lambda_\nu(1-\nu) + \lambda_\sigma(\bar{\sigma} - \sigma) \quad (57)$$

where

$$\begin{aligned} a' &= (1+r)s + \tau\pi(z',\delta'|z,k) + (1-\sigma)(1-\tau)\left\{\pi(z',\delta'|z,k) - \nu \inf_{z',\delta'}\left[\pi(z',\delta'|z,k)\right]\right\} \\ k &= \phi a - k^F + \frac{1-\tau}{1+r}\left\{\sigma\mathbb{E}_{z',\delta'}\left[\pi(z',\delta'|z,k)\right] + (1-\sigma)\nu\inf_{z',\delta'}\left[\pi(z',\delta'|z,k)\right]\right\} \\ \underline{\phi} &= \frac{k^F}{a}. \end{aligned} \quad (58)$$

To simplify notations, let's suppress arguments of functions and operators unless there is ambiguity. The corresponding Kuhn-Tucker conditions are as follows. For the optimal private saving, s,

$$\begin{aligned} \lambda_s s &= 0, \\ \lambda_s &= u'\left((1-\phi)a - s\right) - (1+r)\beta\mathbb{E}V_a \\ &\geq 0. \end{aligned} \quad (59)$$

For the optimal private finance, $k^C = \phi a$,

$$\begin{aligned} \lambda_\phi(\phi - \underline{\phi}) &= 0, \\ \lambda_\phi &= a\underbrace{\left[u'\left((1-\phi)a - s\right) - (1+r)\beta\mathbb{E}V_a\right]}_{=\lambda_s} - a\beta\mathbb{E}\left[V_a \cdot \{-(1+r) + AB\}\right] \\ &\geq 0 \end{aligned} \quad (60)$$

where

$$\begin{aligned} A &\equiv \left[1 - \frac{1-\tau}{1+r}\left\{\sigma\mathbb{E}_{z',\delta'}\left[\frac{d}{dk}\pi(z',\delta'|z,k)\right] + (1-\sigma)\nu\inf_{z',\delta'}\left[\frac{d}{dk}\pi(z',\delta'|z,k)\right]\right\}\right]^{-1} \\ B &\equiv \tau\frac{d}{dk}\pi(z',\delta'|z,k) + (1-\sigma)(1-\tau)\left\{\frac{d}{dk}\pi(z',\delta'|z,k) - \nu\inf_{z',\delta'}\left[\frac{d}{dk}\pi(z',\delta'|z,k)\right]\right\}. \end{aligned} \quad (61)$$

For the optimal external debt finance, $k^D = \frac{1-\tau}{1+r}\nu \inf \pi$,

$$
\begin{aligned}
&\lambda_\nu(1-\nu) = 0, \\
&\qquad \lambda_\nu = (1-\sigma)\frac{1-\tau}{1+r}\inf \pi \beta \mathbb{E}\left[V_a \cdot \{-(1+r) + AB\}\right] \\
&\qquad\quad = \underbrace{(1-\sigma)\frac{1-\tau}{1+r}\inf \pi A}_{=\frac{dk}{d\nu}} \cdot \underbrace{\left\{ \underbrace{\beta \mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi' - (1+r)\}\right]}_{\text{Marginal Value of Expected Return}} - \underbrace{(1-\sigma+\sigma\tau)\beta\mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi' - \pi'\}\right]}_{\text{Marginal Cost of Risk}} \right\}}_{\text{Marginal Expected Value of Investment}} \\
&\qquad\quad \geq 0
\end{aligned}
\tag{62}
$$

where

$$
\pi' \equiv \frac{d}{dk}\pi(z', \delta'|z, k).
$$

Lastly, given $\sigma > 0$, the optimal external equity finance, $k^E = \frac{\sigma}{1+r}\left\{(1-\tau)\mathbb{E}\pi - (1+r)k^D\right\}$, satisfies the following conditions.

$$
\begin{aligned}
&\lambda_\sigma(\bar{\sigma} - \sigma) = 0, \\
&\qquad \lambda_\sigma = \underbrace{(1-\tau)\beta\mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi - \pi\}\right]}_{\substack{=\beta\mathbb{E}\left[V_a \cdot \frac{da'}{d\sigma}\Big|_{dk(\nu,\sigma)=0}\right]>0 \\ \text{Marginal Value of Risk Sharing} \\ \text{Given the Fixed Amount of Capital } k}} + \underbrace{\underbrace{\frac{1-\tau}{1+r}\left(\mathbb{E}\pi - \nu \inf \pi\right) A}_{=\frac{dk}{d\sigma}} \cdot \beta\mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi' - (1+r) - (1-\sigma+\sigma\tau)(\mathbb{E}\pi' - \pi')\}\right]}_{=|J|\cdot\lambda_\nu \text{ where } |J| = \left|\frac{d\nu}{d\sigma}\right|_{dk(\nu,\sigma)=0} = \frac{\mathbb{E}\pi - \nu \inf \pi}{(1-\sigma)\inf \pi} > 0} \\
&\qquad\quad \geq 0
\end{aligned}
\tag{63}
$$

Proof. From the Kuhn-Tucker condition for λ_ϕ,

$$
\lambda_s = \frac{1}{a}\left\{\lambda_\phi + |J|\,\lambda_\nu\right\} \text{ where } |J| = \left|\frac{d\nu}{d\phi}\right|_{dk(\phi,\nu)=0} > 0.
$$

Given the assumption that a firm is allowed to invest in a risk-free asset, the external debt finance k^D is only bounded above such that $\lambda_\nu \geq 0$. If $\lambda_\nu > 0$, $\lambda_s > 0$ and the optimal private saving is bounded below such that $s = 0$. If $\lambda_\nu = 0$, $\lambda_s = \frac{\lambda_\phi}{a}$ and the optimal private saving and the optimal private finance are undetermined because the marginal costs of them are aligned such that $\mathbb{1}_{\lambda_s > 0} = \mathbb{1}_{\lambda_\phi > 0}$. Thus, the zero private saving, $s = 0$, is weakly preferred and the optimization can be achieved by choosing $\{\phi, \nu, \sigma\}$ with $s = 0$.

Given Condition 1 and $\lambda_\nu \geq 0$, the marginal value of external equity finance is always greater than zero such that

$$\lambda_\sigma = (1-\tau)\beta\mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi - \pi\}\right] + \left|\frac{d\nu}{d\sigma}\right|_{dk(\nu,\sigma)=0} \cdot \lambda_\nu$$
$$> 0.$$

Thus, given $\sigma > 0$, the optimal external equity finance is bounded above such that $\sigma = \bar{\sigma}$. □

Figure 34 shows that the optimal external equity finance is binding. Given the entrepreneur's managerial talent and wealth, (z, a), there is a downward sloping curve on which the marginal expected value of investment is zero such that $\lambda_\nu(\sigma, k(\sigma,\nu), a'(\sigma,\nu)|s,\phi) = 0$. From the Proposition 1, the marginal value of external equity finance is always positive on the curve such that $\lambda_\sigma|_{\lambda_\nu=0} = \beta\mathbb{E}\left[V_a \cdot \frac{da'}{d\sigma}\Big|_{dk=0}\right] = (1-\tau)\beta\mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi - \pi\}\right] > 0$ because of the positive marginal benefit of risk sharing through external equity finance. The entrepreneur, thus, sells her firm's shares as many as possible until the constraint for the external equity finance is binding such that $\sigma = \bar{\sigma}_{SA}$.

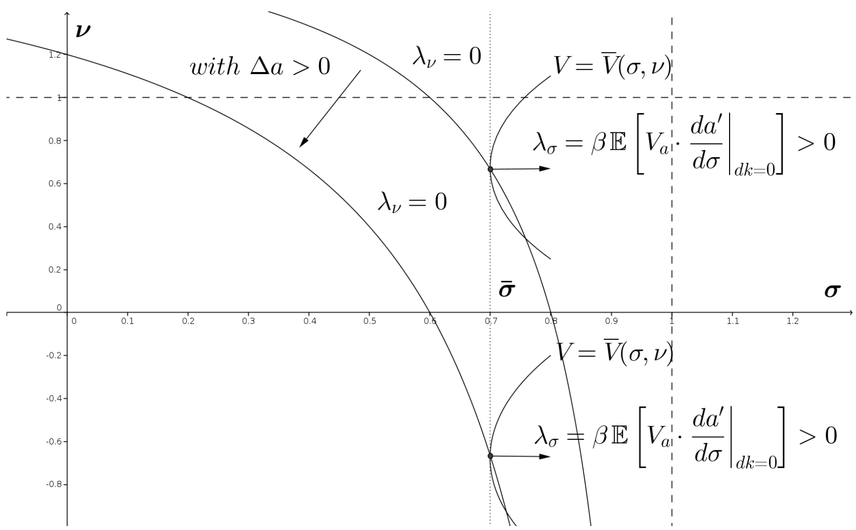

Figure 34: Risk Sharing and Binding External Equity Finance

Second, Figure 35 shows how the optimal private saving becomes zero. The risk-free investment opportunity keeps the marginal opportunity cost of private saving greater than or equal to that of private finance such that $a\lambda_s \geq \lambda_\phi \geq 0$. Given $a\lambda_s \geq \lambda_\phi$, the indifference curve $V = \overline{V}(\phi, s)$ cuts from below the line of constant marginal opportunity cost of private saving, $\lambda_s = \overline{\lambda}_s(c, a')$, which is achieved by $dc(s,\phi) = da'(s, \nu(s,\phi), k(\phi,\nu)) = dk(\phi, \nu(s,\phi)) = 0$. Thus, the indifference curve is pushed down until the borrowing constraint of an entrepreneur is binding such that $s = 0$.

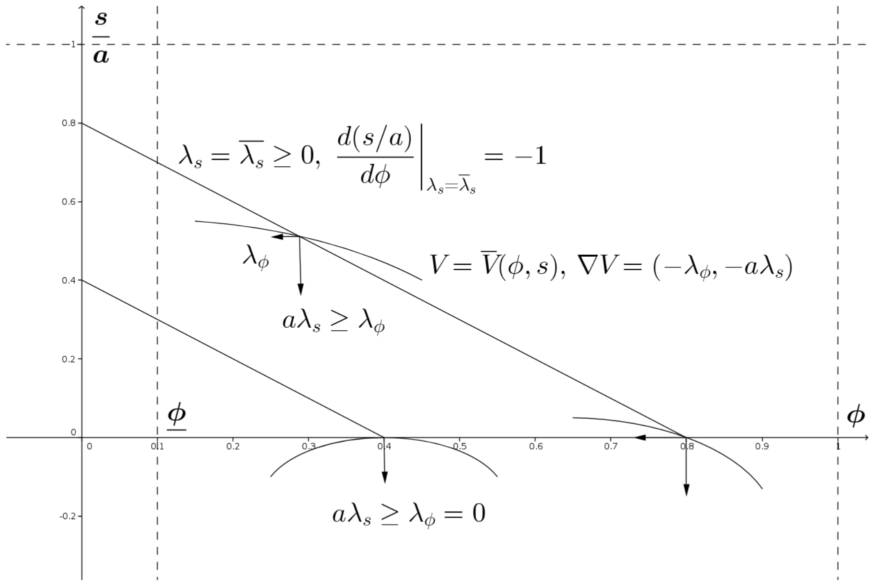

Figure 35: Non-Negative Marginal Expected Value of Investment and Binding Private Borrowing

F Proof of Proposition 2

Let's define $\phi \in \left[\underline{\phi}, 1\right]$, $\nu_1 \leq 1$, and $\nu_2 \leq 1$ such that

$$\begin{aligned}
k_1^C &= \phi a, \\
k_1^D &= \frac{1-\tau}{1+r}\nu_1 \left[\inf_{z_1', \delta_1'} \left[\pi(z_1', \delta_1' | z_1, k_1^*)\right] + (1-\sigma_2)\left\{(1-\tau)\inf_{z_2', \delta_2'} \left[\pi(z_2', \delta_2' | z_2, k_2)\right] - (1+r)k_2^D\right\}\right], \\
k_2^D &= \frac{1-\tau}{1+r}\nu_2 \inf_{z_2', \delta_2'} \pi\left[(z_2', \delta_2' | z_2, k_2)\right].
\end{aligned} \tag{64}$$

Then, given (z_2, w^M), a business-group entrepreneur with (z_1, a) solves the following problem.

$$\begin{aligned}
\mathcal{L}(z_1, a | z_2, w^M) = u\left((1-\phi)a - s\right) + \beta \mathbb{E}_{z_1', z_2', \delta_1', \delta_2'}\left[V(z_1', a') | z_1\right] + \lambda_s s + \lambda_\phi(\phi - \underline{\phi}) + \lambda_{k_2^C}\left(k_2^C - k^F - w_M\right) \\
+ \lambda_{\nu_1}(1-\nu_1) + \lambda_{\nu_2}(1-\nu_2) + \lambda_{\sigma_1}(\bar{\sigma} - \sigma_1) + \lambda_{\sigma_2}(\bar{\sigma} - \sigma_2)
\end{aligned} \tag{65}$$

where

$$
\begin{aligned}
a' &= (1+r)s + \tau\pi(z_1', \delta_1'|z_1, k_1^*) \\
&\quad + (1-\sigma_1)(1-\tau)\left[\pi(z_1', \delta_1'|z_1, k_1^*) - \nu_1\left\{\inf_{z_1',\delta_1'}[\pi(z_1', \delta_1'|z_1, k_1^*)] + (1-\sigma_2)(1-\nu_2)(1-\tau)\inf_{z_2',\delta_2'}[\pi(z_2', \delta_2'|z_2, k_2)]\right\}\right] \\
&\quad + \tau\pi(z_2', \delta_2'|z_2, k_2) + (1-\sigma_1+\sigma_1\tau)(1-\sigma_2)(1-\tau)\left\{\pi(z_2', \delta_2'|z_2, k_2) - \nu_2\inf_{z_2',\delta_2'}[\pi(z_2', \delta_2'|z_2, k_2)]\right\} \\
k_1^* &= \phi a - k^F - k_2^C + \frac{1-\tau}{1+r}\left\{\sigma_1\mathbb{E}_{z_1',\delta_1'}[\pi(z_1', \delta_1'|z_1, k_1^*)] + (1-\sigma_1)\nu_1\inf_{z_1',\delta_1'}[\pi(z_1', \delta_1'|z_1, k_1^*)]\right\} \\
&\quad + \frac{(1-\tau)^2(1-\sigma_2)}{1+r}\left[\sigma_1\left\{\mathbb{E}_{z_2',\delta_2'}[\pi(z_2', \delta_2'|z_2, k_2)] - \nu_2\inf_{z_2',\delta_2'}[\pi(z_2', \delta_2'|z_2, k_2)]\right\} + (1-\sigma_1)\nu_1(1-\nu_2)\inf_{z_2',\delta_2'}[\pi(z_2', \delta_2'|z_2, k_2)]\right] \\
k_2 &= k_2^C - k^F - w^M + \frac{1-\tau}{1+r}\left\{\sigma_2\mathbb{E}_{z_2',\delta_2'}[\pi(z_2', \delta_2'|z_2, k_2)] + (1-\sigma_2)\nu_2\inf_{z_2',\delta_2'}[\pi(z_2', \delta_2'|z_2, k_2)]\right\}.
\end{aligned} \tag{66}
$$

The corresponding Kuhn-Tucker conditions are as follows. Let's suppress arguments of functions and operators for simplicity unless there is ambiguity. For the optimal private saving, s,

$$
\begin{aligned}
\lambda_s s &= 0 \\
\lambda_s &= u'((1-\phi)a - s) - (1+r)\beta\mathbb{E}V_a \\
&\geq 0.
\end{aligned} \tag{67}
$$

For the optimal private finance of Firm 1, $k_1^C = \phi a$,

$$
\begin{aligned}
\lambda_\phi(\phi - \underline{\phi}) &= 0 \\
\lambda_\phi &= a\underbrace{[u'((1-\phi)a - s) - (1+r)\beta\mathbb{E}V_a]}_{=\lambda_s} - a\beta\mathbb{E}[V_a \cdot \{-(1+r) + A_1B_1\}] \\
&\geq 0
\end{aligned} \tag{68}
$$

where

$$
\begin{aligned}
A_1 &\equiv \left[1 - \frac{1-\tau}{1+r}\left\{\sigma_1\mathbb{E}_{z_1',\delta_1'}\pi_1' + (1-\sigma_1)\nu_1\inf_{z_1',\delta_1'}\pi_1'\right\}\right]^{-1} \\
B_1 &\equiv \tau\pi_1' + (1-\sigma_1)(1-\tau)(\pi_1' - \nu_1\inf\pi_1') \\
\pi_1' &\equiv \frac{d}{dk_1^*}\pi_1(z_1', \delta_1'|z_1, k_1^*).
\end{aligned} \tag{69}
$$

For the optimal external debt finance of Firm 1, $k_1^D = \frac{1-\tau}{1+r}\nu_1 \left[\inf \pi_1 + (1-\sigma_2)(1-\nu_2)(1-\tau)\inf \pi_2\right]$,

$$
\begin{aligned}
&\lambda_{\nu_1}(1-\nu_1) = 0 \\
&\qquad \lambda_{\nu_1} = (1-\sigma_1)\frac{1-\tau}{1+r}\left\{\inf \pi_1 + (1-\sigma_2)(1-\tau)(1-\nu_2)\inf \pi_2\right\} \cdot \beta\mathbb{E}\left[V_a \cdot \{-(1+r) + A_1 B_1\}\right] \\
&\qquad\quad = \underbrace{(1-\sigma_1)\frac{1-\tau}{1+r}\left\{\inf \pi_1 + (1-\sigma_2)(1-\tau)(1-\nu_2)\inf \pi_2\right\} A_1}_{=\frac{dk_1^*}{d\nu_1}} \\
&\qquad\qquad \cdot \underbrace{\left\{\underbrace{\beta\mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi_1' - (1+r)\}\right]}_{\text{Marginal Value of Expected Return}} - \underbrace{(1-\sigma_1+\sigma_1\tau)\beta\mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi_1' - \pi_1'\}\right]}_{\text{Marginal Cost of Risk}}\right\}}_{\text{Marginal Expected Value of Investment through Firm 1}} \\
&\qquad\quad \geq 0.
\end{aligned}
\tag{70}
$$

For the optimal external equity finance of Firm 1,

$$
k_1^E = \frac{\sigma_1}{1+r}\left[(1-\tau)\mathbb{E}\pi_1 + (1-\tau)(1-\sigma_2)\left\{(1-\tau)\mathbb{E}\pi_2 - (1+r)k_2^D\right\} - (1+r)k_1^D\right],
$$

$$
\begin{aligned}
&\lambda_{\sigma_1}(\bar{\sigma} - \sigma_1) = 0 \\
&\qquad \lambda_{\sigma_1} = \underbrace{(1-\tau)\beta\mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi_1 - \pi_1\}\right] + (1-\tau)^2(1-\sigma_2)\beta\mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi_2 - \pi_2\}\right]}_{\substack{=\beta\mathbb{E}\left[V_a \cdot \frac{da'}{d\sigma_1}\Big|_{dk_1^*(\sigma_1,\nu_1)=0}\right]>0 \\ \text{Marginal Value of Risk Sharing Through Firm 1 Given Capital } (k_1^*, k_2)}} \\
&\qquad\qquad + \underbrace{\underbrace{\left[\frac{1-\tau}{1+r}\left\{\mathbb{E}\pi_1 - \nu_1\left(\inf \pi_1 + (1-\sigma_2)(1-\tau)(1-\nu_2)\inf \pi_2\right)\right\} + \frac{(1-\tau)^2}{1+r}(1-\sigma_2)\left\{\mathbb{E}\pi_2 - \nu_2 \inf \pi_2\right\}\right] A_1}_{=\frac{dk_1^*}{d\sigma_1}} \\
&\qquad\qquad\quad \cdot \left\{\beta\mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi_1' - (1+r)\}\right] - (1-\sigma_1+\sigma_1\tau)\beta\mathbb{E}\left[V_a \cdot \{\mathbb{E}\pi_1' - \pi_1'\}\right]\right\}}_{=|J|\cdot\lambda_{\nu_1} \text{ where } |J| = \left|\frac{d\nu_1}{d\sigma_1}\right|_{dk_1^*(\sigma_1,\nu_1)=0}} \\
&\qquad \geq 0.
\end{aligned}
\tag{71}
$$

For the optimal internal equity finance from Firm 1 to Firm 2, k_2^C,

$$
\begin{aligned}
&\lambda_{k_2^C}(k_2^C - k^F - w^M) = 0 \\
&\qquad \lambda_{k_2^C} = \beta\mathbb{E}\left[V_a \cdot \{A_1 B_1 - A_{12} A_2 A_1 B_1 - A_2 B_2\}\right] \\
&\qquad\quad \geq 0
\end{aligned}
\tag{72}
$$

where

$$
\begin{aligned}
A_{12} &\equiv \frac{(1-\tau)^2(1-\sigma_2)}{1+r}\left\{\sigma_1\left(\mathbb{E}_{z_2',\delta_2'}\pi_2' - \nu_2 \inf_{z_2',\delta_2'} \pi_2'\right) + (1-\sigma_1)\nu_1(1-\nu_2)\inf_{z_2',\delta_2'}\pi_2'\right\} \\
A_2 &\equiv \left[1 - \frac{1-\tau}{1+r}\left\{\sigma_2 \mathbb{E}_{z_2',\delta_2'}\pi_2' + (1-\sigma_2)\nu_2 \inf_{z_2',\delta_2'}\pi_2'\right\}\right]^{-1} \\
B_2 &\equiv \tau\pi_2' + (1-\sigma_1+\sigma_1\tau)(1-\sigma_2)(1-\tau)(\pi_2' - \nu_2\inf_{z_2',\delta_2'}\pi_2') - (1-\tau)^2(1-\sigma_1)(1-\sigma_2)\nu_1(1-\nu_2)\inf_{z_2',\delta_2'}\pi_2'.
\end{aligned}
\tag{73}
$$

For the optimal external debt finance of Firm 2, $k_2^D = \frac{1-\tau}{1+r}\nu_2 \inf \pi_2$,

$$
\begin{aligned}
&\lambda_{\nu_2}(1-\nu_2) = 0 \\
&\qquad \lambda_{\nu_2} = \underbrace{\frac{1-\tau}{1+r}(1-\sigma_2)\inf\pi_2\left\{\tau + (1-\tau)(1-\sigma_1)(1-\nu_1)\right\}A_1}_{=\frac{dk_1^*}{d\nu_2}\Big|_{dk_2(k_2^C,\nu_2)=0} \quad \to 0 \text{ as } \tau\to 0 \text{ with } \nu_1=1} \cdot \underbrace{\beta\mathbb{E}\left[V_a\cdot\left\{(\mathbb{E}\pi_1' - (1+r)) - (1-\sigma_1+\sigma_1\tau)(\mathbb{E}\pi_1' - \pi_1')\right\}\right]}_{\text{Marginal Expected Value of Investment through Firm 1}} \\
&\qquad\qquad \underbrace{-\frac{1-\tau}{1+r}(1-\sigma_2)\inf\pi_2}_{=\frac{dk_2^C}{d\nu_2}\Big|_{dk_2=0}} \cdot \underbrace{\beta\mathbb{E}\left[V_a\cdot\left\{A_1B_1 - A_{12}A_2A_1B_1 - A_2B_2\right)\right]}_{=\lambda_{k_2^C}} \\
&\qquad\qquad \geq 0.
\end{aligned}
\tag{74}
$$

For the optimal external equity finance of Firm 2, $k_2^E = \frac{\sigma_2}{1+r}\left\{(1-\tau)\mathbb{E}\pi_2 - (1+r)k_2^D\right\}$,

$$
\begin{aligned}
&\lambda_{\sigma_2}(\bar{\sigma} - \sigma_2) = 0 \\
&\lambda_{\sigma_2} = \underbrace{\beta\mathbb{E}\left[V_a\cdot\frac{1-\tau}{1+r}\left(\mathbb{E}\pi_2 - \nu_2\inf\pi_2\right)\left\{\left(-1 + A_{12}A_2\right)A_1B_1 + A_2B_2 + \left\{\tau + (1-\tau)(1-\sigma_1)(1-\nu_1)\right\}\left\{-(1+r) + A_1B_1\right\}\right\}\right]}_{=|J|\cdot\lambda_{\nu_2} \text{ where } |J| = \left|\frac{d\nu_2}{d\sigma_2}\right|_{dk_2(\nu_2,\sigma_2)=0} = \frac{\mathbb{E}\pi_2 - \nu_2\inf\pi_2}{(1-\sigma_2)\inf\pi_2}} \\
&\quad + \underbrace{\frac{(1-\tau)^2}{1+r}(1-\sigma_1)\nu_1(\mathbb{E}\pi_2 - \inf\pi_2)A_1}_{=\frac{dk_1^*}{d\sigma_2}\Big|_{dk_2(\nu_2,\sigma_2)=0}} \cdot \underbrace{\beta\mathbb{E}\left[V_a\cdot\left\{(\mathbb{E}\pi_1' - (1+r)) - (1-\sigma_1+\sigma_1\tau)(\mathbb{E}\pi_1' - \pi_1')\right\}\right]}_{\text{Marginal Expected Value of Investment through Firm 1}} \\
&\quad + \underbrace{\beta\mathbb{E}\left[V_a\cdot(1-\tau)(1-\sigma_1+\sigma_1\tau)\left\{\mathbb{E}\pi_2 - \pi_2\right\}\right]}_{\substack{=\beta\mathbb{E}\left[V_a\cdot\frac{da'}{d\sigma_2}\Big|_{dk_1^*=dk_2=0}\right] \\ \text{Marginal Value of Risk Sharing Through Firm 2 Given Capital } (k_1^*,k_2)}} \\
&\geq 0.
\end{aligned}
\tag{75}
$$

Proof. From the Kuhn-Tucker condition for λ_ϕ,

$$\lambda_s = \frac{1}{a}\left\{\lambda_\phi + |J|\,\lambda_{\nu_1}\right\} \text{ where } |J| = \left|\frac{d\nu_1}{d\phi}\right|_{dk_1^*=0} > 0.$$

Given the assumption that firms are allowed to invest in a risk-free asset, the external debt finance of Firm 1 is only bounded above such that $\lambda_{\nu_1} \geq 0$. If $\lambda_{\nu_1} > 0$, $\lambda_s > 0$ and the optimal private saving is bounded below such that $s = 0$. If $\lambda_{\nu_1} = 0$, $\mathbb{1}_{\lambda_s} = \mathbb{1}_{\lambda_\phi}$ and the optimal private saving and the optimal private finance are undetermined unless they are binding together. Thus, the zero private saving is weakly preferred and the optimization can be achieved with $s = 0$.

From the Kuhn-Tucker condition for λ_{ν_2},

$$\lambda_{\nu_1} = C \cdot \lambda_{k_2^C} + D \cdot \lambda_{\nu_2}, \quad C, D > 0 \text{ given } \tau > 0.$$

Since firms are allowed to invest in a risk-free asset, the external debt finance of Firm 2 is only bounded above such that $\lambda_{\nu_2} \geq 0$. If $\lambda_{\nu_2} > 0$, $\lambda_{\nu_1} > 0$ and the optimal external debt finance of Firm 1 is bounded above such that $\nu_1 = 1$. If $\lambda_{\nu_2} = 0$, $\mathbb{1}_{\lambda_{\nu_1}} = \mathbb{1}_{\lambda_{k_2^C}}$ and the optimal external debt finance of Firm 1 and the optimal internal equity finance are undetermined unless they are binding together. Thus, the full external debt finance of Firm 1 is weakly preferred and the optimization can be achieved with $\nu_1 = 1$.

Given Condition 2 and $\lambda_{\nu_1}, \lambda_{\nu_2} \geq 0$, the marginal values of external equity finance of Firm 1 and Firm 2 are always greater than zero such that,

$$\lambda_{\sigma_1} \geq \beta\mathbb{E}\left[V_a \cdot \left.\frac{da'}{d\sigma_1}\right|_{dk_1^*=0}\right] > 0,$$

$$\lambda_{\sigma_2} \geq \beta\mathbb{E}\left[V_a \cdot \left.\frac{da'}{d\sigma_2}\right|_{dk_1^*=dk_2=0}\right] > 0.$$

Thus, the optimal external equity finance is binding such that $(\sigma_1, \sigma_2) = (\bar{\sigma}, \bar{\sigma})$. $\square$

The intuition of Proposition 2 is similar to that of Proposition 1. Given the non-negative value of investment, the risk sharing motive makes an entrepreneur to sell both her shares of Firm 1 and Firm 1's shares of Firm 2 as many as possible. Thus, the constraints for the external equity finance of Firm 1 and Firm 2 are binding.

Moreover, the risk-free investment opportunity of firms makes an entrepreneur to take advantage of external debt finance of Firm 1 and carry it over into Firm 2. It is entrepreneur's relegated saving in the sense that the risk-free cash flow of Firm 2 is diverted out to the entrepreneur due to financial frictions. Note that financial frictions are required to link λ_{ν_1} and λ_{ν_2}. If $\tau = 0$, the Kuhn-Tucker conditions are collapsed into $\lambda_{\nu_2} = \lambda_{k_2^C} = 0$ regardless of λ_{ν_1} and the full external debt finance of Firm 1 is not guaranteed anymore.

The following Figure 36 shows that the borrowing constraint for Firm 1 is binding. The risk-free investment opportunity of Firm 2 keeps the marginal value of external debt finance of Firm 1 is greater than or equal to the marginal opportunity cost of internal equity finance such that $\lambda_{\nu_1} \geq C\lambda_{k_2^C} \geq 0$. Given $\lambda_{\nu_1} \geq C\lambda_{k_2^C}$, the indifference curve $V = \overline{V}(\nu_1, k_2^C)$ cuts from above the curve of constant marginal value of external debt finance of Firm 1, $\lambda_{\nu_1} = \overline{\lambda}_{\nu_1}(k_1^*, a')$, which is achieved by $dk_1^*(\nu_1, k_2^C, \nu_2(\nu_1, k_2^C), k_2(k_2^C, \nu_2)) = dk_2(k_2^C, \nu_2(\nu_1, k_2^C)) = da'(\nu_1, \nu_2(\nu_1, k_2^C), k_1^*, k_2) = 0$. Thus, the indifference curve is pushed up until the borrowing constraint of Firm 1 is binding such that $\nu_1 = 1$.

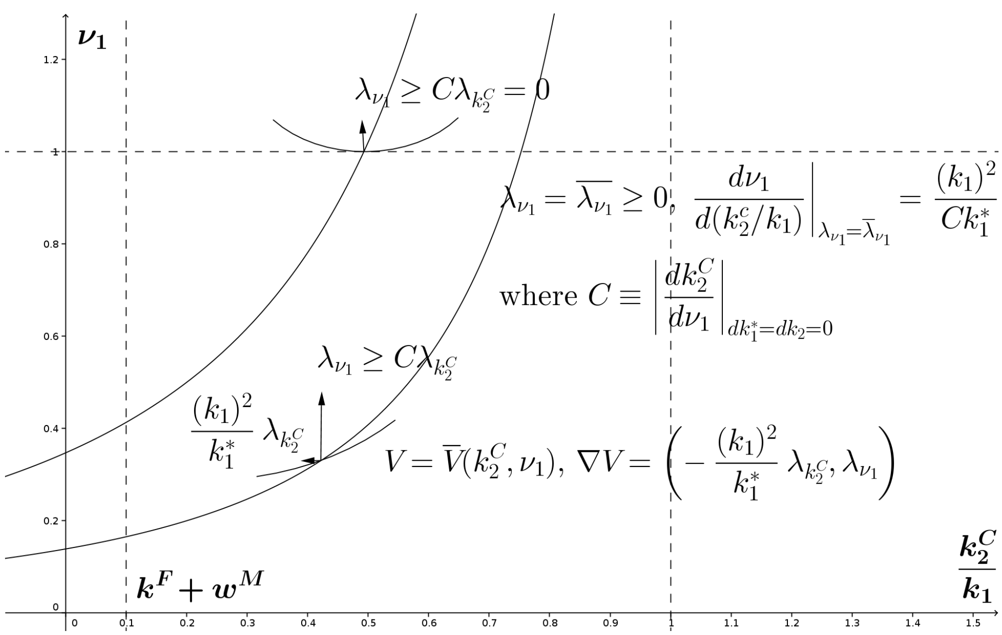

Figure 36: Non-negative Marginal Expected Value of Investment and Binding External Debt Finance of Firm 1

G Some Algebra

The following algebra is omitted in the above entrepreneur's problem for brevity.

A Stand-Alone Entrepreneur's Problem

From $\lambda_\nu \geq 0$,

$$\begin{aligned}
-(1+r)+AB &= A\left[-(1+r)A^{-1}+B\right] \\
&= A\left[-(1+r)+(1-\tau)\left\{\sigma\mathbb{E}\pi'+(1-\sigma)\nu\inf\pi'\right\}+(1-\sigma+\sigma\tau)\pi'-(1-\sigma)(1-\tau)\nu\inf\pi'\right] \\
&= A\left[\mathbb{E}\pi'-(1+r)-(1-\sigma+\sigma\tau)(\mathbb{E}\pi'-\pi')\right]
\end{aligned} \tag{76}$$

where

$$\begin{aligned}
A &\equiv \left[1-\frac{1-\tau}{1+r}\left\{\sigma\mathbb{E}\pi'+(1-\sigma)\nu\inf\pi'\right\}\right]^{-1} \\
B &\equiv \tau\pi'+(1-\sigma)(1-\tau)\left\{\pi'-\nu\inf\pi'\right\}.
\end{aligned} \tag{77}$$

From $\lambda_\sigma \geq 0$,

$$\begin{aligned}
A^{-1}dk(\nu,\sigma) &= \frac{1-\tau}{1+r}(1-\sigma)\inf\pi d\nu+\frac{1-\tau}{1+r}\left(\mathbb{E}\pi-\nu\inf\pi\right)d\sigma \\
\left.\frac{d\nu}{d\sigma}\right|_{dk(\nu,\sigma)=0} &= -\frac{\mathbb{E}\pi-\nu\inf\pi}{(1-\sigma)\inf\pi}
\end{aligned} \tag{78}$$

and

$$\begin{aligned}
\left.da'(\nu,\sigma)\right|_{dk=0} &= -(1-\sigma)(1-\tau)\inf\pi d\nu-(1-\tau)(\pi-\nu\inf\pi)d\sigma \\
\left.\frac{da'(\nu,\sigma)}{d\sigma}\right|_{dk=0} &= -(1-\sigma)(1-\tau)\inf\pi\cdot\left.\frac{d\nu}{d\sigma}\right|_{dk=0}-(1-\tau)\left(\pi-\nu\inf\pi\right) \\
&= (1-\tau)(\mathbb{E}\pi-\nu\inf\pi)-(1-\tau)(\pi-\nu\inf\pi) \\
&= (1-\tau)(\mathbb{E}\pi-\pi).
\end{aligned} \tag{79}$$

In the proof of Proposition 1,

$$\begin{aligned}
A^{-1}dk(\phi,\nu) &= ad\phi+\frac{1-\tau}{1+r}(1-\sigma)\inf\pi d\nu \\
|J| &= \left|\frac{d\nu}{d\phi}\right|_{dk(\phi,\nu)=0} \\
&= \frac{a}{\frac{1-\tau}{1+r}(1-\sigma)\inf\pi}.
\end{aligned} \tag{80}$$

The line of constant marginal opportunity cost of private saving, $\lambda_s=\lambda_s(c(s,\phi),a'(s,\nu(s,\phi),k(\phi,\nu))|\sigma)$, is derived by solving for the following system of equations

$$\begin{aligned} dc(s,\phi) &= -ds - ad\phi = 0 \\ A^{-1}dk(\phi,\nu) &= ad\phi + \frac{1-\tau}{1+r}(1-\sigma)\inf \pi d\nu = 0 \\ da'(s,\nu)\big|_{dk=0} &= (1+r)ds - (1-\sigma)(1-\tau)\inf \pi d\nu = 0 \end{aligned} \tag{81}$$

such that

$$ad\phi = -ds = -\frac{1-\tau}{1+r}(1-\sigma)\inf \pi d\nu. \tag{82}$$

Note that $da' = 0$ is redundant with $dc = dk = 0$.

A Business-Group Entrepreneur's Problem

From $\lambda_{\sigma_1} \geq 0$,

$$\begin{aligned} A_1^{-1}dk_1^*(\nu_1,\sigma_1)\big|_{dk_2=0} &= \left\{\frac{1-\tau}{1+r}(1-\sigma_1)\inf \pi_1 + \frac{(1-\tau)^2(1-\sigma_2)}{1+r}(1-\sigma_1)(1-\nu_2)\inf \pi_2\right\} d\nu_1 \\ &+ \left\{\frac{1-\tau}{1+r}(\mathbb{E}\pi_1 - \nu_1 \inf \pi_1) + \frac{(1-\tau)^2(1-\sigma_2)}{1+r}(\mathbb{E}\pi_2 - \nu_2\inf \pi_2) - \frac{(1-\tau)^2(1-\sigma_2)}{1+r}\nu_1(1-\nu_2)\inf \pi_2\right\} d\sigma_1 \\ da'(\nu_1,\sigma_1)\big|_{dk_1^*=dk_2=0} &= \left\{-(1-\sigma_1)(1-\tau)\inf \pi_1 - (1-\tau)^2(1-\sigma_1)(1-\sigma_2)(1-\nu_2)\inf \pi_2\right\} d\nu_1 \\ &+ \left\{-(1-\tau)(\pi_1 - \nu_1\inf \pi_1) + (1-\tau)^2(1-\sigma_2)\nu_1(1-\nu_2)\inf \pi_2 - (1-\tau)^2(1-\sigma_2)(\pi_2 - \nu_2 \inf \pi_2)\right\} d\sigma_1. \end{aligned} \tag{83}$$

Adding to the bottom equation the upper equation multiplied by $(1+r)$ with taking $dk_1^* = 0$,

$$\begin{aligned} da'\big|_{dk_1^*=dk_2=0} &= \left\{(1-\tau)(\mathbb{E}\pi_1 - \pi_1) + (1-\tau)^2(1-\sigma_2)(\mathbb{E}\pi_2 - \pi_2)\right\} d\sigma_1 \\ \frac{da'}{d\sigma_1}\bigg|_{dk_1^*=dk_2=0} &= \left\{(1-\tau)(\mathbb{E}\pi_1 - \pi_1) + (1-\tau)^2(1-\sigma_2)(\mathbb{E}\pi_2 - \pi_2)\right\}. \end{aligned} \tag{84}$$

From λ_{ν_2},

$$\begin{aligned} A_2^{-1}dk_2(k_2^C,\nu_2) &= dk_2^C + \frac{1-\tau}{1+r}(1-\sigma_2)\inf \pi_2 d\nu_2 \\ \frac{dk_2^C}{d\nu_2}\bigg|_{dk_2(k_2^C,\nu_2)=0} &= -\frac{1-\tau}{1+r}(1-\sigma_2)\inf \pi_2 \end{aligned} \tag{85}$$

and

$$
\begin{aligned}
A_1^{-1} dk_1^*(k_2^C,\nu_2)\Big|_{dk_2=0} &= -dk_2^C + \frac{(1-\tau)^2(1-\sigma_2)}{1+r} \inf \pi_2 \left\{-\sigma_1 - (1-\sigma_1)\nu_1\right\} d\nu_2 \\
\frac{dk_1^*(k_2^C,\nu_2)}{d\nu_2}\Bigg|_{dk_2=0} &= -A_1 \left.\frac{dk_2^C}{d\nu_2}\right|_{dk_2=0} - \frac{(1-\tau)^2(1-\sigma_2)}{1+r} \inf \pi_2 \left\{\sigma_1 + (1-\sigma_1)\nu_1\right\} A_1 \\
&= \frac{1-\tau}{1+r}(1-\sigma_2) \inf \pi_2 \left\{\tau + (1-\tau)(1-\sigma_1)(1-\nu_1)\right\} A_1.
\end{aligned} \tag{86}
$$

From λ_{σ_2},

$$
\begin{aligned}
A_1^{-1} dk_1^*(\nu_2,\sigma_2)\Big|_{dk_2=0} &= -\frac{(1-\tau)^2(1-\sigma_2)}{1+r} \left\{\sigma_1 + (1-\sigma_1)\nu_1\right\} \inf \pi_2 d\nu_2 \\
&\quad - \frac{(1-\tau)^2}{1+r} \left\{\sigma_1(\mathbb{E}\pi_2 - \nu_2 \inf \pi_2) + (1-\sigma_1)\nu_1(1-\nu_2)\inf \pi_2\right\} d\sigma_2 \\
A_2^{-1} dk_2(\nu_2,\sigma_2) &= \frac{1-\tau}{1+r}(1-\sigma_2)\inf \pi_2 d\nu_2 + \frac{1-\tau}{1+r}(\mathbb{E}\pi_2 - \nu_2 \inf \pi_2) d\sigma_2.
\end{aligned} \tag{87}
$$

Adding to the upper equation the bottom equation multiplied by $(1-\tau)\left\{\sigma_1 + (1-\sigma_1)\nu_1\right\}$ with taking $dk_2 = 0$,

$$
\begin{aligned}
A_1^{-1} dk_1^*(\nu_2,\sigma_2)\Big|_{dk_2=0} &= \frac{(1-\tau)^2}{1+r} \left[(\mathbb{E}\pi_2 - \nu_2 \inf \pi_2)\left\{-\sigma_1 + \sigma_1 + (1-\sigma_1)\nu_1\right\} - (1-\sigma_1)\nu_1(1-\nu_2)\inf \pi_2\right] d\sigma_2 \\
&= \frac{(1-\tau)^2(1-\sigma_1)\nu_1}{1+r}(\mathbb{E}\pi_2 - \inf \pi_2) d\sigma_2 \\
\frac{dk_1^*(\nu_2,\sigma_2)}{d\sigma_2}\Bigg|_{dk_2=0} &= \frac{(1-\tau)^2(1-\sigma_1)\nu_1}{1+r}(\mathbb{E}\pi_2 - \inf \pi_2) A_1.
\end{aligned} \tag{88}
$$

By adding up the following two equations with taking $dk_1^* = dk_2 = 0$,

$$
\begin{aligned}
dk_1^*(k_2^C,\nu_2,\sigma_2) &= -dk_2^C - \frac{(1-\tau)^2}{1+r} \left\{\sigma_1(\mathbb{E}\pi_2 - \nu_2 \inf \pi_2) + (1-\sigma_1)\nu_1(1-\nu_2)\inf \pi_2\right\} d\sigma_2 \\
&\quad + \frac{(1-\tau)^2(1-\sigma_2)}{1+r} \left\{-\sigma_1 \inf \pi_2 - (1-\sigma_1)\nu_1 \inf \pi_2\right\} d\nu_2 \\
dk_2(k_2^C,\nu_2,\sigma_2) &= dk_2^C + \frac{1-\tau}{1+r}(\mathbb{E}\pi_2 - \nu_2 \inf \pi_2) d\sigma_2 + \frac{1-\tau}{1+r}(1-\sigma_2)\inf \pi_2 d\nu_2,
\end{aligned} \tag{89}
$$

we can derive

$$
\left.\frac{d\nu_2}{d\sigma_2}\right|_{dk_1^*=dk_2=0} = \frac{-(1-\sigma_1+\sigma_1\tau)(\mathbb{E}\pi_2 - \nu_2 \inf \pi_2) + (1-\tau)(1-\sigma_1)\nu_1(1-\nu_2)\inf \pi_2}{(1-\sigma_2)\inf \pi_2 \left\{\tau + (1-\tau)(1-\sigma_1)(1-\nu_1)\right\}}. \tag{90}
$$

Then, by substituting for $\left.\frac{d\nu_2}{d\sigma_2}\right|_{dk_1^*=dk_2=0}$,

$$\begin{aligned}
\left.\frac{da'(\nu_2,\sigma_2)}{d\sigma_2}\right|_{dk_1^*=dk_2=0} &= \Big\{(1-\tau)^2(1-\sigma_1)\nu_1 - (1-\sigma_1+\sigma_1\tau)(1-\tau)\Big\}(1-\sigma_2)\inf\pi_2 \cdot \left.\frac{d\nu_2}{d\sigma_2}\right|_{dk_1^*=dk_2=0} \\
&\quad + (1-\tau)^2(1-\sigma_1)\nu_1(1-\nu_2)\inf\pi_2 - (1-\sigma_1+\sigma_1\tau)(1-\tau)(\pi_2-\nu_2\inf\pi_2) \\
&= -(1-\tau)^2(1-\sigma_1)\nu_1(1-\nu_2)\inf\pi_2 + (1-\sigma_1+\sigma_1\tau)(1-\tau)(\mathbb{E}\pi_2-\nu_2\inf\pi_2) \\
&\quad + (1-\tau)^2(1-\sigma_1)\nu_1(1-\nu_2)\inf\pi_2 - (1-\sigma_1+\sigma_1\tau)(1-\tau)(\pi_2-\nu_2\inf\pi_2) \\
&= (1-\sigma_1+\sigma_1\tau)(1-\tau)(\mathbb{E}\pi_2-\pi_2).
\end{aligned} \tag{91}$$

Lastly, the curve of constant marginal value of external debt finance of Firm 1,

$$\lambda_{\nu_1} = \overline{\lambda}_{\nu_1}(k_1^*(\nu_1,k_2^C,\nu_2(\nu_1,k_2^C),k_2(k_2^C,\nu_2)), a'(\nu_1,\nu_2(\nu_1,k_2^C),k_1^*,k_2(k_2^C,\nu_2))),$$

is derived by solving for the following system of equations with taking $dk_1^*=dk_2=da'=0$

$$\begin{aligned}
A_1^{-1}dk_1^*(\nu_1,k_2^C,\nu_2) &= -dk_2^C + \frac{1-\tau}{1+r}(1-\sigma_1)\left\{\inf\pi_1 + (1-\sigma_1)(1-\tau)(1-\nu_2)\inf\pi_2\right\}d\nu_1 \\
&\quad + \frac{(1-\tau)^2(1-\sigma_2)}{1+r}\inf\pi_2\left\{-\sigma_1-(1-\sigma_1)\nu_1\right\}d\nu_2 \\
A_2^{-1}dk_2(k_2^C,\nu_2) &= dk_2^C + \frac{1-\tau}{1+r}(1-\sigma_2)\inf\pi_2 d\nu_2 \\
da'(\nu_1,\nu_2) &= -(1-\sigma_1)(1-\tau)\left\{\inf\pi_1 + (1-\sigma_2)(1-\nu_2)(1-\tau)\inf\pi_2\right\}d\nu_1 \\
&\quad - (1-\sigma_2)(1-\tau)\inf\pi_2\left\{(1-\sigma_1+\sigma_1\tau)-(1-\sigma_1)(1-\tau)\nu_1\right\}d\nu_2
\end{aligned} \tag{92}$$

such that

$$\begin{aligned}
dk_2^C &= \frac{\frac{1-\tau}{1+r}(1-\sigma_1)\left\{\inf\pi_1 + (1-\sigma_2)(1-\nu_2)(1-\tau)\inf\pi_2\right\}}{\tau+(1-\sigma_1)(1-\nu_1)(1-\tau)}d\nu_1 \\
&= -\frac{1-\tau}{1+r}(1-\sigma_2)\inf\pi_2 d\nu_2.
\end{aligned} \tag{93}$$

Note that $da'=0$ is redundant with $dk_1^*=dk_2=0$.